3 A factory produces a certain type of electrical component. It is known that 15% of the components produced are faulty. A random sample of 200 components is chosen.

Use an approximation to find the probability that more than 40 of these components are faulty. [5]

$X \sim B(200, 0.15)$

$\mu = 200 \times 0.15$ $\qquad$ $\sigma^2 = 30 \times 0.85$

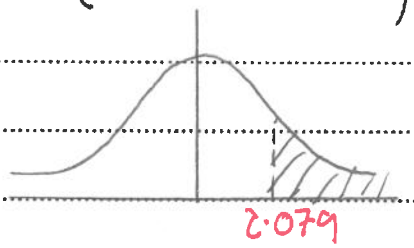
$= 30$ $\qquad$ $= 25.5$

$X \sim N(30, 25.5)$

$P(X > 40) \rightarrow P(X > 40.5)$ (continuity correction)

$P\left(Z > \frac{40.5 - 30}{\sqrt{25.5}}\right)$

$= P(Z > 2.079)$



$= 1 - \Phi(2.079)$

$= 1 - 0.9812$

$= 0.0188$

2 Anil is a candidate in an election. He received 40% of the votes. A random sample of 120 voters is chosen.

Use an approximation to find the probability that, of the 120 voters, between 36 and 54 inclusive voted for Anil. [5]

$V \sim B(120, 0.4)$

$\mu = 120 \times 0.4$ $\quad\quad \sigma^2 = 48 \times 0.6$

$= 48$ $\quad\quad = 28.8$

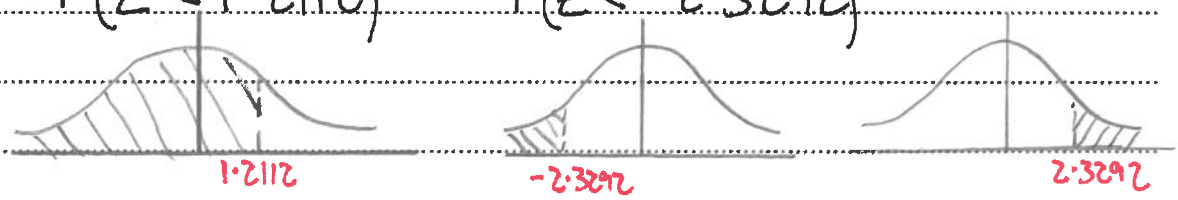
$V \sim N(48, 28.8)$

$P(36 \leq V \leq 54) \rightarrow P(35.5 < V < 54.5)$ (continuity correction)

$P\left(\frac{35.5 - 48}{\sqrt{28.8}} < Z < \frac{54.5 - 48}{\sqrt{28.8}}\right)$

$= P(-2.3292 < Z < 1.2112)$

$= P(Z < 1.2112) - P(Z < -2.3292)$



$= \Phi(1.211) - (1 - \Phi(2.329))$

$= 0.8871 - (1 - 0.9900)$

$= 0.8771$

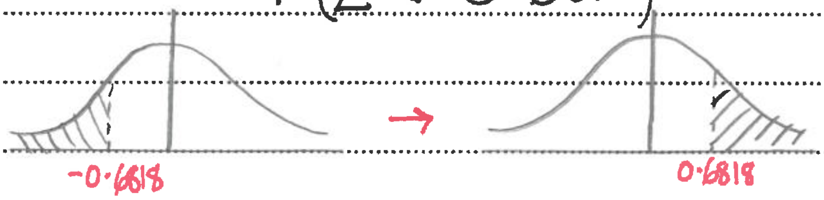
3 A farmer sells eggs. The weights, in grams, of the eggs can be modelled by a normal distribution with mean 80.5 and standard deviation 6.6. Eggs are classified as small, medium or large according to their weight. A small egg weighs less than 76 grams and 40% of the eggs are classified as medium.

(a) Find the percentage of eggs that are classified as small. [3]

$$E \sim N(80.5, 6.6^2)$$

$$P(E < 76) = P\left(Z < \frac{76 - 80.5}{6.6}\right)$$

$$= P(Z < -0.6818)$$



$$= 1 - \Phi(0.682)$$

$$= 1 - 0.7524$$

$$= 0.2476 \rightarrow 24.76\%$$

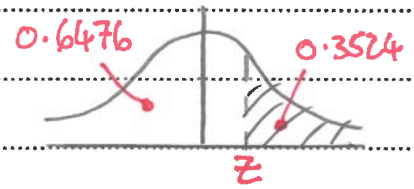
(b) Find the least possible weight of an egg classified as large. [3]

proportion of small + medium eggs $= 0.2476 + 0.4 = 0.6476$

proportion of large eggs $= 1 - 0.6476 = 0.3524$

$$P(E > w) = 0.3524$$

$$P\left(Z > \frac{w - 80.5}{6.6}\right) = 0.3524$$



$0.6476 = \Phi(0.379)$ (closest value)

$$\frac{w - 80.5}{6.6} = 0.379 \rightarrow w - 80.5 = 2.5014$$

$$w = 83.0\text{g}$$

150 of the eggs for sale last week were weighed.

(c) Use an approximation to find the probability that more than 68 of these eggs were classified as medium. [5]

P(medium)

$X \sim B(150, 0.4)$

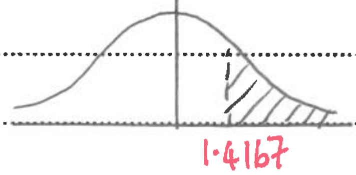
$\mu = 150 \times 0.4 = 60$ $\quad\quad$ $\sigma^2 = 60 \times 0.6 = 36$

$X \sim N(60, 36)$

$P(X > 68) \rightarrow P(X > 68.5)$ (continuity correction)

$P\left(Z > \frac{68.5 - 60}{\sqrt{36}}\right)$

$= P(Z > 1.4167)$



$= 1 - \Phi(1.417)$

$= 1 - 0.9217$

$= 0.0783$

2 In a large college, 32% of the students have blue eyes. A random sample of 80 students is chosen.

Use an approximation to find the probability that fewer than 20 of these students have blue eyes. [5]

$X \sim B(80, 0.32)$

$\mu = 80 \times 0.32$ $\sigma^2 = 25.6(0.68)$

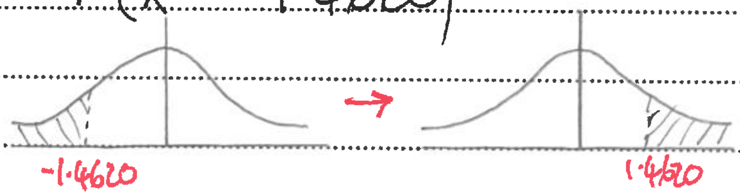
$= 25.6$ $= 17.408$

$X \sim N(25.6, 17.408)$

$P(X < 20) \rightarrow P(X < 19.5)$ (continuity correction)

$P\left(X < \frac{19.5 - 25.6}{\sqrt{17.408}}\right)$

$= P(X < -1.4620)$



$= 1 - \Phi(1.462)$

$= 1 - 0.9282$

$= \underline{0.0718}$

5 The lengths of Western bluebirds are normally distributed with mean 16.5 cm and standard deviation 0.6 cm.

A random sample of 150 of these birds is selected.

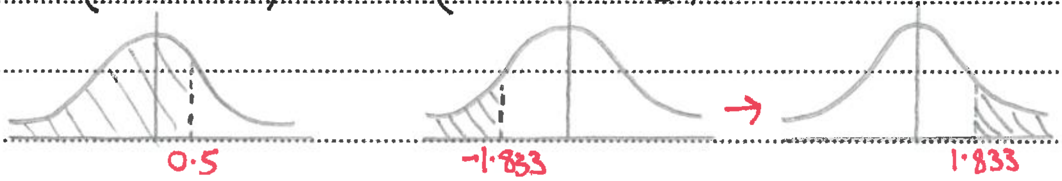
(a) How many of these 150 birds would you expect to have length between 15.4 cm and 16.8 cm? [4]

$P(15.4 < L < 16.8)$

$P\left(\frac{15.4 - 16.5}{0.6} < Z < \frac{16.8 - 16.5}{0.6}\right)$

$P(-1.833 < Z < 0.5)$

$= P(Z < 0.5) - P(Z < -1.833)$



$= \Phi(0.5) - (1 - \Phi(1.833))$

$= 0.6915 - (1 - 0.9668)$

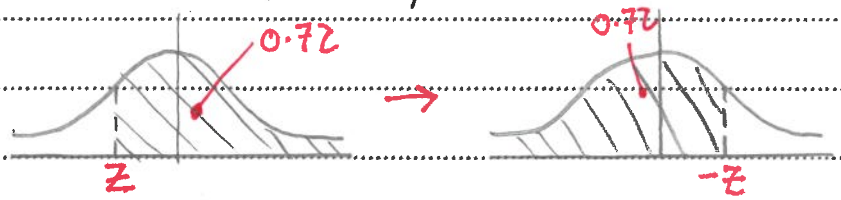
$= \underline{0.6583}$ → $150 \times 0.6583 = \underline{99 \text{ birds}}$

The lengths of Eastern bluebirds are normally distributed with mean 18.4 cm and standard deviation σ cm. It is known that 72% of Eastern bluebirds have length greater than 17.1 cm.

(b) Find the value of σ. [3]

$P(L > 17.1) = 0.72$

$P\left(Z > \frac{17.1 - 18.4}{\sigma}\right) = 0.72$



$0.72 = \Phi(0.583)$ from table

$z = -0.583$

$$\frac{17.1 - 18.4}{\sigma} = -0.583$$

$$-1.3 = -0.583\sigma$$

$$\sigma = 2.23$$

A random sample of 120 Eastern bluebirds is chosen.

(c) Use an approximation to find the probability that fewer than 80 of these 120 bluebirds have length greater than 17.1 cm. [5]

$$L \sim B(120, 0.72)$$

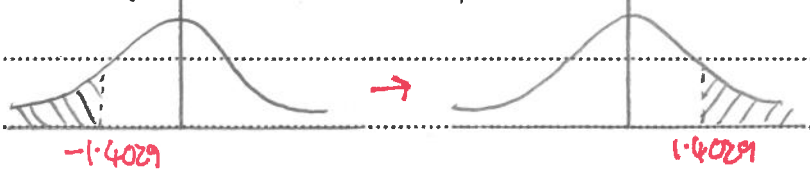
$$\mu = 120 \times 0.72 = 86.4 \qquad \sigma^2 = 86.4(0.28) = 24.192$$

$$L \sim N(86.4, 24.192)$$

$P(L < 80) \rightarrow P(L < 79.5)$ (continuity correction)

$$P\left(Z < \frac{79.5 - 86.4}{\sqrt{24.192}}\right)$$

$$= P(Z < -1.4029)$$



$$= 1 - \Phi(1.403)$$

$$= 1 - 0.9196$$

$$= 0.0804$$

2 The residents of Persham were surveyed about the reliability of their internet service. 12% rated the service as 'poor', 36% rated it as 'satisfactory' and 52% rated it as 'good'.

A random sample of 8 residents of Persham is chosen.

(a) Find the probability that more than 2 and fewer than 8 of them rate their internet service as poor or satisfactory. [3]

$R \sim B(8, 0.48)$

poor + satisfactory

$P(2<X<8) = 1 - (P(0) + P(1) + P(2) + P(8))$

$= 1 - ({}^8C_0 \times 0.48^0 \times 0.52^8 + {}^8C_1 \times 0.48^1 \times 0.52^7$

$+ {}^8C_2 \times 0.48^2 \times 0.52^6 + {}^8C_8 \times 0.48^8 \times 0.52^0)$

$= \underline{0.825}$

A random sample of 125 residents of Persham is now chosen.

(b) Use an approximation to find the probability that more than 72 of these residents rate their internet service as good. [5]

$R \sim B(125, 0.52)$

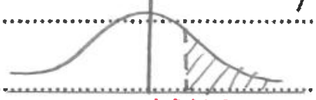
$\mu = 125 \times 0.52$ $\quad\quad \sigma^2 = 65(0.48)$

$= 65$ $\quad\quad = 31.2$

$R \sim N(65, 31.2)$

$P(N>72) \rightarrow P(N>72.5)$ (continuity correction)

$P\left(Z > \frac{72.5-65}{\sqrt{31.2}}\right) = P(Z > 1.3427)$



1.3427

$= 1 - \Phi(1.343)$

$= 1 - 0.9104$

$= \underline{0.0896}$

4 The 13 00 train from Jahor to Keman runs every day. The probability that the train arrives late in Keman is 0.35.

(a) For a random sample of 7 days, find the probability that the train arrives late on fewer than 3 days. [3]

$L \sim B(7, 0.35)$

$P(L < 3) = P(0) + P(1) + P(2)$

$= {}^7C_0 \times 0.35^0 \times 0.65^7 + {}^7C_1 \times 0.35^1 \times 0.65^6 + {}^7C_2 \times 0.35^2 \times 0.65^5$

$= \underline{0.532}$

A random sample of 142 days is taken.

(b) Use an approximation to find the probability that the train arrives late on more than 40 days. [5]

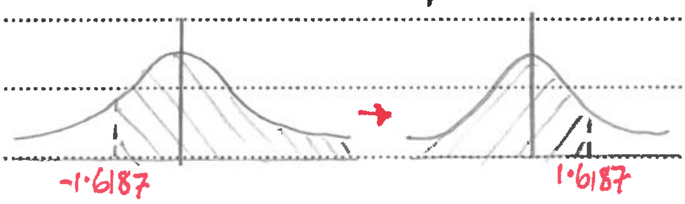
$L \sim B(142, 0.35)$

$\mu = 142 \times 0.35 = 49.7$ $\qquad \sigma^2 = 49.7(0.65) = 32.305$

$L \sim N(49.7, 32.305)$

$P(L > 40) \rightarrow P(L > 40.5)$ (continuity correction)

$P\left(Z > \frac{40.5 - 49.7}{\sqrt{32.305}}\right) = P(Z > -1.6187)$



$= \Phi(1.619)$

$= \underline{0.9472}$

5 In Greenton, 70% of the adults own a car. A random sample of 8 adults from Greenton is chosen.

(a) Find the probability that the number of adults in this sample who own a car is less than 6. [3]

$G \sim B(8, 0.7)$

$P(G < 6) = 1 - (P(6) + P(7) + P(8))$

$= 1 - ({}^8C_6 \times 0.7^6 \times 0.3^2 + {}^8C_7 \times 0.7^7 \times 0.3^1 + {}^8C_8 \times 0.7^8 \times 0.3^0)$

$= \underline{0.448}$

A random sample of 120 adults from Greenton is now chosen.

(b) Use an approximation to find the probability that more than 75 of them own a car. [5]

$G \sim B(120, 0.7)$

$\mu = 120 \times 0.7$ $\sigma^2 = 84(0.3)$

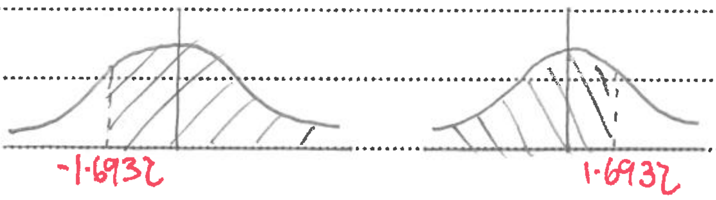
$= 84$ $= 25.2$

$G \sim N(84, 25.2)$

$P(G > 75) \rightarrow P(G > 75.5)$ (continuity correction)

$P\left(Z > \dfrac{75.5 - 84}{\sqrt{25.2}}\right)$

$= P(Z > -1.6932)$



$= \Phi(1.693)$

$= \underline{0.9548}$

5 In a large college, 28% of the students do not play any musical instrument, 52% play exactly one musical instrument and the remainder play two or more musical instruments.

A random sample of 12 students from the college is chosen.

(a) Find the probability that more than 9 of these students play at least one musical instrument. [3]

$M \sim B(12, 0.72)$

one instrument + two or more instruments

$P(M > 9) = P(10) + P(11) + P(12)$

$= {}^{12}C_{10} \times 0.72^{10} \times 0.28^{2} + {}^{12}C_{11} \times 0.72^{11} \times 0.28^{1} + {}^{12}C_{12} \times 0.72^{12} \times 0.28^{0}$

$= \underline{0.304}$

A random sample of 90 students from the college is now chosen.

(b) Use an approximation to find the probability that fewer than 40 of these students play exactly one musical instrument. [5]

$X \sim B(90, 0.52)$ ← exactly one instrument

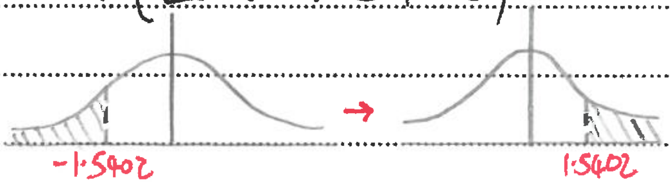
$\mu = 90 \times 0.52$ $\quad \sigma^2 = 46.8(0.48)$

$= 46.8$ $\quad = 22.464$

$P(X < 40) \rightarrow P(X < 39.5)$ (continuity correction)

$P\left(Z < \frac{39.5 - 46.8}{\sqrt{22.464}}\right)$

$= P(Z < -1.5402)$



$= 1 - \Phi(1.540)$

$= 1 - 0.9382$

$= 0.0618$

6 At a company's call centre, 90% of callers are connected immediately to a representative.

A random sample of 12 callers is chosen.

(a) Find the probability that fewer than 10 of these callers are connected immediately. [3]

$C \sim B(12, 0.9)$

$P(C < 10) = 1 - (P(10) + P(11) + P(12))$

$= 1 - ({}^{12}C_{10} \times 0.9^{10} \times 0.1^{2} + {}^{12}C_{11} \times 0.9^{11} \times 0.1 + {}^{12}C_{12} \times 0.9^{12} \times 0.1^{0})$

$= \underline{0.111}$

A random sample of 80 callers is chosen.

(b) Use an approximation to find the probability that more than 69 of these callers are connected immediately. [5]

$C \sim B(80, 0.9)$

$\mu = 80 \times 0.9$ $\quad\quad$ $\sigma^2 = 72(0.1)$

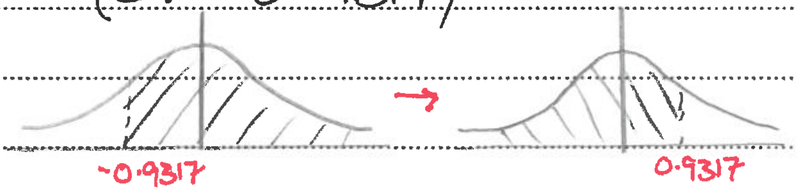
$= 72$ $\quad\quad$ $= 7.2$

$C \sim N(72, 7.2)$

$P(C > 69) \rightarrow P(C > 69.5)$ (continuity correction)

$= P\left(Z > \frac{69.5 - 72}{\sqrt{7.2}}\right)$

$= P(Z > -0.9317)$



$= \Phi(0.932)$

$= 0.8243$

(c) Justify the use of your approximation in part **(b)**. [1]

$np = 80 \times 0.9$ $\quad\quad$ $nq = 80 \times 0.1$

$= 72$ $\quad\quad$ $= 8$

Both are greater than 5 so we can use the Normal approximation

6 In Questa, 60% of the adults travel to work by car.

(a) A random sample of 12 adults from Questa is taken.

Find the probability that the number who travel to work by car is less than 10. [3]

$C \sim B(12, 0.6)$

$P(C<10) = 1 - (P(10) + P(11) + P(12))$

$= 1 - ({}^{12}C_{10} \times 0.6^{10} \times 0.4^{2} + {}^{12}C_{11} \times 0.6^{11} \times 0.4^{1} + {}^{12}C_{12} \times 0.6^{12} \times 0.4^{0})$

$= 0.917$

(b) A random sample of 150 adults from Questa is taken.

Use an approximation to find the probability that the number who travel to work by car is less than 81. [5]

$C \sim B(150, 0.6)$

$\mu = 150 \times 0.6$ $\quad \sigma^2 = 90(0.4)$

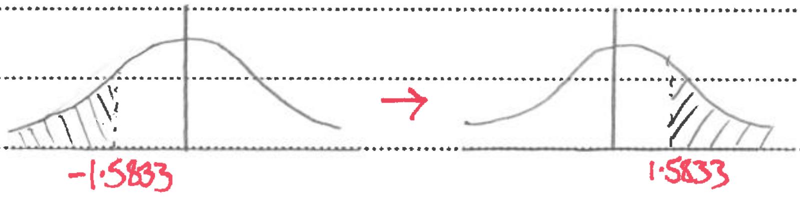
$= 90$ $\quad = 36$

$C \sim N(90, 36)$

$P(C < 81) \rightarrow P(C < 80.5)$

$P\left(Z < \frac{80.5 - 90}{\sqrt{36}}\right)$

$= P(Z < -1.5833)$



$= 1 - \Phi(1.583)$

$= 1 - 0.9433$

$= \underline{0.0567}$

(c) Justify the use of your approximation in part (b). [1]

$np = 150 \times 0.6$
$= 90$

$nq = 150 \times 0.4$
$= 60$

Both are greater than 5 so we can use the Normal approximation.

7 On any given day, the probability that Moena messages her friend Pasha is 0.72.

(a) Find the probability that for a random sample of 12 days Moena messages Pasha on no more than 9 days. [3]

$M \sim B(12, 0.72)$

$P(M \leqslant 9) = 1 - (P(10) + P(11) + P(12))$

$= 1 - ({}^{12}C_{10} \times 0.72^{10} \times 0.28^{2} + {}^{12}C_{11} \times 0.72^{11} \times 0.28^{1} + {}^{12}C_{12} \times 0.72^{12} \times 0.28^{0})$

$= \underline{0.696}$

(b) Moena messages Pasha on 1 January. Find the probability that the next day on which she messages Pasha is 5 January. [1]

$M \sim Geo(0.72)$

= Probability that first success is on fourth trial:

2nd Jan ✗ 3rd Jan ✗ 4th Jan ✗ 5th Jan ✓

$= 0.72 \times 0.28^{3}$

$= \underline{0.0158}$

(c) Use an approximation to find the probability that in any period of 100 days Moena messages Pasha on fewer than 64 days. [5]

$M \sim B(100, 0.72)$

$\mu = 100 \times 0.72 = 72$ $\qquad \sigma^2 = 72(0.28) = 20.16$

$M \sim N(72, 20.16)$

$P(M < 64) \rightarrow P(M < 63.5)$ (continuity correction)

$P\left(Z < \frac{63.5 - 72}{\sqrt{20.16}}\right)$

$= P(Z < -1.8931)$

-1.8931 $\qquad$ 1.8931

$= 1 - \Phi(1.893)$

$= 1 - 0.9708$

$= 0.0292$

5 A pair of fair coins is thrown repeatedly until a pair of tails is obtained. The random variable X denotes the number of throws required to obtain a pair of tails.

(a) Find the expected value of X. [1]

$X \sim Geo\left(\frac{1}{4}\right)$

$P(T \cap T) = \frac{1}{2} \times \frac{1}{2}$

$\mu = \frac{1}{\frac{1}{4}}$

$= 4$

(b) Find the probability that exactly 3 throws are required to obtain a pair of tails. [1]

$P(X = 3) = \frac{1}{4} \times \left(\frac{3}{4}\right)^2$

$= \frac{9}{64}$

(c) Find the probability that fewer than 6 throws are required to obtain a pair of tails. [2]

$P(X<6) = P(X \leq 5)$

$= 1 - q^5$

probability of 5 failures

$= 1 - \left(\frac{3}{4}\right)^5$

$= 0.763$

On a different occasion, a pair of fair coins is thrown 80 times.

(d) Use an approximation to find the probability that a pair of tails is obtained more than 25 times. [5]

$X \sim B(80, \frac{1}{4})$

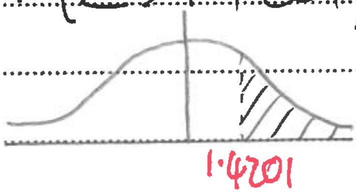
$\mu = 80 \times \frac{1}{4} = 20$ $\quad\quad \sigma^2 = 20(\frac{3}{4}) = 15$

$X \sim N(20, 15)$

$P(X > 25) \rightarrow P(X > 25.5)$ (continuity correction)

$P\left(Z > \frac{25.5 - 20}{\sqrt{15}}\right)$

$= P(Z > 1.4201)$



$= 1 - \Phi(1.420)$

$= 1 - 0.9222$

$= 0.0778$

5 The probability that a driver passes an advanced driving test is 0.3 on any given attempt.

(a) Dipak keeps taking the test until he passes. The random variable X denotes the number of attempts required for Dipak to pass the test.

(i) Find $P(2 \leqslant X \leqslant 6)$. [2]

$X \sim Geo(0.3)$

$P(2 \leq X \leq 6) = P(X \leq 6) - P(X \leq 1)$

$= (1 - q^6) - (1 - q^1)$

6 failures 1 failure

$= (1 - 0.7^6) - (1 - 0.7)$

$= 0.582$

(ii) Find $E(X)$. [1]

$E(X) = \mu = \frac{1}{0.3}$

$= \frac{10}{3}$

Five friends will each take their advanced driving test tomorrow.

(b) Find the probability that at least three of them will pass tomorrow. [3]

$X \sim B(5, 0.3)$

$P(B \geqslant 3) = P(3) + P(4) + P(5)$

$= {}^5C_3 \times 0.3^3 \times 0.7^2 + {}^5C_4 \times 0.3^4 \times 0.7 + {}^5C_5 \times 0.3^5$

$= 0.1323 + 0.02835 + 0.00243$

$= 0.163$

75 people will take their advanced driving test next week.

(c) Use an approximation to find the probability that more than 20 of them will pass next week. [5]

$X \sim B(75, 0.3)$

$\mu = 75 \times 0.3$ $\sigma^2 = 22.5 \times 0.7$

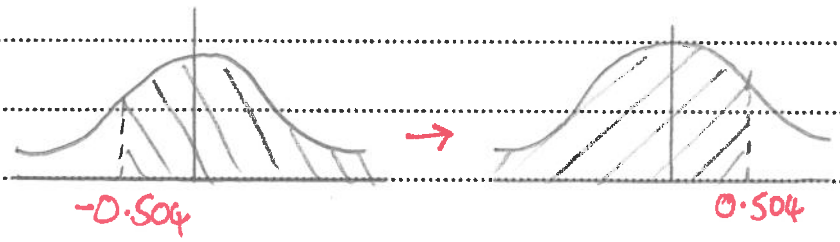
$= 22.5$ $= 15.75$

$X \sim N(22.5, 15.75)$

$P(X > 20) \rightarrow P(X > 20.5)$ (continuity correction)

$P\left(Z > \frac{20.5 - 22.5}{\sqrt{15.75}}\right)$

$= P(Z > -0.5040)$



$= \Phi(0.504)$

$= 0.6929$

5 Every day Richard takes a flight between Astan and Bejin. On any day, the probability that the flight arrives early is 0.15, the probability that it arrives on time is 0.55 and the probability that it arrives late is 0.3.

(a) Find the probability that on each of 3 randomly chosen days, Richard's flight does not arrive late. [1]

$P(L') = 0.15 + 0.55$

$= 0.7$

$P(L'L'L') = 0.7 \times 0.7 \times 0.7$

$= 0.343$

(b) Find the probability that for 9 randomly chosen days, Richard's flight arrives early at least 3 times. [3]

$F \sim B(9, 0.15)$

$P(F \geqslant 3) = 1 - (P(0) + P(1) + P(2))$

$= 1 - ({}^9C_0 \times 0.15^0 \times 0.85^9 + {}^9C_1 \times 0.15^1 \times 0.85^8 + {}^9C_2 \times 0.15^2 \times 0.85^7)$

$= 0.141$

(c) 60 days are chosen at random.

Use an approximation to find the probability that Richard's flight arrives early at least 12 times. [5]

$F \sim B(60, 0.15)$

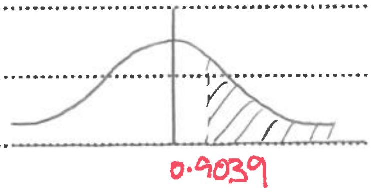
$\mu = 60 \times 0.15 = 9$ $\quad\quad \sigma^2 = 9(0.85) = 7.65$

$F \sim N(9, 7.65)$

$P(F \geq 12) \rightarrow P(F > 11.5)$ (continuity correction)

$P\left(Z > \frac{11.5 - 9}{\sqrt{7.65}}\right)$

$= P(Z > 0.9039)$



$= 1 - \Phi(0.904)$

$= 1 - 0.8169$

$= 0.1831$

6 Eli has four fair 4-sided dice with sides labelled 1, 2, 3, 4. He throws all four dice at the same time. The random variable X denotes the number of 2s obtained.

(a) Show that $P(X = 3) = \frac{3}{64}$. [2]

$X \sim B(4, \frac{1}{4})$

$P(X=3) = {}^4C_3 \times \left(\frac{1}{4}\right)^3 \times \frac{3}{4}$

$= 4 \times \frac{1}{64} \times \frac{3}{4}$

$= \frac{3}{64}$ QED

(b) Complete the following probability distribution table for X. [2]

x	0	1	2	3	4
$P(X = x)$	$\frac{81}{256}$	$27/64$	$27/128$	$\frac{3}{64}$	$\frac{1}{256}$

$P(X=1) = {}^4C_1 \times \left(\frac{1}{4}\right)^1 \times \left(\frac{3}{4}\right)^3$

$= 4 \times \frac{1}{4} \times \frac{27}{64}$

$= \frac{27}{64}$

$P(X=2) = {}^4C_2 \times \left(\frac{1}{4}\right)^2 \times \left(\frac{3}{4}\right)^2$

$= 6 \times \frac{1}{16} \times \frac{9}{16}$

$= \frac{27}{128}$

(c) Find E(X). [2]

$E(x) = 0 \times \frac{81}{256} + 1 \times \frac{27}{64} + 2 \times \frac{27}{128} + 3 \times \frac{3}{64} + 4 \times \frac{1}{256}$

$= 0 + \frac{27}{64} + \frac{9}{28} + \frac{27}{64} + \frac{1}{64}$

$= 1$

Eli throws the four dice at the same time on 96 occasions.

(d) Use an approximation to find the probability that he obtains at least two 2s on fewer than 20 of these occasions. [5]

$P(\text{at least two 2s}) = \frac{27}{128} + \frac{3}{64} + \frac{1}{256} = \frac{67}{256}$

$X \sim B\left(96, \frac{67}{256}\right)$

$\mu = 96 \times \frac{67}{256}$ $\qquad \sigma^2 = 25.125\left(\frac{189}{256}\right)$

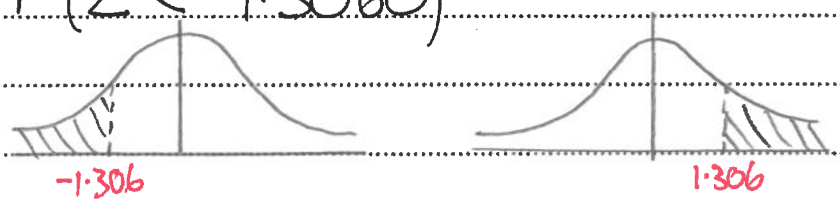
$= 25.125$ $\qquad = 18.549$ STO

$X \sim N(25.125, 18.549)$

$P(X < 20) \rightarrow P(X < 19.5)$ (continuity correction)

$P\left(Z < \frac{19.5 - 25.125}{\sqrt{18.549}}\right)$

$= P(Z < -1.3060)$



$= 1 - \Phi(1.306)$

$= 1 - 0.9042$

$= 0.0958$

7 In the region of Arka, the total number of households in the three villages Reeta, Shan and Teber is 800. Each of the households was asked about the quality of their broadband service. Their responses are summarised in the following table.

		Quality of broadband service		
		Excellent	Good	Poor
Village	Reeta	75	118	32
	Shan	223	177	40
	Teber	12	60	63

440

(a) **(i)** Find the probability that a randomly chosen household is in Shan and has poor broadband service. [1]

$\frac{40}{800} = \frac{1}{20}$

(ii) Find the probability that a randomly chosen household has good broadband service given that the household is in Shan. [2]

$P(G \cap S) = P(G|S) \times P(S)$

$P(G|S) = \frac{P(G \cap S)}{P(S)}$

$= \frac{\frac{177}{800}}{\frac{440}{800}} = \frac{177}{440}$

In the whole of Arka there are a large number of households. A survey showed that 35% of households in Arka have no broadband service.

(b) **(i)** 10 households in Arka are chosen at random.

Find the probability that fewer than 3 of these households have no broadband service. [3]

$A \sim B(10, 0.35)$

$P(A<3) = P(0) + P(1) + P(2)$

$= {}^{10}C_0 \times 0.35^0 \times 0.65^{10} + {}^{10}C_1 \times 0.35^1 \times 0.65^9 + {}^{10}C_2 \times 0.35^2 \times 0.65^8$

$= 0.262$

(ii) 120 households in Arka are chosen at random.

Use an approximation to find the probability that more than 32 of these households have no broadband service. [5]

$A \sim B(120, 0.35)$

$\mu = 120 \times 0.35$ $\quad\quad \sigma^2 = 42(0.65)$

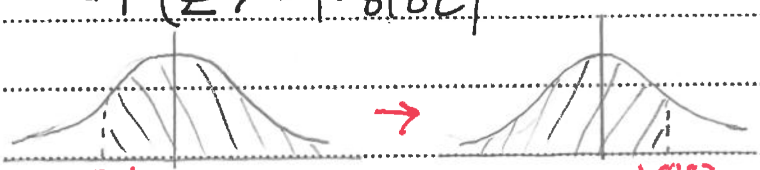
$= 42$ $\quad\quad = 27.3$

$A \sim N(42, 27.3)$

$P(A > 32) \rightarrow P(A > 32.5)$ (continuity correction)

$P\left(Z > \frac{32.5 - 42}{\sqrt{27.3}}\right)$

$= P(Z > -1.8182)$



-1.8182 $\rightarrow$ 1.8182

$= \Phi(1.818)$

$= \underline{0.9655}$

7 There are 400 students at a school in a certain country. Each student was asked whether they preferred swimming, cycling or running and the results are given in the following table.

	Swimming	Cycling	Running	
Female	104	50	66	
Male	31	57	92	180
	135			

A student is chosen at random.

(a) **(i)** Find the probability that the student prefers swimming. [1]

$\frac{135}{400} = \frac{27}{80}$

(ii) Determine whether the events 'the student is male' and 'the student prefers swimming' are independent, justifying your answer. [2]

If independent, $P(M) \times P(S) = P(M \cap S)$

$P(M) = \frac{180}{400}$

$= \frac{9}{20}$

$P(S) = \frac{27}{80}$

$P(M \cap S) = \frac{31}{400}$

$P(M) \times P(S) = \frac{9}{20} \times \frac{27}{80}$

$= \frac{243}{1600} \neq \frac{31}{400}$

so not independent

On average at all the schools in this country 30% of the students do not like any sports.

(b) **(i)** 10 of the students from this country are chosen at random.

Find the probability that at least 3 of these students do not like any sports. [3]

$S \sim B(10, 0.3)$

$P(S \geqslant 3) = 1 - (P(0) + P(1) + P(2))$

$= 1 - ({}^{10}C_0 \times 0.3^0 \times 0.7^{10} + {}^{10}C_1 \times 0.3^1 \times 0.7^9$

$+ {}^{10}C_2 \times 0.3^2 \times 0.7^8)$

$= \underline{0.617}$

(ii) 90 students from this country are now chosen at random.

Use an approximation to find the probability that fewer than 32 of them do not like any sports. [5]

$S \sim B(90, 0.3)$

$\mu = 90 \times 0.3$ $\quad\quad \sigma^2 = 27(0.7)$

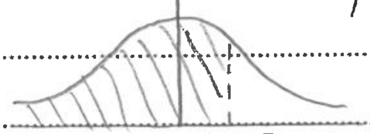
$= 27$ $\quad\quad = 18.9$

$S \sim N(27, 18.9)$

$P(S < 32) \rightarrow P(S < 31.5)$ (continuity correction)

$P\left(Z < \frac{31.5 - 27}{\sqrt{18.9}}\right)$

$= P(Z < 1.0351)$



1·0351

$= \Phi(1.035) = \underline{0.8497}$

