1 Becky sometimes works in an office and sometimes works at home. The random variable X denotes the number of days that she works at home in any given week. It is given that

$$P(X = x) = kx(x + 1),$$

where k is a constant and $x = 1, 2, 3$ or 4 only.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

$x = 1: \ k \times 1 \times (1+1) = k \times 2 = 2k$

$x = 2: \ k \times 2 \times (2+1) = 2k \times 3 = 6k$

$x = 3: \ k \times 3 \times (3+1) = 3k \times 4 = 12k$

$x = 4: \ k \times 4 \times (4+1) = 4k \times 5 = 20k$

$\Sigma P = 1: \ 2k + 6k + 12k + 20k = 1$

$40k = 1$

$k = 0.025$

x	1	2	3	4
$P(X=x)$	0.05	0.15	0.3	0.5

(b) Find $E(X)$ and $Var(X)$. [3]

$E(X) = 1 \times 0.05 + 2 \times 0.15 + 3 \times 0.3 + 4 \times 0.5$

$= 0.05 + 0.3 + 0.9 + 2$

$= 3.25$ (note: mistake in mark-scheme: 3.05 is incorrect)

$Var(X) = 1^2 \times 0.05 + 2^2 \times 0.15 + 3^2 \times 0.3 + 4^2 \times 0.5 - (E(X))^2$

$= 0.05 + 0.6 + 2.7 + 8 - (3.25)^2$

$= 11.35 - 10.5625$

$= 0.7875$

2 The weights of large bags of pasta produced by a company are normally distributed with mean 1.5 kg and standard deviation 0.05 kg.

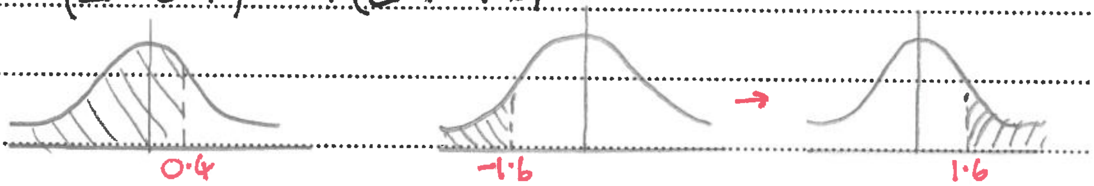
(a) Find the probability that a randomly chosen large bag of pasta weighs between 1.42 kg and 1.52 kg. [3]

$P(1.42 < W < 1.52)$

$P\left(\frac{1.42 - 1.5}{0.05} < Z < \frac{1.52 - 1.5}{0.05}\right)$

$P(-1.6 < Z < 0.4)$

$= P(Z < 0.4) - P(Z < -1.6)$



$= \Phi(0.4) - (1 - \Phi(1.6))$

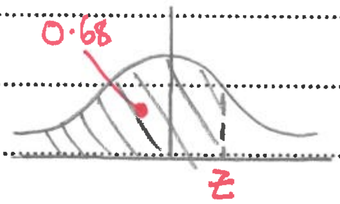
$= 0.6554 - (1 - 0.9452) \quad = 0.6006$

The weights of small bags of pasta produced by the company are normally distributed with mean 0.75 kg and standard deviation σ kg. It is found that 68% of these small bags have weight less than 0.9 kg.

(b) Find the value of σ. [3]

$P(W < 0.9) = 0.68$

$P\left(Z < \frac{0.9 - 0.75}{\sigma}\right) = 0.68$



$0.68 = \Phi(0.468)$ from table (nearest value)

$Z = 0.468$

$\frac{0.9 - 0.75}{\sigma} = 0.468$

$0.15 = 0.468\sigma$

$\sigma = 0.321$

3 Tim has two bags of marbles, A and B.

Bag A contains 8 white, 4 red and 3 yellow marbles.

Bag B contains 6 white, 7 red and 2 yellow marbles.

Tim also has an ordinary fair 6-sided dice. He rolls the dice. If he obtains a 1 or 2, he chooses two marbles at random from bag A, without replacement. If he obtains a 3, 4, 5 or 6, he chooses two marbles at random from bag B, without replacement.

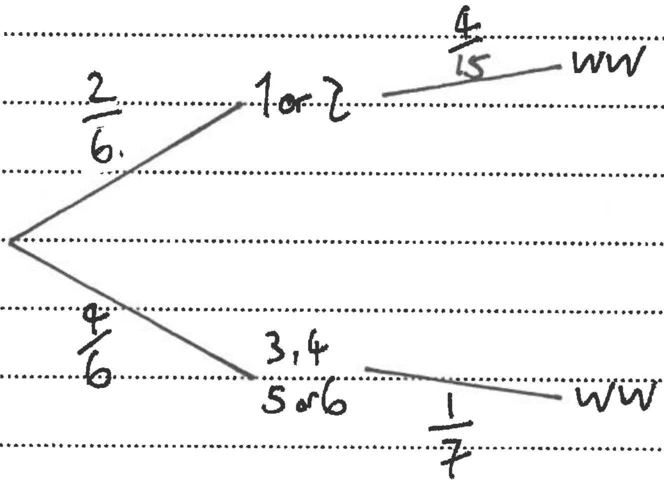
(a) Find the probability that both marbles are white. [3]

Bag A:

$P(WW) = \frac{8}{15} \times \frac{7}{14} = \frac{4}{15}$

Bag B:

$P(WW) = \frac{6}{15} \times \frac{5}{14} = \frac{1}{7}$



$\frac{2}{6} \times \frac{4}{15} = \frac{4}{45}$

$\frac{4}{6} \times \frac{1}{7} = \frac{2}{21}$

$\frac{4}{45} + \frac{2}{21} = \frac{58}{315}$

(b) Find the probability that the two marbles come from bag B given that one is white and one is red. [4]

$P(B \cap WR) = P(B|WR) \times P(WR)$

or RW

$P(B|WR) = \dfrac{P(B \cap WR)}{P(WR)}$

Bag A:

$P(WR)$ in any order $= P(WR) + P(RW)$

$= \frac{8}{15} \times \frac{4}{14} + \frac{4}{15} \times \frac{8}{14}$

$= \frac{32}{105}$

or RW

$P(A \cap WR) = \frac{2}{6} \times \frac{32}{105}$

$= \frac{32}{315}$

Bag B:

$P(WR)$ in any order $= P(WR) + P(RW)$

$= \frac{6}{15} \times \frac{7}{14} + \frac{7}{15} \times \frac{6}{14}$

$= \frac{2}{5}$

or RW

$P(B \cap WR) = \frac{4}{6} \times \frac{2}{5}$

$= 4/15$

$P(B|WR) = \dfrac{\frac{4}{15}}{\frac{32}{315} + \frac{4}{15}}$

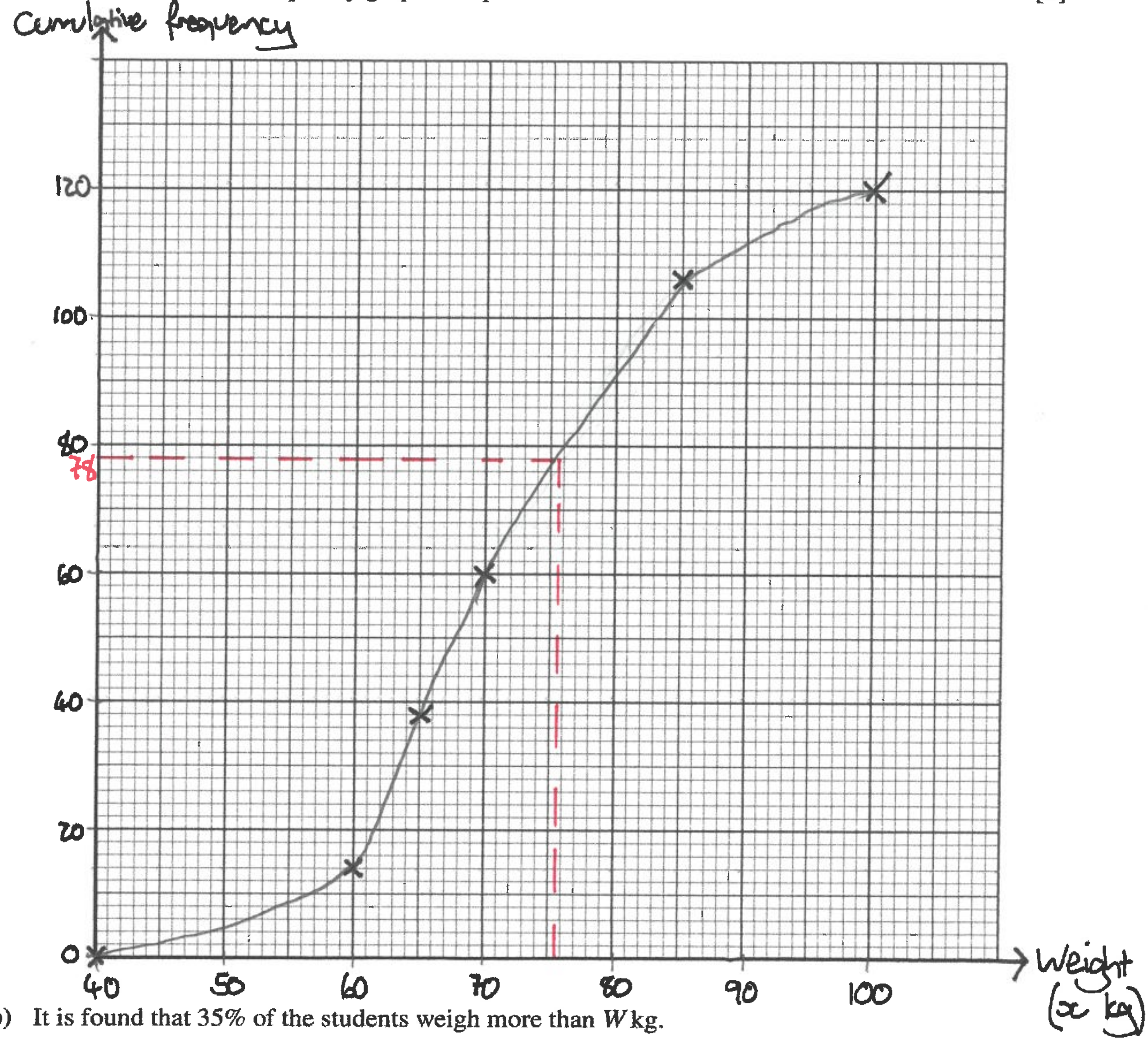
or RW

$= \frac{21}{29}$

4 The weights, x kg, of 120 students in a sports college are recorded. The results are summarised in the following table.

Weight (x kg)	$x \leqslant 40$	$x \leqslant 60$	$x \leqslant 65$	$x \leqslant 70$	$x \leqslant 85$	$x \leqslant 100$
Cumulative frequency	0	14	38	60	106	120

(a) Draw a cumulative frequency graph to represent this information. [2]



(b) It is found that 35% of the students weigh more than W kg.

Use your graph to estimate the value of W. [2]

35% are more than W, so 65% are less than W.

$0.65 \times 120 = 78$

$W \simeq 75.5$ kg

(c) Calculate estimates for the mean and standard deviation of the weights of the 120 students. [6]

Mid-point (x)	Frequency (f)	$f \times x$
20	0	0
50	14	700
62.5	24	1500
67.5	22	1485
77.5	46	3565
92.5	14	1295
	$\Sigma f = 120$	$\Sigma fx = 8545$

$$\bar{x} = \frac{8545}{120}$$

$$= \underline{71.2 \text{ kg}}$$ STO

unrounded

$$\text{Var} = \frac{50^2 \times 14 + 62.5^2 \times 24 + 67.5^2 \times 22 + 77.5^2 \times 46 + 92.5^2 \times 14}{120} - 71.2^2$$

$$\text{Var} = 138.23$$

$$\sigma = \sqrt{\text{Var}}$$

$$= \underline{11.8}$$

5 The probability that a driver passes an advanced driving test is 0.3 on any given attempt.

(a) Dipak keeps taking the test until he passes. The random variable X denotes the number of attempts required for Dipak to pass the test.

(i) Find $P(2 \leqslant X \leqslant 6)$. [2]

$X \sim Geo(0.3)$

$P(2 \leqslant X \leqslant 6) = P(X \leqslant 6) - P(X \leqslant 1)$

$= (1 - q^6) - (1 - q^1)$

6 failures 1 failure

$= (1 - 0.7^6) - (1 - 0.7)$

$= \underline{0.582}$

(ii) Find $E(X)$. [1]

$E(X) = \mu = \frac{1}{0.3}$

$= \underline{\frac{10}{3}}$

Five friends will each take their advanced driving test tomorrow.

(b) Find the probability that at least three of them will pass tomorrow. [3]

$X \sim B(5, 0.3)$

$P(B \geqslant 3) = P(3) + P(4) + P(5)$

$= {}^5C_3 \times 0.3^3 \times 0.7^2 + {}^5C_4 \times 0.3^4 \times 0.7 + {}^5C_5 \times 0.3^5$

$= 0.1323 + 0.02835 + 0.00243$

$= \underline{0.163}$

75 people will take their advanced driving test next week.

(c) Use an approximation to find the probability that more than 20 of them will pass next week. [5]

$X \sim B(75, 0.3)$

$\mu = 75 \times 0.3$ $\qquad$ $\sigma^2 = 22.5 \times 0.7$

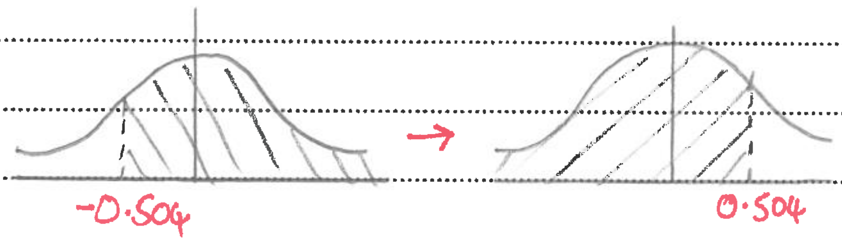
$= 22.5$ $\qquad$ $= 15.75$

$X \sim N(22.5, 15.75)$

$P(X > 20) \rightarrow P(X > 20.5)$ (continuity correction)

$P\left(Z > \frac{20.5 - 22.5}{\sqrt{15.75}}\right)$

$= P(Z > -0.5040)$



$= \Phi(0.504)$

$= 0.6929$

6 Jai and his wife Kaz are having a party. Jai has invited five friends and each friend will bring his wife.

(a) At the beginning of the party, the 12 people will stand in a line for a photograph.

(i) How many different arrangements are there of the 12 people if Jai stands next to Kaz and each friend stands next to his own wife? [3]

Treat each couple as one object:

M_1W_1 M_2W_2 M_3W_3 M_4W_4 M_5W_5 M_6W_6

$6! = 720$

But in each couple, they could be standing MW or WM, so:

$6! \times 2! \times 2! \times 2! \times 2! \times 2! \times 2! = \underline{46080}$

(ii) How many different arrangements are there of the 12 people if Jai and Kaz occupy the two middle positions in the line, with Jai's five friends on one side and the five wives of the friends on the other side? [2]

2! (JK or KJ)

5! (friends in any order) 5! (wives in any order)

But the friends could be either on the left, with the wives on the right, or vice-versa, so $\times 2!$

$\rightarrow 2! \times 5! \times 5! \times 2! = \underline{57600}$

(b) For a competition during the party, the 12 people are divided at random into a group of 5, a group of 4 and a group of 3.

Find the probability that Jai and Kaz are in the same group as each other. [5]

J and K in group of 5:

group of 3

$${}^{2}C_{2} \times {}^{10}C_{3} \times {}^{7}C_{4} \times {}^{3}C_{3} = 4200$$

J and K

3 more people for group of 5

pick 4 people from remaining 7 for group of 4

J and K in group of 4:

$$\underbrace{{}^{2}C_{2} \times {}^{10}C_{2}}_{\text{group of 4}} \times \underbrace{{}^{8}C_{5}}_{\text{group of 5}} \times \underbrace{{}^{3}C_{3}}_{\text{group of 3}} = 2520$$

J and K in group of 3:

$$\underbrace{{}^{2}C_{2} \times {}^{10}C_{1}}_{\text{group of 3}} \times \underbrace{{}^{9}C_{5}}_{\text{group of 5}} \times \underbrace{{}^{4}C_{4}}_{\text{group of 4}} = 1260$$

Number of ways with J and K together $= 4200 + 2520 + 1260$
$= \underline{7980}$

Without any restrictions:

$$\underbrace{{}^{12}C_{5}}_{\text{group of 5}} \times \underbrace{{}^{7}C_{4}}_{\text{group of 4}} \times \underbrace{{}^{3}C_{3}}_{\text{group of 3}} = 27\ 720$$

$$\text{probability} = \frac{7980}{27\ 720} = \underline{\frac{19}{66}}$$