1 A competitor in a throwing event has three attempts to throw a ball as far as possible. The random variable X denotes the number of throws that exceed 30 metres. The probability distribution table for X is shown below.

x	0	1	2	3
$P(X = x)$	0.4	p	r	0.15

(a) Given that $E(X) = 1.1$, find the value of p and the value of r. [3]

$\sum P = 1$: $0.4 + p + r + 0.15 = 1$

$0.55 + p + r = 1$

$p + r = 0.45$ (1)

$E(X) = 1.1$:

$0 \times 0.4 + 1 \times p + 2 \times r + 3 \times 0.15 = 1.1$

$0 + p + 2r + 0.45 = 1.1$

$p + 2r = 0.65$ (2)

(2) − (1): $\underline{r = 0.2}$

→ (1): $p + 0.2 = 0.45$

$\underline{p = 0.25}$

(b) Find the numerical value of $\text{Var}(X)$. [2]

$\text{Var}(X) = 0^2 \times 0.4 + 1^2 \times 0.25 + 2^2 \times 0.2 + 3^2 \times 0.15 - (E(x))^2$

$= 0 + 0.25 + 0.8 + 1.35 - 1.1^2$

$= 2.4 - 1.21$

$= \underline{1.19}$

2 George has a fair 5-sided spinner with sides labelled 1, 2, 3, 4, 5. He spins the spinner and notes the number on the side on which the spinner lands.

(a) Find the probability that it takes fewer than 7 spins for George to obtain a 5. [2]

$P(5) = \frac{1}{5}$

$X \sim Geo\left(\frac{1}{5}\right)$

$P(X < 7) = P(X \leq 6)$

$= 1 - q^6$

← probability of 6 failures

$= 1 - \left(\frac{4}{5}\right)^6$

$= 0.738$

George spins the spinner 10 times.

(b) Find the probability that he obtains a 5 more than 4 times but fewer than 8 times. [3]

$X \sim B\left(10, \frac{1}{5}\right)$

$P(4 < X < 8) = P(5 \leq X \leq 7)$

$= P(5) + P(6) + P(7)$

$= {}^{10}C_5 \times \left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^5 + {}^{10}C_6 \times \left(\frac{1}{5}\right)^6\left(\frac{4}{5}\right)^4 + {}^{10}C_7 \times \left(\frac{1}{5}\right)^7\left(\frac{4}{5}\right)^3$

$= 0.0327$

3 A factory produces a certain type of electrical component. It is known that 15% of the components produced are faulty. A random sample of 200 components is chosen.

Use an approximation to find the probability that more than 40 of these components are faulty. [5]

$X \sim B(200, 0.15)$

$\mu = 200 \times 0.15$ $\quad\quad$ $\sigma^2 = 30 \times 0.85$

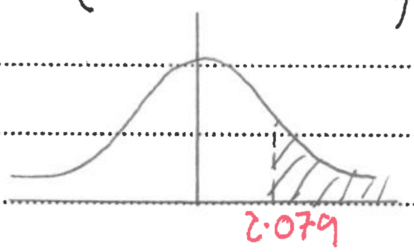
$= 30$ $\quad\quad$ $= 25.5$

$X \sim N(30, 25.5)$

$P(X > 40) \rightarrow P(X > 40.5)$ (continuity correction)

$P\left(Z > \frac{40.5 - 30}{\sqrt{25.5}}\right)$

$= P(Z > 2.079)$



$= 1 - \Phi(2.079)$

$= 1 - 0.9812$

$= 0.0188$

4 The heights, in cm, of the 11 players in each of two teams, the Aces and the Jets, are shown in the following table.

Aces	180	174	169	182	181	166	173	182	168	171	164
Jets	175	174	188	168	166	174	181	181	170	188	190

(a) Draw a back-to-back stem-and-leaf diagram to represent this information with the Aces on the left-hand side of the diagram. [4]

Aces Q_1		Jets
9 8 (Q_1) 6 4	16	6 8
4 3 (Q_2) 1	17	0 4 4 5
2 2 1 (Q_3) 0	18	1 1 8 8
	19	0

Key: 4 | 16 | 6 means 164 cm for the Aces and 166 cm for the Jets

(b) Find the median and the interquartile range of the heights of the players in the Aces. [3]

Q_1: $\frac{11+1}{4} = 3^{rd}$ Q_2: $\frac{11+1}{2} = 6^{th}$ Q_3: $\frac{3(11+1)}{4} = 9^{th}$

$Q_1 = 168$ $Q_2 = 173$ $Q_3 = 181$

Median = 173cm IQR = 181 − 168

= 13cm

(c) Give one comment comparing the spread of the heights of the Aces with the spread of the heights of the Jets. [1]

Range for Aces = 18cm Range for Jets = 24cm

So the Jets' heights are more spread than the Aces.

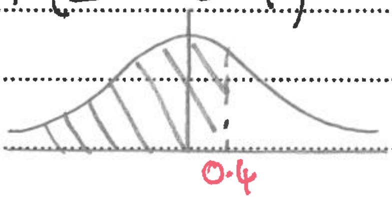
5 (a) The heights of the members of a club are normally distributed with mean 166 cm and standard deviation 10 cm.

(i) Find the probability that a randomly chosen member of the club has height less than 170 cm. [2]

$P(H < 170)$

$P\left(Z < \frac{170 - 166}{10}\right)$

$P(Z < 0.4)$



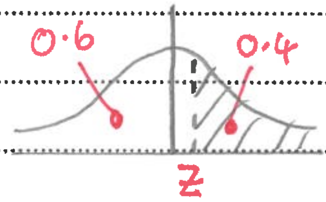
$= \Phi(0.4)$

$= \underline{0.6554}$

(ii) Given that 40% of the members have heights greater than h cm, find the value of h correct to 2 decimal places. [3]

$P(H > h) = 0.4$

$P\left(Z > \frac{h - 166}{10}\right) = 0.4$



$0.6 = \Phi(0.253)$ from table (nearest value)

$z = 0.253$

$\frac{h - 166}{10} = 0.253$

$h - 166 = 2.53$

$\underline{h = 168.53 \text{ cm}}$

(b) The random variable X is normally distributed with mean μ and standard deviation σ.

Given that $\sigma = \frac{2}{3}\mu$, find the probability that a randomly chosen value of X is positive. [3]

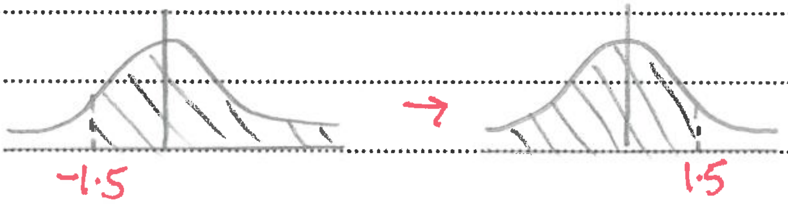
$P(X > 0)$

$P\left(Z > \frac{0 - \mu}{\frac{2\mu}{3}}\right)$

$P\left(Z > \frac{-\mu \times 3}{\frac{2\mu}{3} \times 3}\right)$

$P\left(Z > \frac{-3\cancel{\mu}}{2\cancel{\mu}}\right)$

$P(Z > -1.5)$



$P(Z > -1.5) = P(Z < 1.5)$

$= \Phi(1.5)$

$= \underline{0.9332}$

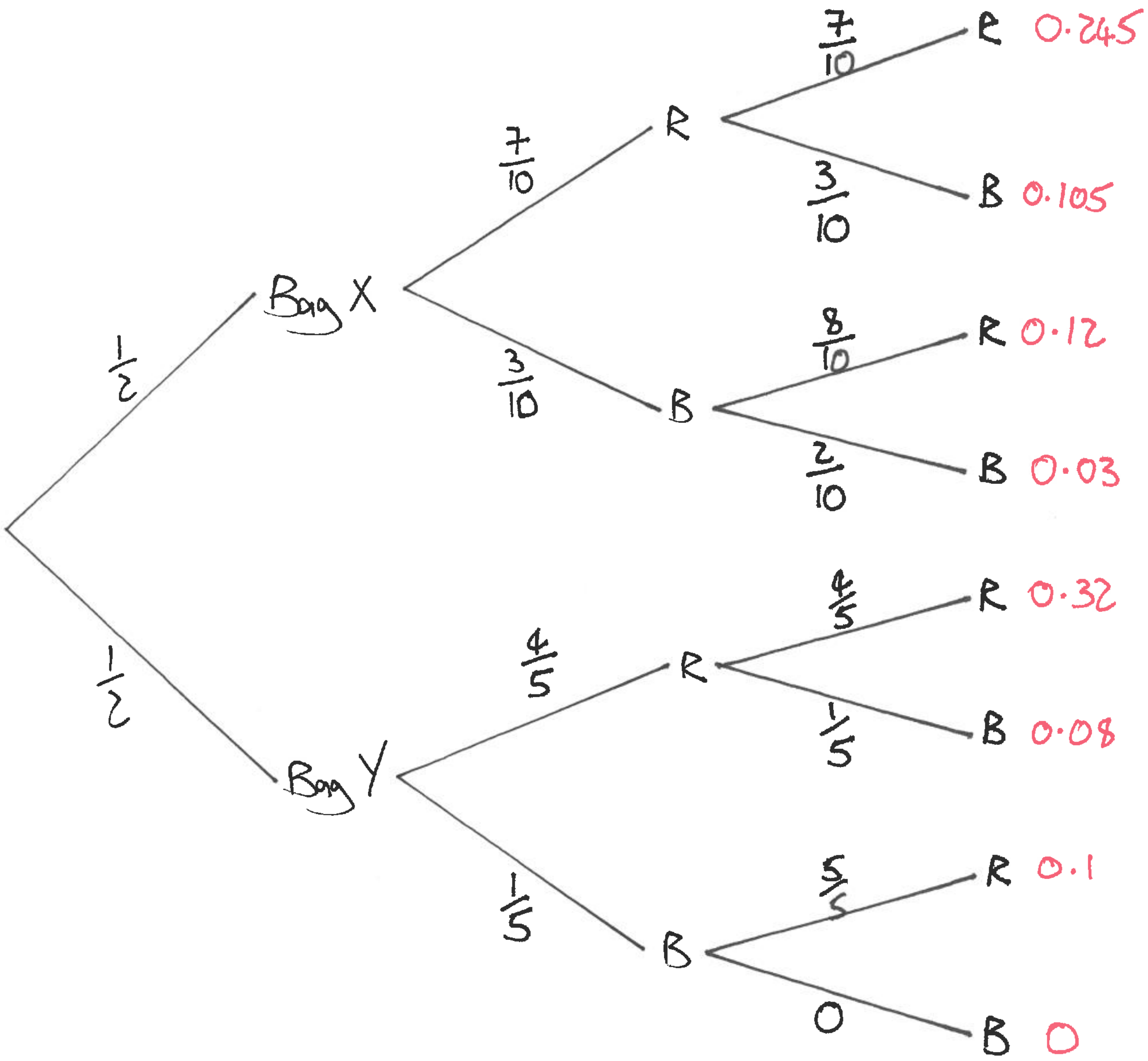
6 Freddie has two bags of marbles.

Bag X contains 7 red marbles and 3 blue marbles.

Bag Y contains 4 red marbles and 1 blue marble.

Freddie chooses one of the bags at random. A marble is removed at random from that bag and not replaced. A new red marble is now added to each bag. A second marble is then removed at random from the same bag that the first marble had been removed from.

(a) Draw a tree diagram to represent this information, showing the probability on each of the branches. [3]



(b) Find the probability that both of the marbles removed from the bag are the same colour. [4]

$P(RR)_X + P(BB)_X + P(RR)_Y + P(BB)_Y$

$= 0.245 + 0.03 + 0.32 + 0$

$= 0.595$

(c) Find the probability that bag Y is chosen given that the marbles removed are **not** both the same colour. [2]

$P(Y \cap \text{Not Same}) = P(Y \mid \text{Not Same}) \times P(\text{Not Same})$

$P(Y \mid \text{Not Same}) = \dfrac{P(Y \cap \text{Not Same})}{P(\text{Not Same})}$

$P(Y \cap \text{Not Same}) = 0.08 + 0.1$
$= 0.18$

$P(\text{Not Same}) = 0.105 + 0.12 + 0.08 + 0.1$
$= 0.405$

$P(Y \mid \text{Not Same}) = \dfrac{0.18}{0.405}$

$= \dfrac{4}{9}$

7 **(a)** Find the number of different arrangements of the 9 letters in the word ANDROMEDA in which no consonant is next to another consonant. (The letters D, M, N and R are consonants and the letters A, E and O are **not** consonants.) [3]

AANDDROME

Arrange vowels: __ __ __ __

$$\frac{4!}{2!} = 12$$

2 As → 2!

Arrange consonants in spaces between vowels:

↑ V ↑ V ↑ V ↑ V ↑

5 spaces → $\frac{^5P_5}{2!}$ ← 5 consonants $= 60$

2 Ds → 2!

$12 \times 60 = 720$

(b) Find the number of different arrangements of the 9 letters in the word ANDROMEDA in which there is an A at each end and the Ds are **not** together. [3]

With Ds together:

A __ __ __ __ __ __ A

fixed DD fixed

one object →

$6! = 720$

With no restrictions (except for As at ends):

A __ __ __ __ __ __ __ A

fixed fixed

$$\frac{7!}{2!} = 2520$$

2Ds → 2!

$2520 - 720 = 1800$

Four letters are selected at random from the 9 letters in the word ANDROMEDA.

(c) Find the probability that this selection contains at least one D and exactly one A. [4]

We are picking letters at random, not looking for different selections, so the As and Ds are treated as distinguishable.

1 D from 2

A D _ _ $\quad {}^2C_1 \times {}^2C_1 \times {}^5C_2 = 40$

1 A from 2 $\qquad$ 2 from NROME

A D D _ $\quad {}^2C_1 \times {}^2C_2 \times {}^5C_1 = 10$

$40 + 10 = 50$

Number of selections without restrictions $= {}^9C_4$

$= 126$

Probability $= \dfrac{50}{126}$

$= \dfrac{25}{63}$