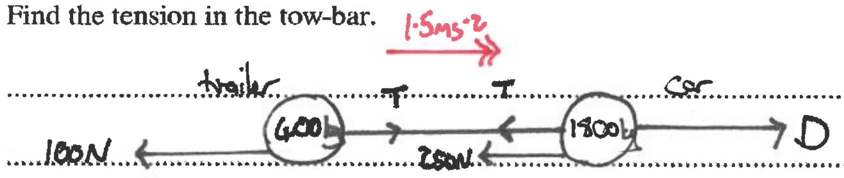
2 A car of mass 1800 kg is towing a trailer of mass 400 kg along a straight horizontal road. The car and trailer are connected by a light rigid tow-bar. The car is accelerating at $1.5\,\mathrm{m\,s^{-2}}$. There are constant resistance forces of 250 N on the car and 100 N on the trailer.

(a) Find the tension in the tow-bar. [2]



Trailer:

$R(\rightarrow)$: $F = ma$

$T - 100 = 400 \times 1.5$

$T - 100 = 600$

$T = 700\,\mathrm{N}$

(b) Find the power of the engine of the car at the instant when the speed is $20\,\mathrm{m\,s^{-1}}$. [3]

Car:

$R(\rightarrow)$: $F = ma$

$D - T - 250 = 1800 \times 1.5$

$D - 700 - 250 = 2700$

$D - 950 = 2700$

$D = 3650\,\mathrm{N}$

$\text{Power} = D \times V$

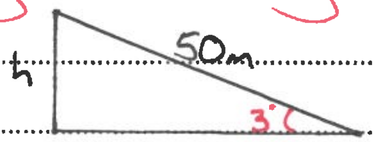
$= 3650 \times 20$

$= 73\,000\,\mathrm{W}$

3 A car of mass m kg is towing a trailer of mass 300 kg down a straight hill inclined at 3° to the horizontal at a constant speed. There are resistance forces on the car and on the trailer, and the total work done against the resistance forces in a distance of 50 m is 40 000 J. The engine of the car is doing no work and the tow-bar is light and rigid.

(a) Find the value of m. [3]

Change in height when travelling 50m:



$\sin 3 = \frac{h}{50}$

$h = 50\sin 3$

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$0 + \frac{1}{2}(m+300)v^2 + 0 = \frac{1}{2}(m+300)v^2 + (m+300)\times 10 \times -50\sin 3 + 40\,000$

$\cancel{\frac{1}{2}(m+300)v^2} = \cancel{\frac{1}{2}(m+300)v^2} + (m+300)\times -500\sin 3 + 40\,000$

$(m+300)\times 500\sin 3 = 40\,000$

$m + 300 = 1528.59$

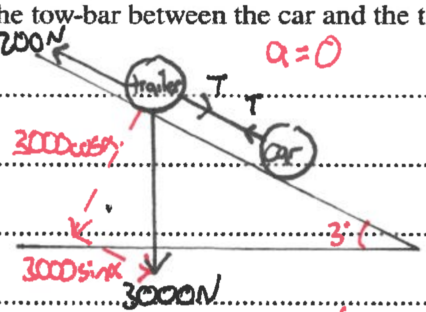
$m = 1228.59$

$= 1230\text{kg}$

The resistance force on the trailer is 200 N.

(b) Find the tension in the tow-bar between the car and the trailer. [2]

Trailer:



$a = 0$

$R(\searrow)$: $F = ma \leftarrow a = 0$ (constant speed)

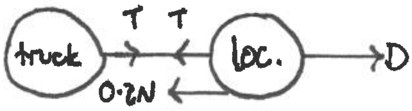
$T + 3000\sin 3 - 200 = 0$

$T + 157.0 - 200 = 0$

$T - 43.0 = 0$

$T = 43.0\text{N}$

4 A toy railway locomotive of mass 0.8 kg is towing a truck of mass 0.4 kg on a straight horizontal track at a constant speed of $2\,\mathrm{m\,s^{-1}}$. There is a constant resistance force of magnitude 0.2 N on the locomotive, but no resistance force on the truck. There is a light rigid horizontal coupling connecting the locomotive and the truck.



(a) State the tension in the coupling. [1]

Truck: $R(\rightarrow)$: $F = ma \leftarrow a = 0$ (constant speed)

$\underline{T = 0\,N}$

(b) Find the power produced by the locomotive's engine. [1]

Locomotive: $R(\rightarrow)$: $F = ma$

$D - T - 0.2 = 0$

$D - 0 - 0.2 = 0$

$\underline{D = 0.2N}$

Power $= D \times v$

$= 0.2 \times 2$

$= \underline{0.4W}$

The power produced by the locomotive's engine is now changed to 1.2 W.

(c) Find the magnitude of the tension in the coupling at the instant that the locomotive begins to accelerate. [5]

Power $= D \times v$

$1.2 = D \times 2$

$\underline{D = 0.6N}$

Locomotive:

$R(\rightarrow)$: $F = ma$

$0.6 - T - 0.2 = 0.8a$

$0.4 - T = 0.8a$ ①

Truck:

$R(\rightarrow)$: $F = ma$

$T = 0.4a$ ②

① + ②: $0.4 = 1.2a$

$a = \frac{1}{3}$

②: $T = 0.4 \times \frac{1}{3}$

$= \underline{0.133N}$

3 A constant resistance of magnitude 1400 N acts on a car of mass 1250 kg.

(a) The car is moving along a straight level road at a constant speed of $28\,\mathrm{m\,s^{-1}}$.

Find, in kW, the rate at which the engine of the car is working. [2]

Constant speed, so $a = 0$ and driving force = resistance = 1400N

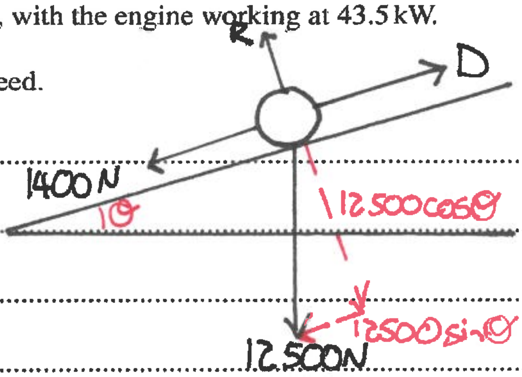
Power = $D \times V$

$= 1400 \times 28$

$= 39\,200\,\mathrm{W}$

(b) The car now travels at a constant speed up a hill inclined at an angle of θ to the horizontal, where $\sin\theta = 0.12$, with the engine working at 43.5 kW.

Find this speed. [3]



a = 0 (constant speed)

$R(\nearrow)$: $D - 1400 - 12500\sin\theta = ma$

$D - 1400 - 12500 \times 0.12 = 0$

$D - 1400 - 1500 = 0$

$D = 2900\,\mathrm{N}$

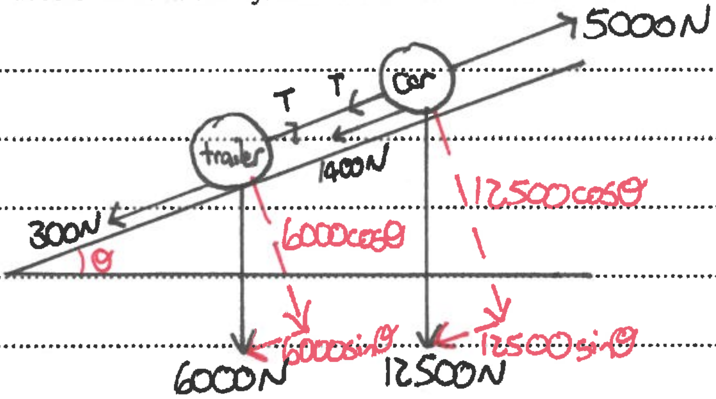
Power = $D \times V$

$43\,500 = 2900v$

$v = 15\,\mathrm{ms^{-1}}$

(c) On another occasion, the car pulls a trailer of mass 600 kg up the same hill. The system of the car and the trailer is modelled as particles connected by a light inextensible cable. The car's engine produces a driving force of 5000 N and the resistance to the motion of the trailer is 300 N. The resistance to the motion of the car remains 1400 N.

Find the acceleration of the system and the tension in the cable. [4]



Car:

R(↗): $5000 - 1400 - 12500\sin\theta - T = 1250a$

$3600 - 12500 \times 0.12 - T = 1250a$

$2100 - T = 1250a$ ①

Trailer:

R(↗): $T - 300 - 6000\sin\theta = 600a$

$T - 300 - 6000 \times 0.12 = 600a$

$T - 1020 = 600a$ ②

①+②: $2100 - 1020 = 1850a$

$1080 = 1850a$

$\underline{a = 0.584\,\text{ms}^{-2}}$ STO

→②: $T - 1020 = 600 \times 0.584$

$T = 350.27 + 1020$

$= \underline{1370\,\text{N}}$

6 A car of mass 1600 kg is pulling a caravan of mass 800 kg. The car and the caravan are connected by a light rigid tow-bar. The resistances to the motion of the car and caravan are 400 N and 250 N respectively.

(a) The car and caravan are travelling along a straight horizontal road.

(i) Given that the car and caravan have a constant speed of $25\,\mathrm{m\,s^{-1}}$, find the power of the car's engine. [2]

Constant speed, so $a = 0$. Consider whole system:

$R(\rightarrow)$: $D - 400 - 250 = 0$

$D = 650\,\mathrm{N}$

Power $= D \times V$

$= 650 \times 25$

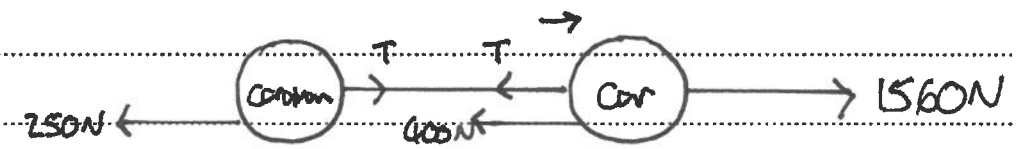
$= \underline{16\,250\,\mathrm{W}}$

(ii) The engine's power is now suddenly increased to 39 kW. Find the instantaneous acceleration of the car and caravan and find the tension in the tow-bar. [5]

Power $= D \times V$

$39000 = D \times 25$

$\underline{D = 1560\,\mathrm{N}}$



Car:

$R(\rightarrow)$: $F = ma$

$1560 - 400 - T = 1600a$

$1160 - T = 1600a$ ①

Caravan:

$R(\rightarrow)$: $F = ma$

$T - 250 = 800a$ ②

cont.

①+②: $1160 - 250 = 2400a$

$910 = 2400a$

$\underline{a = 0.379\,\text{ms}^{-2}}$ STO

→②: $T - 250 = 800 \times 0.379$

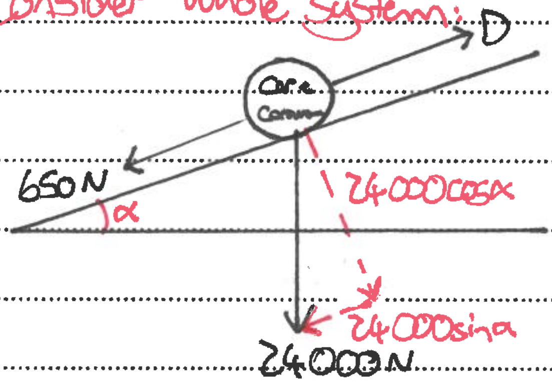
$T - 250 = 303.3$

$\underline{T = 553\,\text{N}}$

(b) The car and caravan now travel up a straight hill, inclined at an angle of $\sin^{-1} 0.05$ to the horizontal, at a constant speed of $v\ \text{m s}^{-1}$. The car's engine is working at 32.5 kW. $\sin\alpha = 0.05$

Find v. [3]

Consider whole system:



$a = 0$ (constant speed)

$R(\nearrow)$: $D - 650 - 24000\sin\alpha = ma$

$D - 650 - 24000 \times 0.05 = 0$

$D - 650 - 1200 = 0$

$\underline{D = 1850\,\text{N}}$

Power $= D \times v$

$32500 = 1850v$

$\underline{v = 17.6\,\text{ms}^{-1}}$

6 A car of mass 1750 kg is pulling a caravan of mass 500 kg. The car and the caravan are connected by a light rigid tow-bar. The resistances to the motion of the car and caravan are 650 N and 150 N respectively.

(a) The car and caravan are moving along a straight horizontal road at a constant speed of $24\,\mathrm{m\,s^{-1}}$.

(i) Find the power of the car's engine. [2]

Constant speed, so $a = 0$ and Driving Force = Resistive Forces.

Whole system:

$a = 0$

$R(\rightarrow): D - 650 - 150 = ma$

$D - 800 = 0$

$D = 800\,\mathrm{N}$

$\text{Power} = D \times V$

$= 800 \times 24$

$= \underline{19\,200\,\mathrm{W}}$

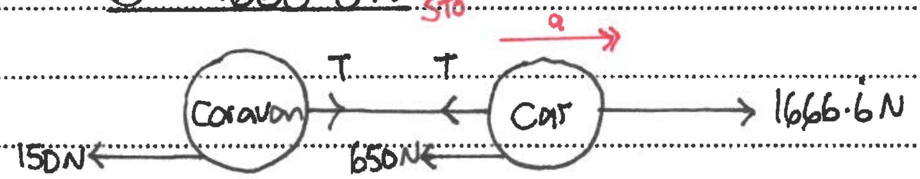
(ii) The engine's power is now suddenly increased to 40 kW.

Find the instantaneous acceleration of the car and caravan and find the tension in the tow-bar. [5]

$\text{Power} = D \times V$

$40\,000 = D \times 24$

$\underline{D = 1666.\dot{6}\,\mathrm{N}}$ STO



Car:

$R(\rightarrow): F = ma$

$1666.6 - 650 - T = 1750a$

$1016.\dot{6} - T = 1750a$ (1)

Caravan:

$R(\rightarrow): F = ma$

$T - 150 = 500a$ (2)

cont

①+②: $1016.\dot{6} - 150 = 2250a$

$866.\dot{6} = 2250a$

$a = 0.385\ ms^{-2}$

→②:

$T - 150 = 500 \times 0.385$

$T - 150 = 192.6$

$T = 342.6$

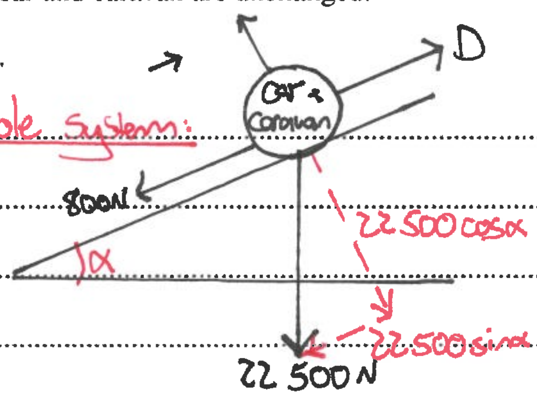
$= 343\ N$

(b) The car and caravan now travel up a straight hill, inclined at an angle $\sin^{-1} 0.14$ to the horizontal, at a constant speed of $v\ \mathrm{m\,s^{-1}}$. The car's engine is working at 31 kW. The resistances to the motion of the car and caravan are unchanged.

$\sin\alpha = 0.14$

Find v. [3]

Whole System:



R(↗): $F = ma$ ← $a = 0$ (constant velocity)

$D - 800 - 22500\sin\alpha = 0$

$D - 800 - 22500 \times 0.14 = 0$

$D = 3950N$

$\text{Power} = D \times v$

$31000 = 3950v$

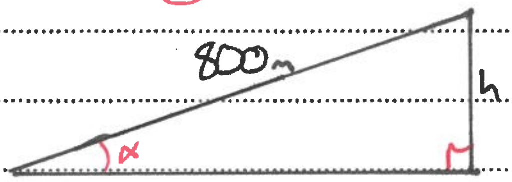
$v = 7.85\ ms^{-1}$

$\sin\alpha = 0.08$

6 A car of mass 1500 kg is pulling a trailer of mass 750 kg up a straight hill of length 800 m inclined at an angle of $\sin^{-1} 0.08$ to the horizontal. The resistances to the motion of the car and trailer are 400 N and 200 N respectively. The car and trailer are connected by a light rigid tow-bar. The car and trailer have speed $30\,\mathrm{m\,s^{-1}}$ at the bottom of the hill and $20\,\mathrm{m\,s^{-1}}$ at the top of the hill.

(a) Use an energy method to find the constant driving force as the car and trailer travel up the hill. [5]

Change in height when travelling 800m:



$\sin\alpha = \frac{h}{800}$

$h = 800\sin\alpha$

$= 800 \times 0.08$

$= \underline{64\text{ m}}$

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

Work = F×d

$800D + \frac{1}{2}\times 2250\times 30^2 + 0 = \frac{1}{2}\times 2250\times 20^2 + 2250\times 10\times 64 + 600\times 800$

F×d

$800D + 1012500 = 450000 + 1440000 + 480000$

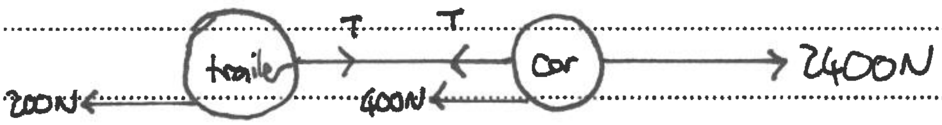
$800D + 1012500 = 2370000$

$800D = 1357500$

$\underline{\underline{D = 1696.875\text{ N}}}$

After reaching the top of the hill the system consisting of the car and trailer travels along a straight level road. The driving force of the car's engine is 2400 N and the resistances to motion are unchanged.

(b) Find the acceleration of the system and the tension in the tow-bar. [4]



Car:

$R(\rightarrow)$: $F = ma$

$2400 - 400 - T = 1500a$

$2000 - T = 1500a$ ①

Trailer:

$R(\rightarrow)$: $F = ma$

$T - 200 = 750a$ ②

① + ②: $2000 - 200 = 2250a$

$1800 = 2250a$

$a = 0.8 \text{ ms}^{-2}$

→ ②: $T - 200 = 750 \times 0.8$

$T - 200 = 600$

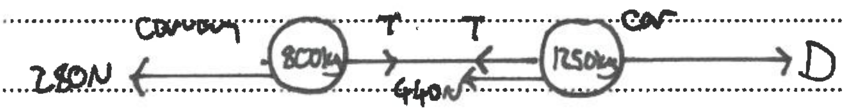
$T = 800 \text{ N}$

5 A car of mass 1250 kg is pulling a caravan of mass 800 kg along a straight road. The resistances to the motion of the car and caravan are 440 N and 280 N respectively. The car and caravan are connected by a light rigid tow-bar.

(a) The car and caravan move along a horizontal part of the road at a constant speed of $30\,\mathrm{m\,s^{-1}}$.

(i) Calculate, in kW, the power developed by the engine of the car. [2]

$a = 0\,\mathrm{ms^{-2}}$



Consider whole system:

$R(\rightarrow):\quad F = ma$ $a = 0$

$D - T + T - 440 - 280 = ma$

$D - 720 = 0$

$D = \underline{720\,N}$

$\text{Power} = D \times V$

$= 720 \times 30$

$= \underline{21600\,W}$

(ii) Given that this power is suddenly decreased by 8 kW, find the instantaneous deceleration of the car and caravan and the tension in the tow-bar. [4]

$\text{New Power} = 21\,600 - 8000$

$= 13\,600\,W$

$\text{Power} = D \times V$

$13\,600 = D \times 30$

$D = \underline{453.\dot{3}\,N}$ STO

Whole system:

$R(\rightarrow):\quad F = ma$

$453.\dot{3} - 440 - 280 = 2050a$

$-266.\dot{6} = 2050a$

$a = \underline{-0.130\,\mathrm{ms^{-2}}}$

Caravan:

$R(\rightarrow):\quad T - 280 = ma$

$T - 280 = 800 \times (-0.130)$

$T - 280 = -104.1$

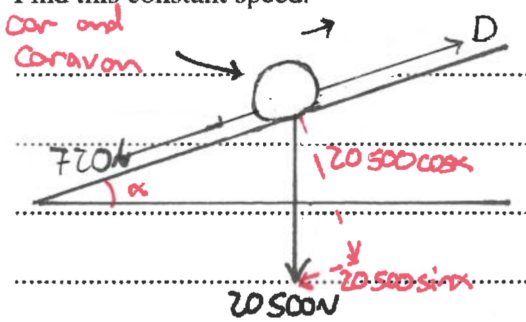
$T = 175.9$

$T = \underline{176\,N}$

$\sin\alpha = 0.06$

(b) The car and caravan now travel along a part of the road inclined at $\sin^{-1} 0.06$ to the horizontal. The car and caravan travel up the incline at constant speed with the engine of the car working at 28 kW.

(i) Find this constant speed. $a = 0\,\text{ms}^{-2}$ [3]



Whole System:

$R(\rightarrow)$: $F = m\underbrace{a}_{=0}$

$D - 720 - 20500\sin\alpha = 0$

$D - 720 - 20500 \times 0.06 = 0$

$D - 720 - 1230 = 0$

$D - 1950 = 0$

$D = \underline{1950\,N}$

Power $= D \times V$

$28000 = 1950 \times V$

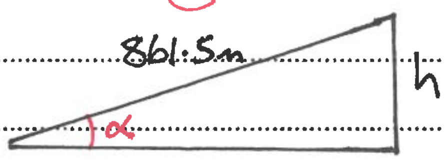
$\underline{V = 14.4\,\text{ms}^{-1}}$ STO

(ii) Find the increase in the potential energy of the caravan in one minute. [2]

Distance travelled in 1 minute: distance $= 14.4 \times 60$

$= 861.5\,\text{m}$ STO

Increase in height in 1 minute:



$\sin\alpha = \dfrac{h}{861.5}$

$h = 861.5 \times 0.06$

$= 51.7\,\text{m}$ STO

Change in PE $= mg\Delta h$

$= 800 \times 10 \times 51.7$

$= 413\,538\,\text{J}$

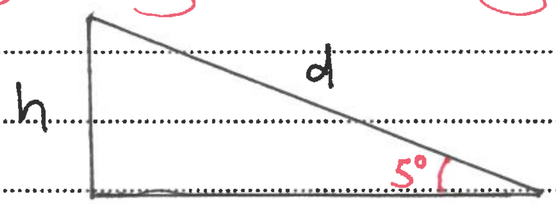
$= \underline{414\,000\,\text{J}}$

5 A car of mass 1400 kg is towing a trailer of mass 500 kg down a straight hill inclined at an angle of 5° to the horizontal. The car and trailer are connected by a light rigid tow-bar. At the top of the hill the speed of the car and trailer is $20\,\text{m s}^{-1}$ and at the bottom of the hill their speed is $30\,\text{m s}^{-1}$.

(a) It is given that as the car and trailer descend the hill, the engine of the car does 150 000 J of work, and there are no resistance forces.

Find the length of the hill. [5]

Change in height when travelling a distance d:



$\sin 5 = \frac{h}{d}$

$h = d\sin 5$

$\text{Work}_{\text{IN}} + KE_{\text{init}} + PE_{\text{init}} = KE_{\text{fin}} + PE_{\text{fin}} + \text{Work}_{\text{out}}$

$150\,000 + \frac{1}{2} \times 1900 \times 20^2 + 0 = \frac{1}{2} \times 1900 \times 30^2 + 1900 \times 10 \times -d\sin 5 + 0$

$150\,000 + 380\,000 = 855\,000 - 19000 d\sin 5$

$530\,000 = 855\,000 - 19000\sin 5 \times d$

$-325\,000 = -19\,000\sin 5 \times d$

$d = \frac{-325\,000}{-19000\sin 5}$

$= 196\text{ m}$

(b) It is given instead that there is a resistance force of 100 N on the trailer, the length of the hill is 200 m, and the acceleration of the car and trailer is constant.

Find the tension in the tow-bar between the car and trailer. [4]

Find acceleration using Suvat:

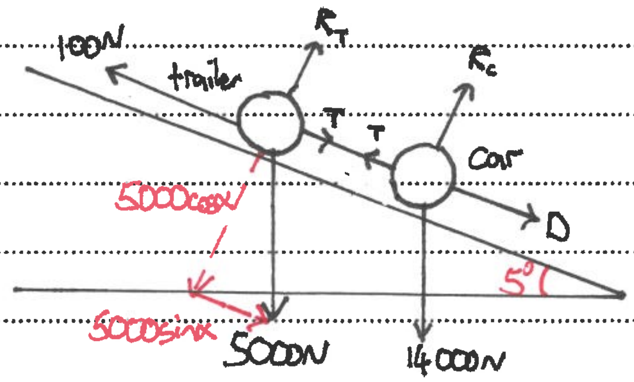
$s = 200$ | $v^2 = u^2 + 2as$

$u = 20$ | $30^2 = 20^2 + 2 \times a \times 200$

$v = 30$ | $900 = 400 + 400a$

$a =$ | $500 = 400a$

$t =$ | $\underline{a = 1.25 \text{ ms}^{-2}}$



Consider Trailer:

$R(\searrow)$: $T + 5000\sin 5 - 100 = ma$

$T + 5000\sin 5 - 100 = 500 \times 1.25$

$T + 5000\sin 5 - 100 = 625$

$T = 725 - 5000\sin 5$

$= \underline{289 \text{ N}}$