1 A competitor in a throwing event has three attempts to throw a ball as far as possible. The random variable X denotes the number of throws that exceed 30 metres. The probability distribution table for X is shown below.

x	0	1	2	3
$P(X = x)$	0.4	p	r	0.15

(a) Given that $E(X) = 1.1$, find the value of p and the value of r. [3]

$\sum P = 1: \quad 0.4 + p + r + 0.15 = 1$

$0.55 + p + r = 1$

$p + r = 0.45$ ①

$E(X) = 1.1:$

$0 \times 0.4 + 1 \times p + 2 \times r + 3 \times 0.15 = 1.1$

$0 + p + 2r + 0.45 = 1.1$

$p + 2r = 0.65$ ②

② − ①: $r = 0.2$

→ ①: $p + 0.2 = 0.45$

$p = 0.25$

(b) Find the numerical value of $\mathrm{Var}(X)$. [2]

$\mathrm{Var}(X) = 0^2 \times 0.4 + 1^2 \times 0.25 + 2^2 \times 0.2 + 3^2 \times 0.15 - (E(x))^2$

$= 0 + 0.25 + 0.8 + 1.35 - 1.1^2$

$= 2.4 - 1.21$

$= 1.19$

1 Becky sometimes works in an office and sometimes works at home. The random variable X denotes the number of days that she works at home in any given week. It is given that

$$P(X = x) = kx(x + 1),$$

where k is a constant and $x = 1, 2, 3$ or 4 only.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

$x = 1: k \times 1 \times (1+1) = k \times 2 = 2k$

$x = 2: k \times 2 \times (2+1) = 2k \times 3 = 6k$

$x = 3: k \times 3 \times (3+1) = 3k \times 4 = 12k$

$x = 4: k \times 4 \times (4+1) = 4k \times 5 = 20k$

$\Sigma P = 1 : 2k + 6k + 12k + 20k = 1$

$40k = 1$

$k = 0.025$

x	1	2	3	4
$P(X = x)$	0.05	0.15	0.3	0.5

(b) Find $E(X)$ and $Var(X)$. [3]

$E(X) = 1 \times 0.05 + 2 \times 0.15 + 3 \times 0.3 + 4 \times 0.5$

$= 0.05 + 0.3 + 0.9 + 2$

$= \underline{3.25}$ (note: mistake in mark-scheme: 3.05 is incorrect)

$Var(X) = 1^2 \times 0.05 + 2^2 \times 0.15 + 3^2 \times 0.3 + 4^2 \times 0.5 - (E(x))^2$

$= 0.05 + 0.6 + 2.7 + 8 - (3.25)^2$

$= 11.35 - 10.5625$

$= \underline{0.7875}$

2 The random variable X can take only the values $-2, -1, 0, 1, 2$. The probability distribution of X is given in the following table.

x	-2	-1	0	1	2
$P(X = x)$	p	p	0.1	q	q

Given that $P(X \geqslant 0) = 3P(X < 0)$, find the values of p and q. [4]

$\Sigma P = 1$: $p + p + 0.1 + q + q = 1$

$2p + 2q = 0.9$ ①

$P(X \geqslant 0) = 3P(X < 0)$:

$0.1 + q + q = 3(p + p)$

$0.1 + 2q = 3(2p)$

$0.1 + 2q = 6p$

$6p - 2q = 0.1$ ②

① + ②: $8p = 1$

$p = \frac{1}{8}$

→ ①: $2\left(\frac{1}{8}\right) + 2q = 0.9$

$0.25 + 2q = 0.9$

$2q = \frac{13}{20}$

$q = \frac{13}{40}$

3 The random variable X takes the values -2, 1, 2, 3. It is given that $P(X = x) = kx^2$, where k is a constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

$x = -2: \quad k(-2)^2 = 4k$

$x = 1: \quad k(1)^2 = k$

$x = 2: \quad k(2)^2 = 4k$

$x = 3: \quad k(3)^2 = 9k$

$\Sigma P = 1: \quad 4k + k + 4k + 9k = 1$

$18k = 1$

$k = \frac{1}{18}$

x	-2	1	2	3
$P(X=x)$	$\frac{4}{18}$	$\frac{1}{18}$	$\frac{4}{18}$	$\frac{9}{18}$

(b) Find $E(X)$ and $Var(X)$. [3]

$E(X) = -2 \times \frac{4}{18} + 1 \times \frac{1}{18} + 2 \times \frac{4}{18} + 3 \times \frac{9}{18}$

$= \frac{-8}{18} + \frac{1}{18} + \frac{8}{18} + \frac{27}{18}$

$= \frac{28}{18} = \underline{\frac{14}{9}}$

$Var(X) = (-2)^2 \times \frac{4}{18} + 1^2 \times \frac{1}{18} + 2^2 \times \frac{4}{18} + 3^2 \times \frac{9}{18} - (E(x))^2$

$= \frac{16}{18} + \frac{1}{18} + \frac{16}{18} + \frac{81}{18} - \left(\frac{14}{9}\right)^2$

$= \frac{114}{18} - \frac{196}{81} = \underline{\frac{317}{81}}$

4 A fair spinner has sides numbered 1, 2, 2. Another fair spinner has sides numbered −2, 0, 1. Each spinner is spun. The number on the side on which a spinner comes to rest is noted. The random variable X is the sum of the numbers for the two spinners.

(a) Draw up the probability distribution table for X. [3]

	1	2	2
−2	−1	0	0
0	1	2	2
1	2	3	3

x	−1	0	1	2	3
$P(X=x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{3}{9}$	$\frac{2}{9}$

(b) Find E(X) and Var(X). [3]

$E(X) = -1\times\frac{1}{9} + 0\times\frac{2}{9} + 1\times\frac{1}{9} + 2\times\frac{3}{9} + 3\times\frac{2}{9}$

$= \frac{-1}{9} + 0 + \frac{1}{9} + \frac{6}{9} + \frac{6}{9}$

$= \frac{4}{3}$

$Var(X) = (-1)^2\times\frac{1}{9} + 0^2\times\frac{2}{9} + 1^2\times\frac{1}{9} + 2^2\times\frac{3}{9} + 3^2\times\frac{2}{9} - \left(\frac{4}{3}\right)^2$

$= \frac{1}{9} + 0 + \frac{1}{9} + \frac{12}{9} + \frac{18}{9} - \frac{16}{9}$

$= \frac{16}{9}$

5 A fair three-sided spinner has sides numbered 1, 2, 3. A fair five-sided spinner has sides numbered 1, 1, 2, 2, 3. Both spinners are spun once. For each spinner, the number on the side on which it lands is noted. The random variable X is the larger of the two numbers if they are different, and their common value if they are the same.

(a) Show that $P(X = 3) = \frac{7}{15}$. [2]

	1	2	3
1	1	2	3
1	1	2	3
2	2	2	3
2	2	2	3
3	3	3	3

$P(X=3) = \frac{7}{15}$

(b) Draw up the probability distribution table for X. [3]

x	1	2	3
$P(X=x)$	$2/15$	$\frac{6}{15}$	$\frac{7}{15}$

(c) Find E(X) and Var(X). [3]

$E(X) = 1 \times \frac{2}{15} + 2 \times \frac{6}{15} + 3 \times \frac{7}{15}$

$= \frac{2}{15} + \frac{12}{15} + \frac{21}{15}$

$= \frac{35}{15} = \frac{7}{3}$

$Var(X) = 1^2 \times \frac{2}{15} + 2^2 \times \frac{6}{15} + 3^2 \times \frac{7}{15} - (E(x))^2$

$= \frac{2}{15} + \frac{24}{15} + \frac{63}{15} - \left(\frac{7}{3}\right)^2$

$= \frac{89}{15} - \frac{49}{9}$

$= \frac{22}{45}$

1 A fair red spinner has edges numbered 1, 2, 2, 3. A fair blue spinner has edges numbered −3, −2, −1, −1. Each spinner is spun once and the number on the edge on which each spinner lands is noted. The random variable X denotes the sum of the resulting two numbers.

(a) Draw up the probability distribution table for X. [3]

	1	2	2	3
−3	−2	−1	−1	0
−2	−1	0	0	1
−1	0	1	1	2
−1	0	1	1	2

x	−2	−1	0	1	2
$P(X=x)$	$\frac{1}{16}$	$3/16$	$5/16$	$5/16$	$2/16$

(b) Given that $E(X) = 0.25$, find the value of $Var(X)$. [2]

$$Var(X) = (-2)^2 \times \frac{1}{16} + (-1)^2 \times \frac{3}{16} + 0^2 \times \frac{5}{16} + 1^2 \times \frac{5}{16} + 2^2 \times \frac{2}{16} - (E(X))^2$$

$$= \frac{4}{16} + \frac{3}{16} + 0 + \frac{5}{16} + \frac{8}{16} - (0.25)^2$$

$$= \frac{20}{16} - \frac{1}{16}$$

$$= \frac{19}{16}$$

4 The random variable X takes each of the values 1, 2, 3, 4 with probability $\frac{1}{4}$. Two independent values of X are chosen at random. If the two values of X are the same, the random variable Y takes that value. Otherwise, the value of Y is the larger value of X minus the smaller value of X.

(a) Draw up the probability distribution table for Y. [4]

	1	2	3	4
1	1	1	2	3
2	1	2	1	2
3	2	1	3	1
4	3	2	1	4

y	1	2	3	4
$P(Y=y)$	$7/16$	$5/16$	$\frac{3}{16}$	$1/16$

(b) Find the probability that $Y = 2$ given that Y is even. [2]

$P(2 \cap E) = P(2|E) \times P(E)$

$P(2|E) = \frac{P(2 \cap E)}{P(E)}$

$P(2 \cap E) = \frac{5}{16}$

$P(E) = \frac{5}{16} + \frac{1}{16}$

$= \frac{6}{16}$

$P(2|E) = \frac{\frac{5}{16}}{6/16}$

$= \frac{5}{6}$

2 Alisha has four coins. One of these coins is biased so that the probability of obtaining a head is 0.6. The other three coins are fair. Alisha throws the four coins at the same time. The random variable X denotes the number of heads obtained.

(a) Show that the probability of obtaining exactly one head is 0.225. [3]

Coins — biased — Probability

A	B	C	D
H	T	T	T
T	H	T	T
T	T	H	T
T	T	T	H

$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times 0.4 \times 3 = 0.15$

$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times 0.6 = 0.075$

P(one head) $= 0.15 + 0.075$

$= \underline{0.225}$ QED

(b) Complete the following probability distribution table for X. [2]

x	0	1	2	3	4
$P(X = x)$	0.05	0.225	0.375	0.275	0.075

Probability of 3 heads:

A	B	C	D[b]	Probability
H	H	H	T	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times 0.4 = 0.05$
H	H	T	H	
H	T	H	H	$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times 0.6 \times 3 = 0.225$
T	H	H	H	

P(three heads) $= 0.05 + 0.225$

$= \underline{0.275}$

P(two heads) $= 1 - 0.05 - 0.225 - 0.275 - 0.075$

$= \underline{0.375}$

(c) Given that $E(X) = 2.1$, find the value of $\mathrm{Var}(X)$. [2]

$\mathrm{Var}(X) = 0^2 \times 0.05 + 1^2 \times 0.225 + 2^2 \times 0.375 + 3^2 \times 0.275$
$+ 4^2 \times 0.075 - (E(X))^2$

$= 0 + 0.225 + 1.5 + 2.475 + 1.2 - 2.1^2$

$= 5.4 - 4.41$

$= \underline{0.99}$

2 A bag contains 5 red balls and 3 blue balls. Sadie takes 3 balls at random from the bag, without replacement. The random variable X represents the number of red balls that she takes.

(a) Show that the probability that Sadie takes exactly 1 red ball is $\frac{15}{56}$. [2]

pick one red from 5 → $\dfrac{{}^5C_1 \times {}^3C_2}{{}^8C_3}$ ← pick two blues from 3

↖ total number of possibilities

$= \dfrac{15}{56}$ QED

(b) Draw up the probability distribution table for X. [3]

0 Reds: $\dfrac{{}^5C_0 \times {}^3C_3}{{}^8C_3} = \dfrac{1}{56}$

1 Red: $\dfrac{15}{56}$ (part (a))

2 Reds: $\dfrac{{}^5C_2 \times {}^3C_1}{{}^8C_3} = \dfrac{15}{28} \left(= \dfrac{30}{56}\right)$

3 Reds: $\dfrac{{}^5C_3 \times {}^3C_0}{{}^8C_3} = \dfrac{5}{28} \left(= \dfrac{10}{56}\right)$

x	0	1	2	3
$P(X=x)$	$1/56$	$15/56$	$30/56$	$10/56$

(c) Given that $E(X) = \frac{15}{8}$, find $Var(X)$. [2]

$Var(X) = 0^2 \times \frac{1}{56} + 1^2 \times \frac{15}{56} + 2^2 \times \frac{30}{56} + 3^2 \times \frac{10}{56} - (E(X))^2$

$= 0 + \frac{15}{56} + \frac{120}{56} + \frac{90}{56} - \left(\frac{15}{8}\right)^2$

$= \frac{225}{56} - \frac{225}{64}$

$= \frac{225}{448}$

6 Three coins A, B and C are each thrown once.

- Coins A and B are each biased so that the probability of obtaining a head is $\frac{2}{3}$.
- Coin C is biased so that the probability of obtaining a head is $\frac{4}{5}$.

(a) Show that the probability of obtaining exactly 2 heads and 1 tail is $\frac{4}{9}$. [3]

A	B	C	
H	H	T	$\frac{2}{3} \times \frac{2}{3} \times \frac{1}{5} = \frac{4}{45}$
H	T	H	$\frac{2}{3} \times \frac{1}{3} \times \frac{4}{5} = \frac{8}{45}$
T	H	H	$\frac{1}{3} \times \frac{2}{3} \times \frac{4}{5} = \frac{8}{45}$

$$\frac{4}{45} + \frac{8}{45} + \frac{8}{45} = \frac{20}{45}$$

$$= \frac{4}{9} \quad \text{QED}$$

The random variable X is the number of heads obtained when the three coins are thrown.

(b) Draw up the probability distribution table for X. [3]

0H: T T T $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{5} = \frac{1}{45}$

1H: A B C

$$\left.\begin{array}{llll} H & T & T & \frac{2}{3} \times \frac{1}{3} \times \frac{1}{5} = \frac{2}{45} \\ T & H & T & \frac{1}{3} \times \frac{2}{3} \times \frac{1}{5} = \frac{2}{45} \\ T & T & H & \frac{1}{3} \times \frac{1}{3} \times \frac{4}{5} = \frac{4}{45} \end{array}\right\} \frac{8}{45}$$

2H: $\frac{4}{9}$ (part (a)) $\left(= \frac{20}{45}\right)$

3H: H H H $\frac{2}{3} \times \frac{2}{3} \times \frac{4}{5} = \frac{16}{45}$

x	0	1	2	3
$P(X=x)$	$\frac{1}{45}$	$\frac{8}{45}$	$\frac{20}{45}$	$\frac{16}{45}$

(c) Given that $E(X) = \frac{32}{15}$, find $\mathrm{Var}(X)$. [2]

$$\mathrm{Var}(X) = 0^2 \times \frac{1}{45} + 1^2 \times \frac{8}{45} + 2^2 \times \frac{20}{45} + 3^2 \times \frac{16}{45} - (E(X))^2$$

$$= 0 + \frac{8}{45} + \frac{80}{45} + \frac{144}{45} - \left(\frac{32}{15}\right)^2$$

$$= \frac{232}{45} - \frac{1024}{225}$$

$$= \frac{136}{225}$$