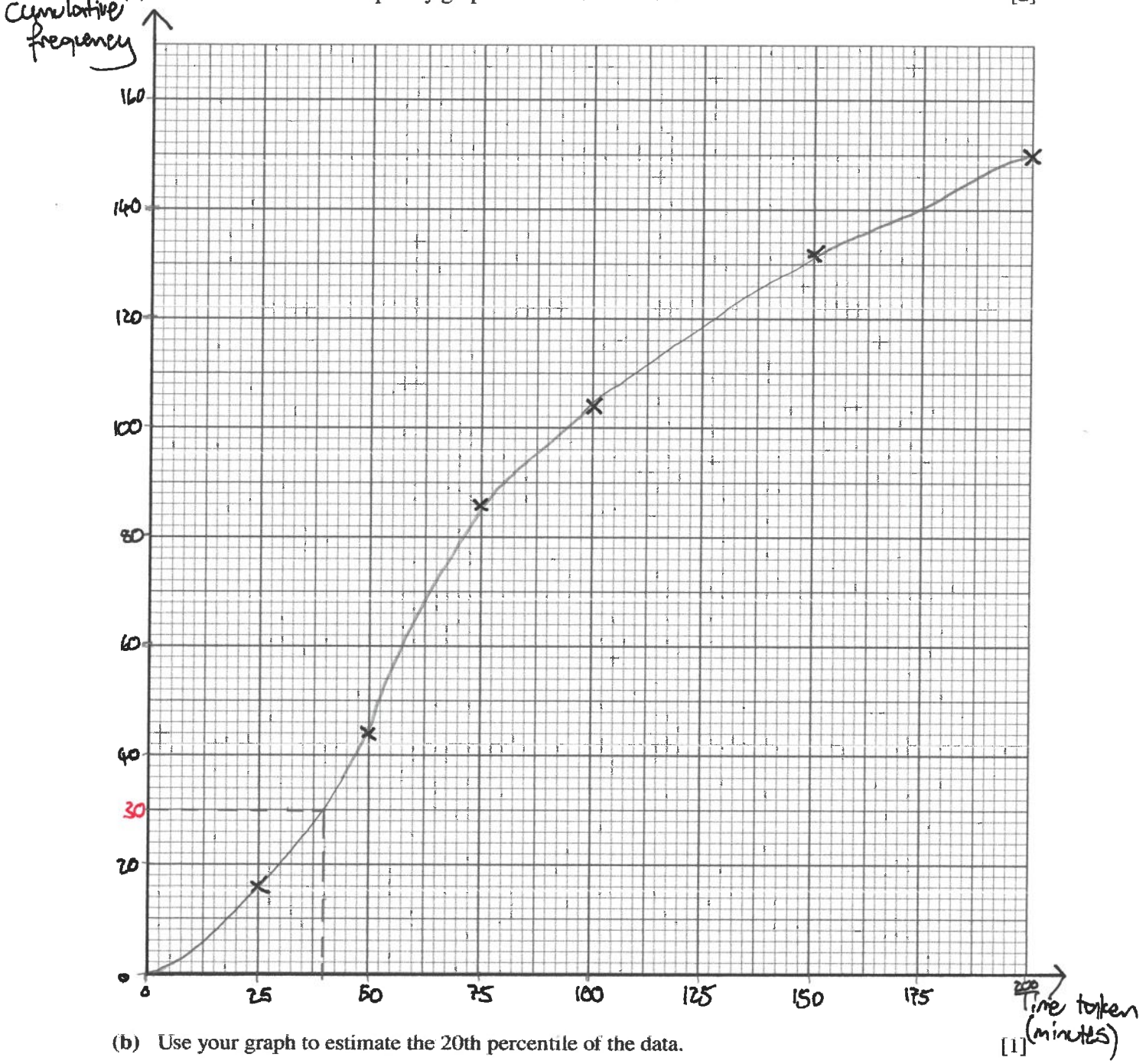
1 The time taken, t minutes, to complete a puzzle was recorded for each of 150 students. These times are summarised in the table.

Time taken (t minutes)	$t \leqslant 25$	$t \leqslant 50$	$t \leqslant 75$	$t \leqslant 100$	$t \leqslant 150$	$t \leqslant 200$
Cumulative frequency	16	44	86	104	132	150

(a) Draw a cumulative frequency graph to illustrate the data. [2]

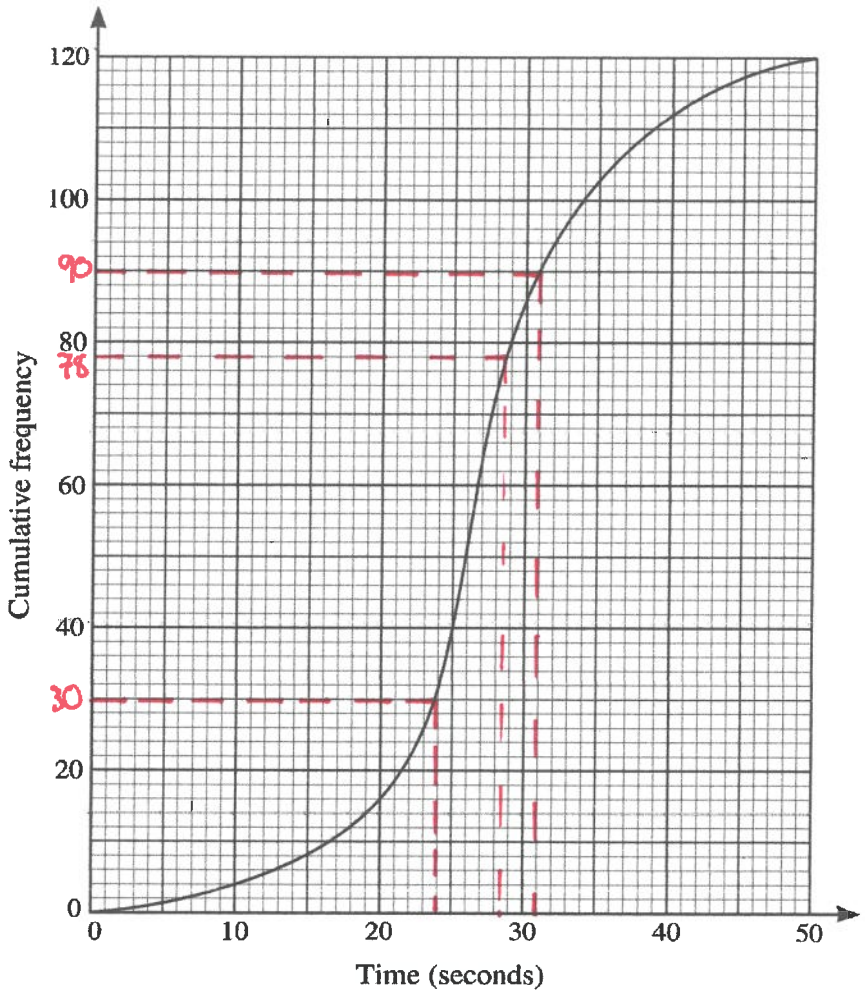


(b) Use your graph to estimate the 20th percentile of the data. [1]

$0.2 \times 150 = 30$

20th percentile = 40

1



The times taken by 120 children to complete a particular puzzle are represented in the cumulative frequency graph.

(a) Use the graph to estimate the interquartile range of the data. [2]

Q_1: $0.25 \times 120 = 30$ Q_3: $0.75 \times 120 = 90$

$Q_1 = 24$ $Q_3 = 31$

$IQR = 31 - 24 = 7$

35% of the children took longer than T seconds to complete the puzzle.

(b) Use the graph to estimate the value of T. [2]

35% took longer than T, so 65% took less than T.

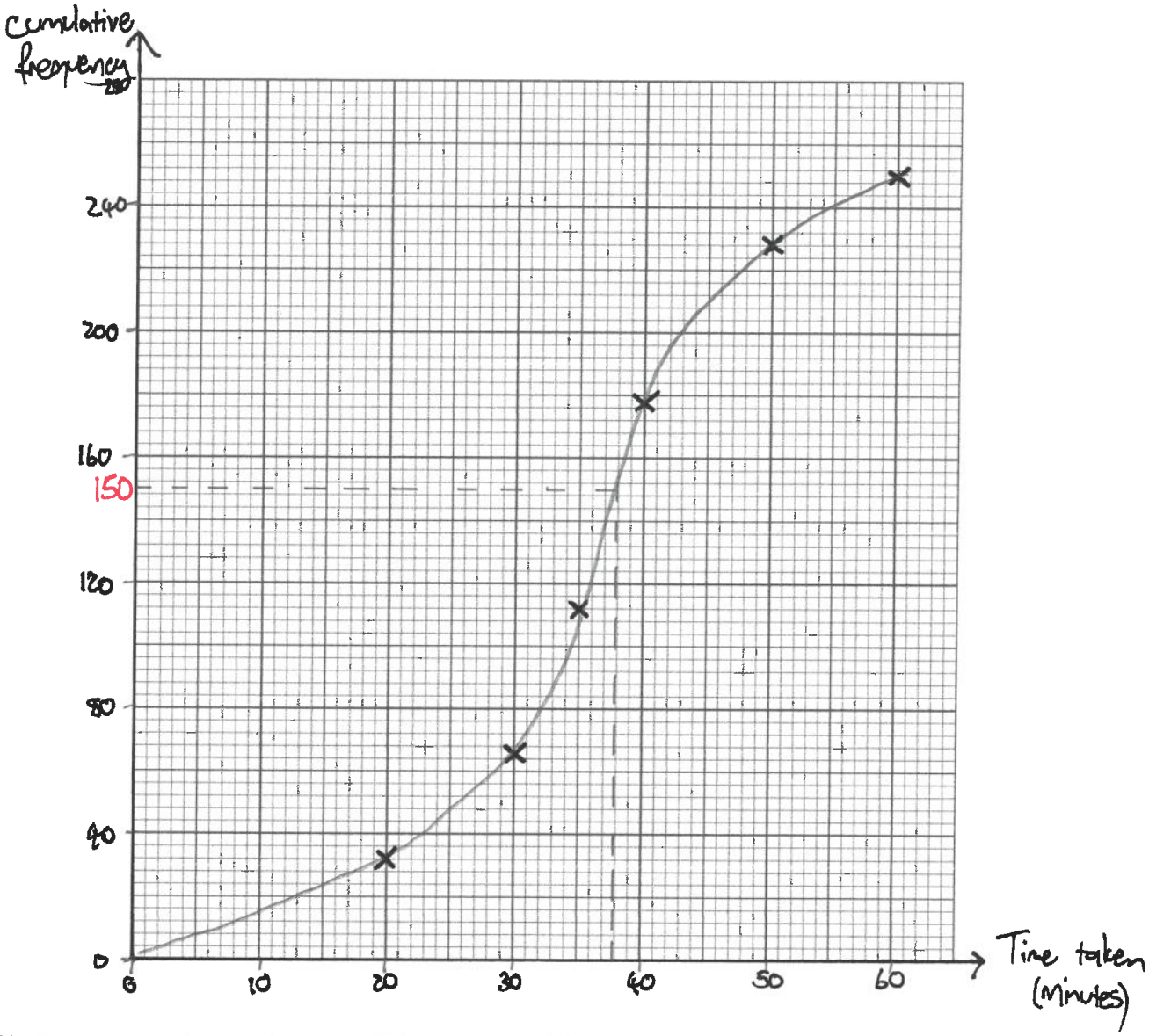
$0.65 \times 120 = 78$

$T \approx 28.5$s

3 The times, t minutes, taken to complete a walking challenge by 250 members of a club are summarised in the table.

Time taken (t minutes)	$t \leqslant 20$	$t \leqslant 30$	$t \leqslant 35$	$t \leqslant 40$	$t \leqslant 50$	$t \leqslant 60$
Cumulative frequency	32	66	112	178	228	250

(a) Draw a cumulative frequency graph to illustrate the data. [2]



(b) Use your graph to estimate the 60th percentile of the data. [1]

0·6 × 250 = 150

60th percentile ≈ 38 mins

It is given that an estimate for the mean time taken to complete the challenge by these 250 members is 34.4 minutes.

(c) Calculate an estimate for the standard deviation of the times taken to complete the challenge by these 250 members. [4]

Mid-point (t)	Frequency (f)
10	32
25	34
32.5	46
37.5	66
45	50
55	22
	$\Sigma f = 250$

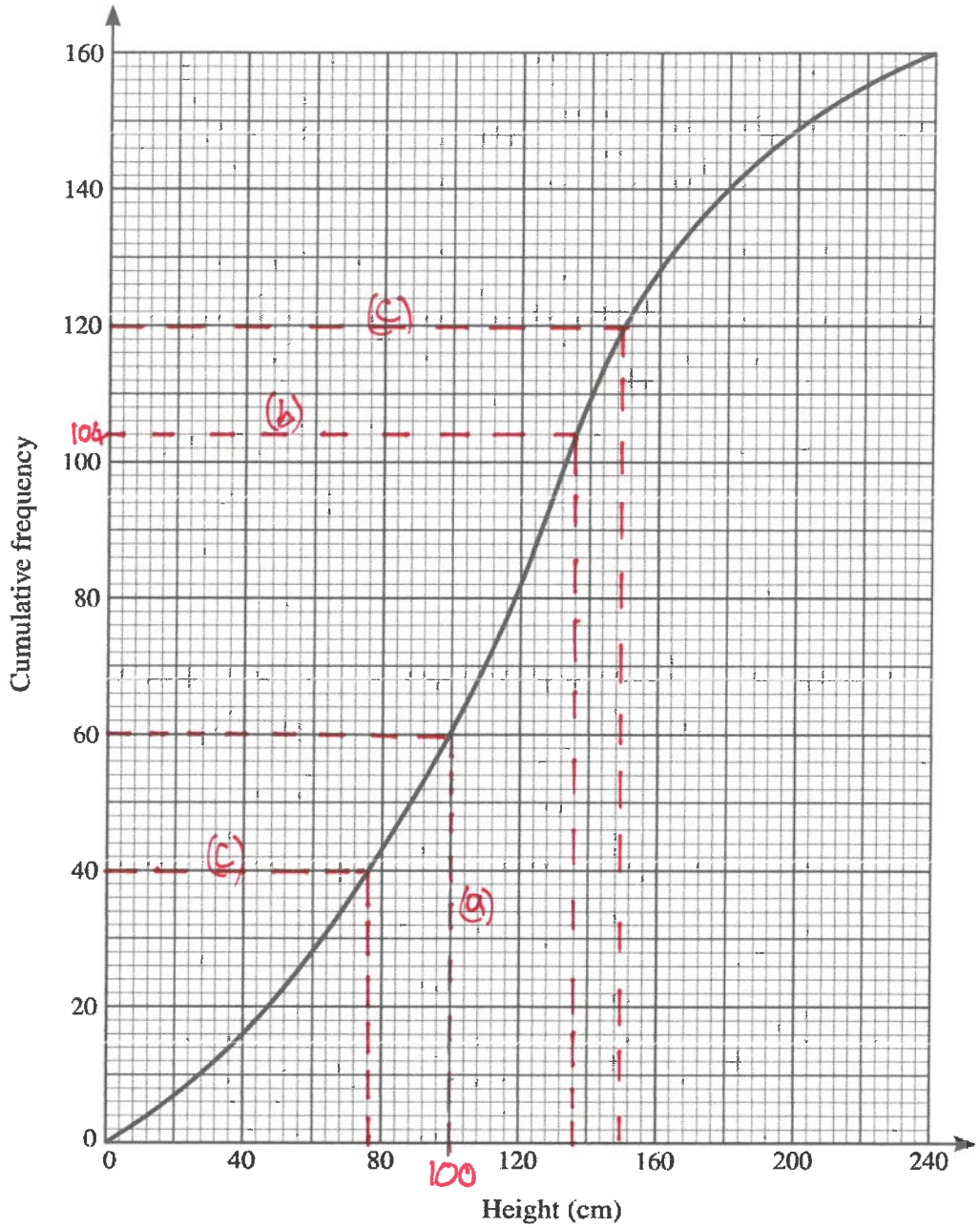
$$\text{Var} = \frac{10^2 \times 32 + 25^2 \times 34 + 32.5^2 \times 46 + 37.5^2 \times 66 + 45^2 \times 50 + 55^2 \times 22}{250} - 34.4^2$$

$$= 151.24$$

$$\sigma = \sqrt{151.24}$$

$$= 12.3$$

1 The heights in cm of 160 sunflower plants were measured. The results are summarised on the following cumulative frequency curve.



(a) Use the graph to estimate the number of plants with heights less than 100 cm. [1]

60

(b) Use the graph to estimate the 65th percentile of the distribution. [2]

$0.65 \times 160 = 104$

65th percentile = 136 cm

(c) Use the graph to estimate the interquartile range of the heights of these plants. [2]

Q_1: $0.25 \times 160 = 40$

$Q_1 = 76$

Q_3: $0.75 \times 160 = 120$

$Q_3 = 150$

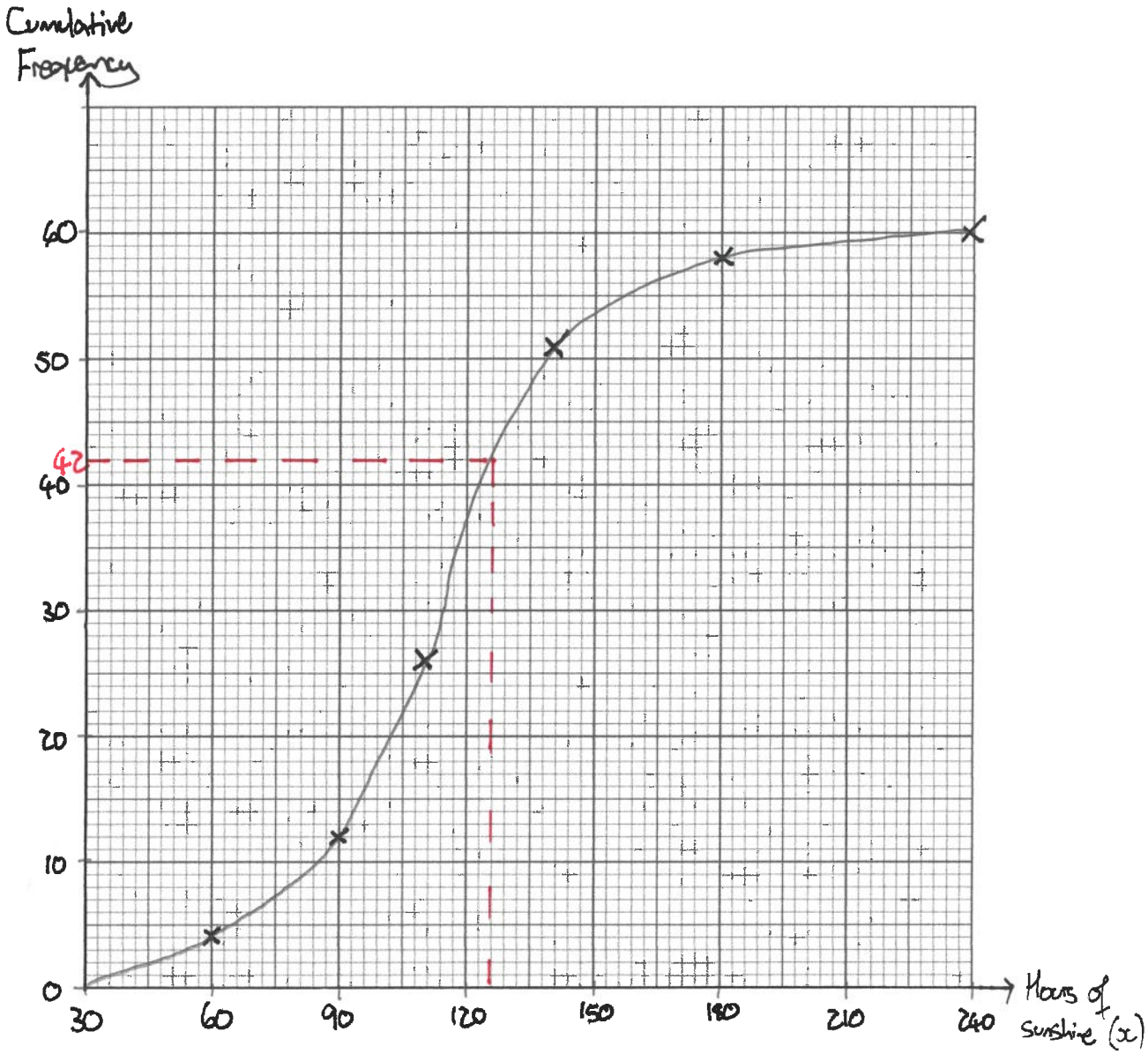
$IQR = 150 - 76$

$= 74$ cm

1 Each year the total number of hours, x, of sunshine in Kintoo is recorded during the month of June. The results for the last 60 years are summarised in the table.

x	$30 \leqslant x < 60$	$60 \leqslant x < 90$	$90 \leqslant x < 110$	$110 \leqslant x < 140$	$140 \leqslant x < 180$	$180 \leqslant x \leqslant 240$
Number of years	4	8	14	25	7	2
cf	4	12	26	51	58	60

(a) Draw a cumulative frequency graph to illustrate the data. [3]



(b) Use your graph to estimate the 70th percentile of the data. [2]

$0.7 \times 60 = 42$

70^{th} percentile = 126 hours

(c) Calculate an estimate for the mean number of hours of sunshine in Kintoo during June over the last 60 years. [3]

Mid-point (x)	Frequency (f)	$f \times x$
45	4	180
75	8	600
100	14	1400
125	25	3125
160	7	1120
210	2	420
	$\Sigma f = 60$	$\Sigma fx = 6845$

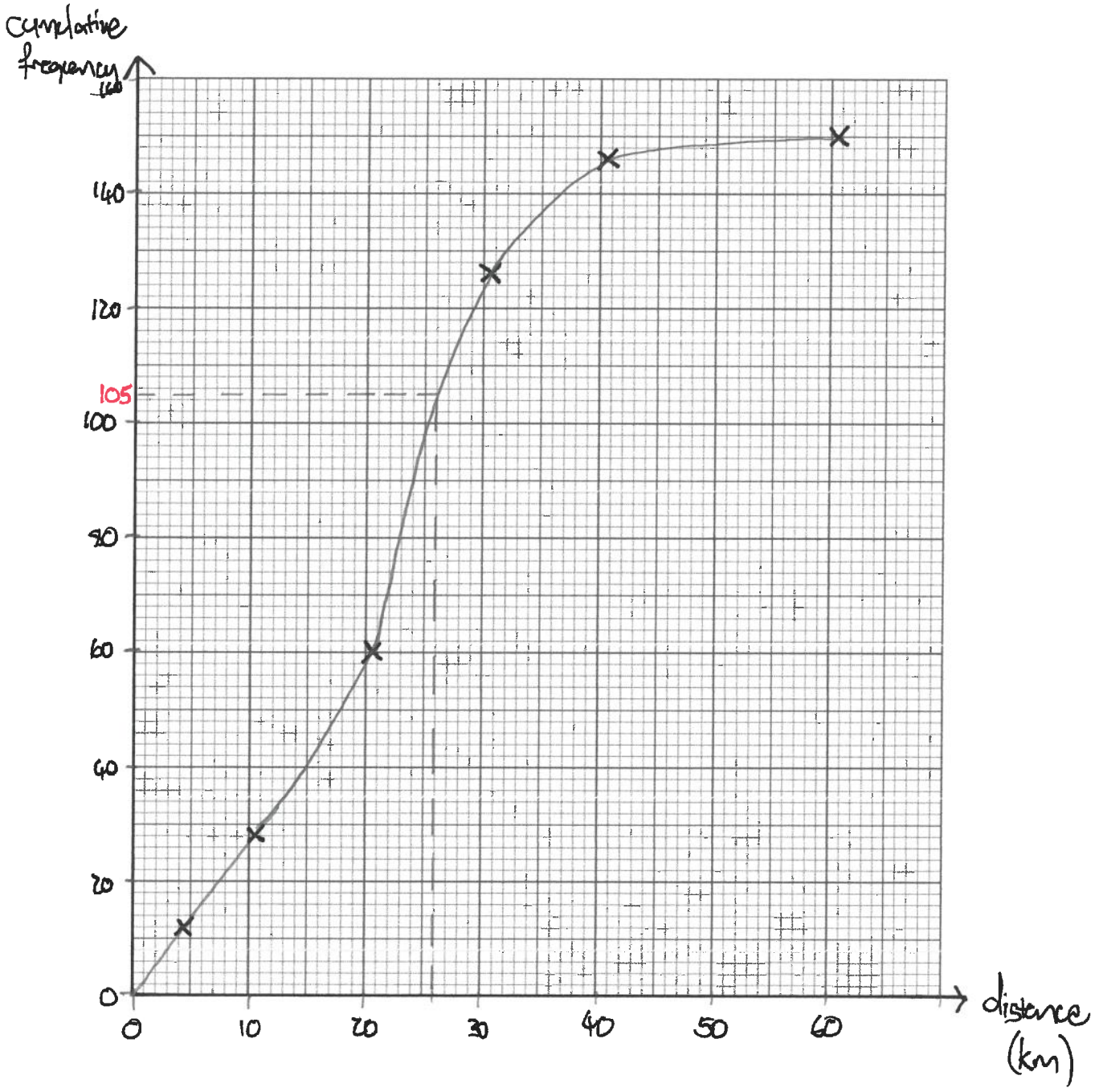
$$\bar{x} = \frac{6845}{60}$$

$$= 114 \text{ hours}$$

5 A driver records the distance travelled in each of 150 journeys. These distances, correct to the nearest km, are summarised in the following table.

		4·5	10·5	20·5	30·5	40·5	60·5
Distance (km)	0 – 4	5 – 10	11 – 20	21 – 30	31 – 40	41 – 60	
Frequency	12	16	32	66	20	4	
cf	12	28	60	126	146	150	

(a) Draw a cumulative frequency graph to illustrate the data. [4]



(b) For 30% of these journeys the distance travelled is d km or more.

Use your graph to estimate the value of d. [2]

30% are d km or more, so 70% are less than d.

$0.7 \times 150 = 105$

$d \simeq 26$ km

(c) Calculate an estimate of the mean distance travelled for the 150 journeys. [3]

Mid-point (x)	Frequency (f)	fx
2·25	12	27
7·5	16	120
15·5	32	496
25·5	66	1683
35·5	20	710
50·5	4	202
	$\Sigma f = 150$	$\Sigma fx = 3238$

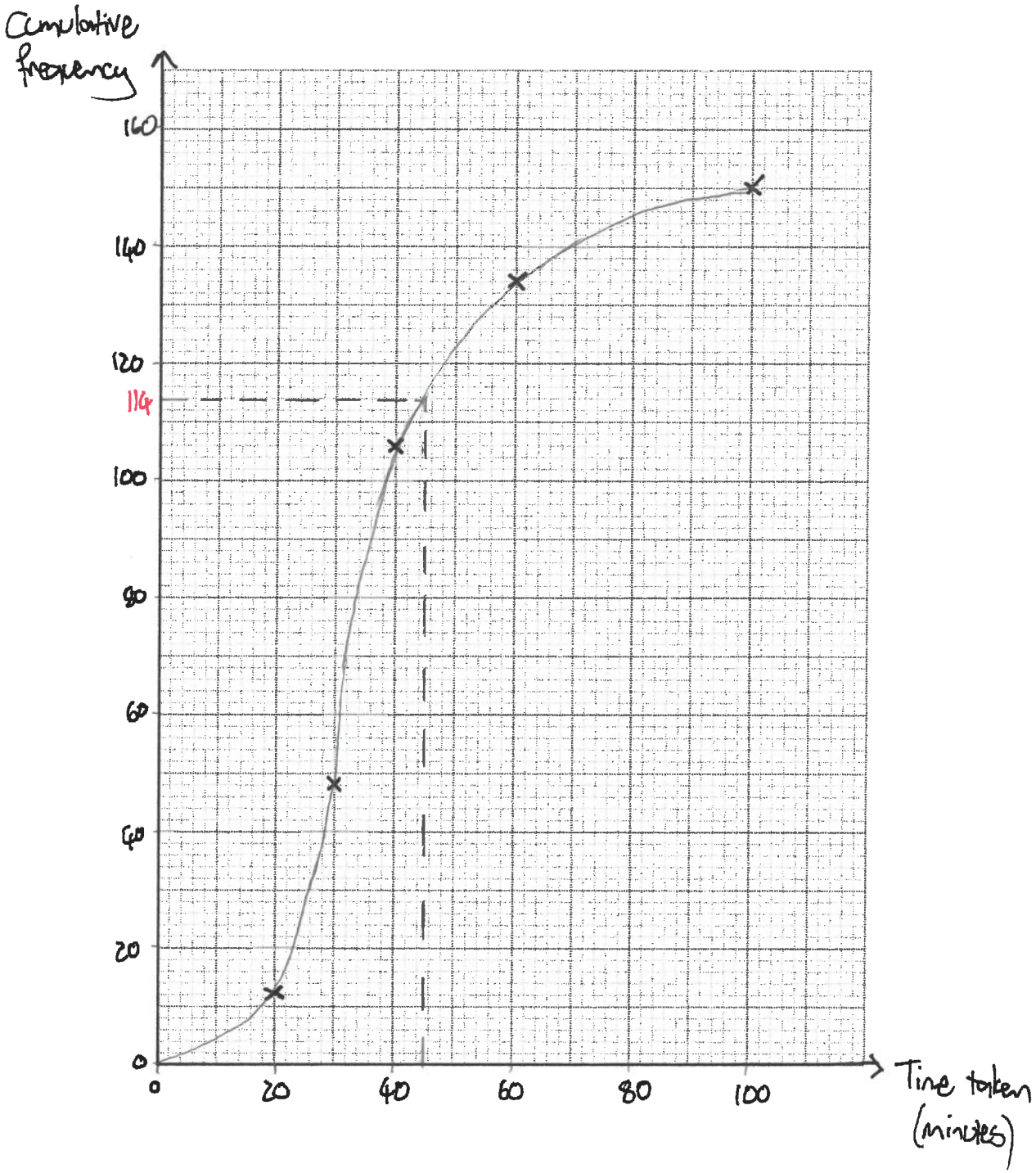
$\bar{x} = \frac{3238}{150}$

$= 21.6$ km

6 The times, t minutes, taken by 150 students to complete a particular challenge are summarised in the following cumulative frequency table.

Time taken (t minutes)	$t \leqslant 20$	$t \leqslant 30$	$t \leqslant 40$	$t \leqslant 60$	$t \leqslant 100$
Cumulative frequency	12	48	106	134	150

(a) Draw a cumulative frequency graph to illustrate the data. [2]



(b) 24% of the students take k minutes or longer to complete the challenge. Use your graph to estimate the value of k. [2]

24% take k mins or longer, so 76% take less than k mins.

$0.76 \times 150 = 114$

$k = 45$ mins

(c) Calculate estimates of the mean and the standard deviation of the time taken to complete the challenge. [6]

Mid-point (t)	Frequency (f)	$f \times t$
10	12	120
25	36	900
35	58	2030
50	28	1400
80	16	1280
	$\Sigma f = 150$	$\Sigma ft = 5730$

$$\bar{t} = \frac{5730}{150} = 38.2$$

$$\text{Var} = \frac{10^2 \times 12 + 25^2 \times 36 + 35^2 \times 58 + 50^2 \times 28 + 80^2 \times 16}{150} - 38.2^2$$

$= 321.76$

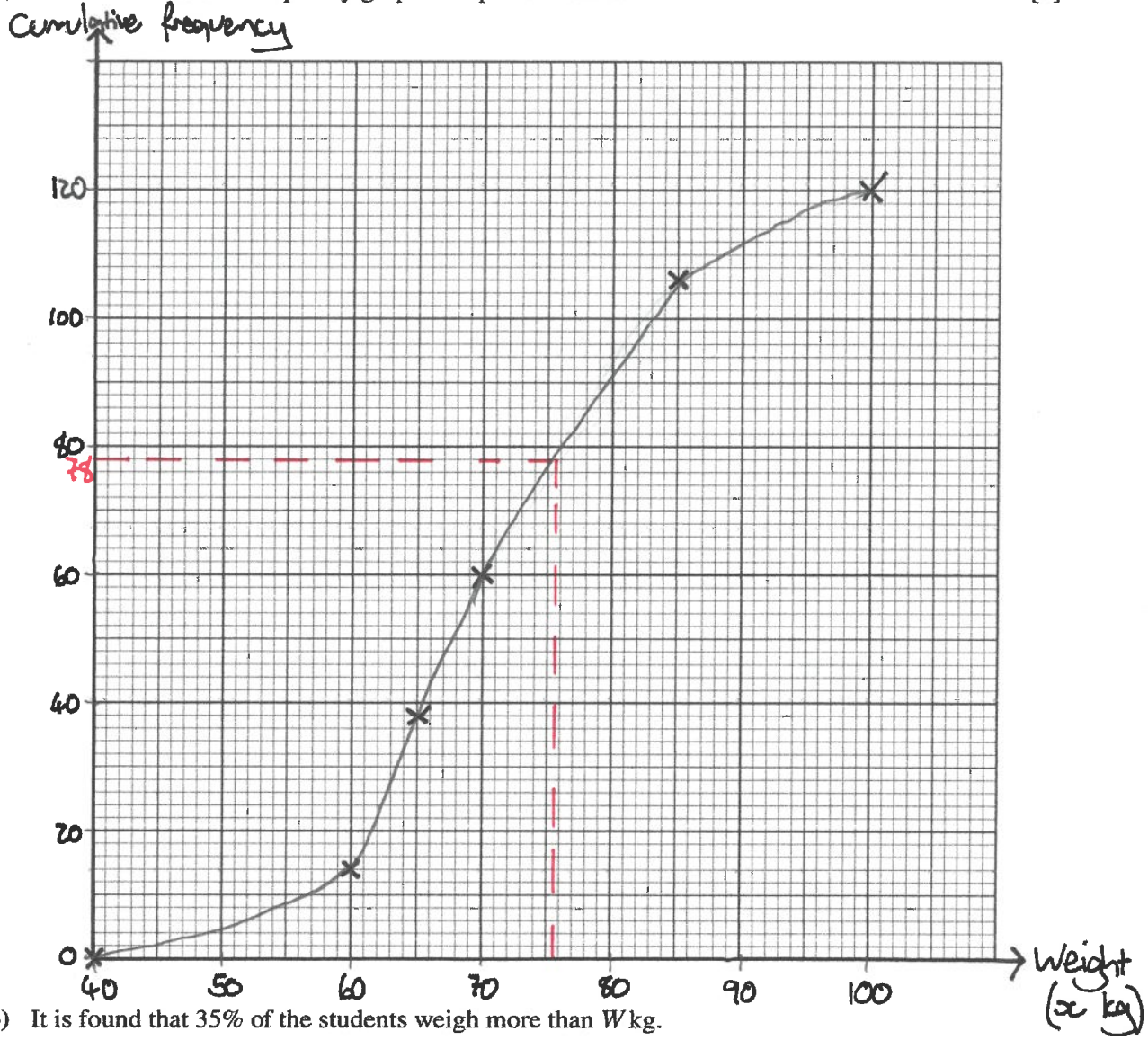
$\sigma = \sqrt{\text{Var}}$

$= 17.9$

4 The weights, x kg, of 120 students in a sports college are recorded. The results are summarised in the following table.

Weight (x kg)	$x \leqslant 40$	$x \leqslant 60$	$x \leqslant 65$	$x \leqslant 70$	$x \leqslant 85$	$x \leqslant 100$
Cumulative frequency	0	14	38	60	106	120

(a) Draw a cumulative frequency graph to represent this information. [2]



(b) It is found that 35% of the students weigh more than W kg.

Use your graph to estimate the value of W. [2]

35% are more than W, so 65% are less than W.

$0.65 \times 120 = 78$

$W \approx 75.5$kg

(c) Calculate estimates for the mean and standard deviation of the weights of the 120 students. [6]

Mid-point (x)	Frequency (f)	fx
20	0	0
50	14	700
62.5	24	1500
67.5	22	1485
77.5	46	3565
92.5	14	1295
	$\Sigma f = 120$	$\Sigma fx = 8545$

$$\bar{x} = \frac{8545}{120}$$

$$= 71.2 \text{ kg} \quad \text{STO}$$

unrounded

$$\text{Var} = \frac{50^2 \times 14 + 62.5^2 \times 24 + 67.5^2 \times 22 + 77.5^2 \times 46 + 92.5^2 \times 14}{120} - 71.2^2$$

$$\text{Var} = 138.23$$

$$\sigma = \sqrt{\text{Var}}$$

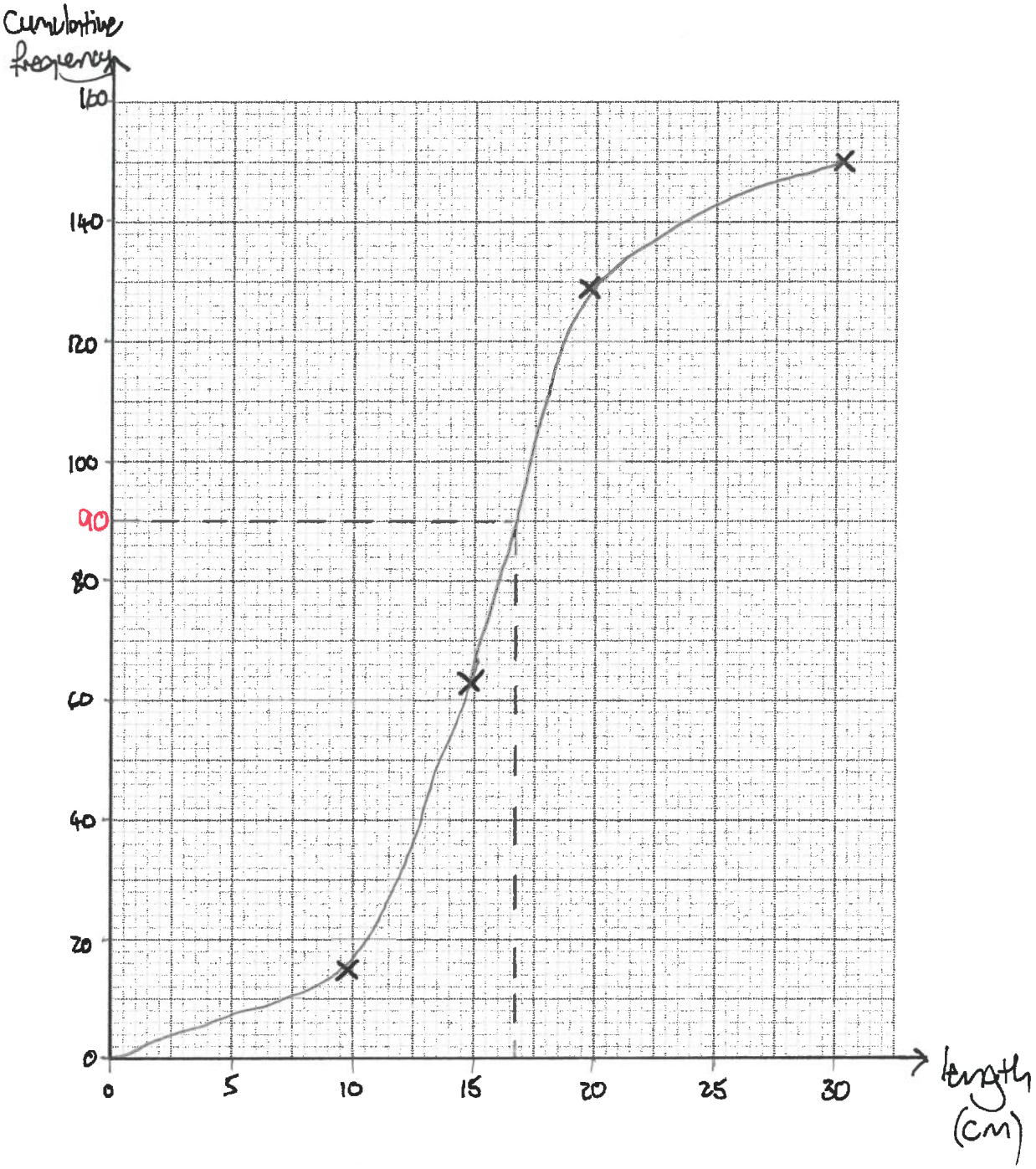
$$= 11.8$$

7 Helen measures the lengths of 150 fish of a certain species in a large pond. These lengths, correct to the nearest centimetre, are summarised in the following table.

9.5 14.5 19.5 30.5

Length (cm)	0 – 9	10 – 14	15 – 19	20 – 30
Frequency	15	48	66	21
cf	15	63	129	150

(a) Draw a cumulative frequency graph to illustrate the data. [4]



(b) 40% of these fish have a length of d cm or more. Use your graph to estimate the value of d. [2]

40% are dcm or more, so 60% are less than d.

$0.6 \times 150 = 90$

$d \approx 16.5$ cm

The mean length of these 150 fish is 15.295 cm.

(c) Calculate an estimate for the variance of the lengths of the fish. [3]

Mid-point (x)	Frequency (f)
4.75	15
12	48
17	66
25	21
	$\Sigma f = 150$

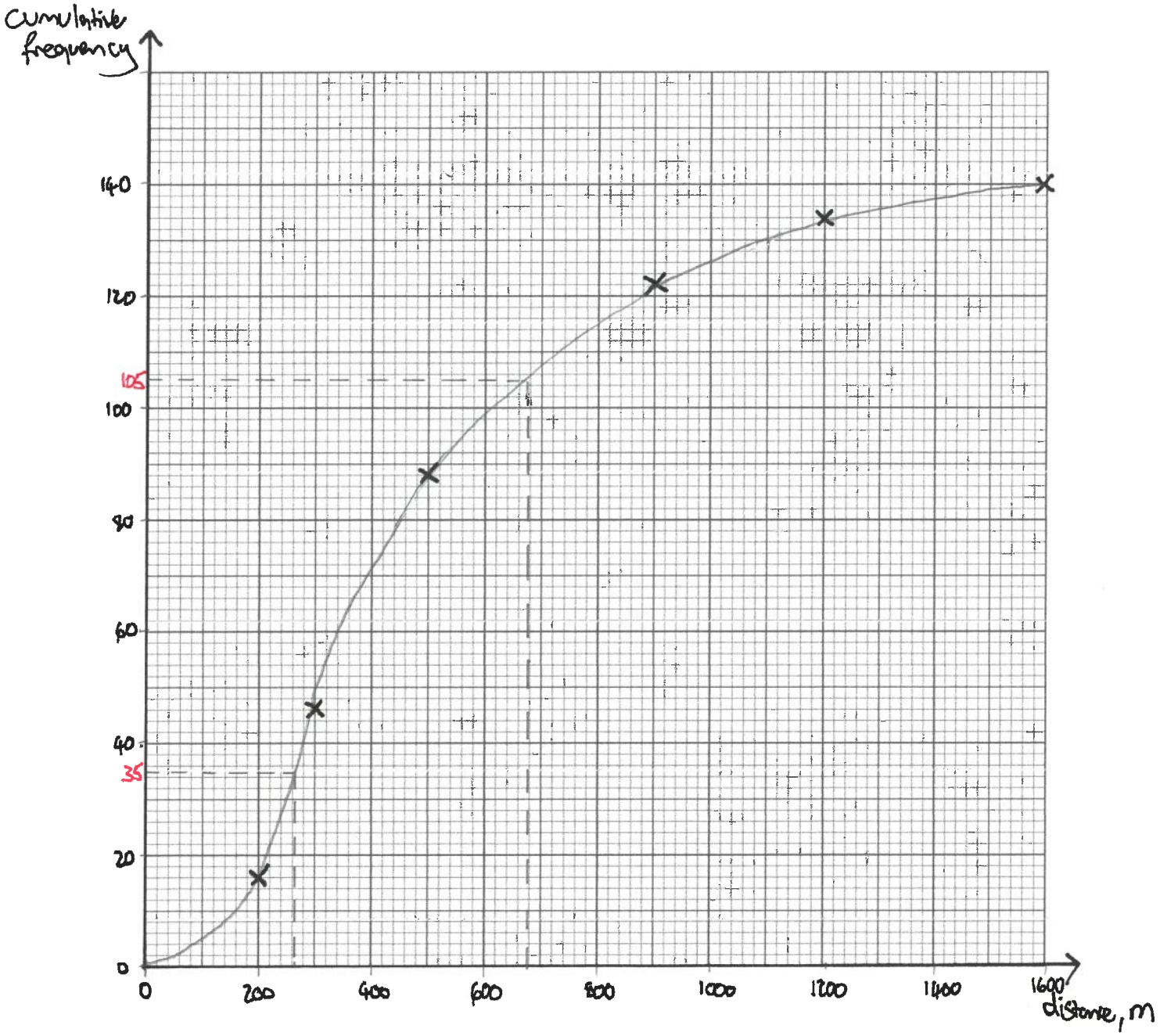
$\text{Var} = \dfrac{4.75^2 \times 15 + 12^2 \times 48 + 17^2 \times 66 + 25^2 \times 21}{150} - 15.295^2$

$= 29.1$

7 The distances, x m, travelled to school by 140 children were recorded. The results are summarised in the table below.

Distance, x m	$x \leqslant 200$	$x \leqslant 300$	$x \leqslant 500$	$x \leqslant 900$	$x \leqslant 1200$	$x \leqslant 1600$
Cumulative frequency	16	46	88	122	134	140

(a) On the grid, draw a cumulative frequency graph to represent these results. [2]



(b) Use your graph to estimate the interquartile range of the distances. [2]

Q_1: $0.25 \times 140 = 35$

$Q_1 = 260$

Q_3: $0.75 \times 140 = 105$

$Q_3 = 680$

$IQR = 680 - 260$

$= 420$

(c) Calculate estimates of the mean and standard deviation of the distances. [6]

Mid-point (x)	Frequency (f)	fx
100	16	1600
250	30	7500
400	42	16 800
700	34	23 800
1050	12	12 600
1400	6	8 400
	$\Sigma f = 140$	$\Sigma fx = 70\,700$

$$\bar{x} = \frac{70\,700}{140} = 505$$

$$\text{Var} = \frac{100^2 \times 16 + 250^2 \times 30 + 400^2 \times 42 + 700^2 \times 34 + 1050^2 \times 12 + 1400^2 \times 6}{140} - 505^2$$

$= 105\,010$

$\sigma = \sqrt{\text{Var}}$

$= 324$