1 A particle $P$ is projected vertically upwards with speed $u\,\mathrm{m\,s^{-1}}$ from a point on the ground. $P$ reaches its greatest height after 3 s.

(a) Find $u$. [1]

↑+ $s =$

$u =$

$v = 0$

$a = -10$

$t = 3$

$v = u + at$

$0 = u + (-10) \times 3$

$0 = u - 30$

$u = \underline{\underline{30\,\mathrm{ms^{-1}}}}$

(b) Find the greatest height of $P$ above the ground. [2]

↑+ $s =$

$u = 30$

$v = 0$

$a = -10$

$t = 3$

$s = \frac{1}{2}(u + v) \times t$

$= \frac{1}{2}(30 + 0) \times 3$

$= \frac{1}{2} \times 30 \times 3$

$= \underline{\underline{45\,\mathrm{m}}}$

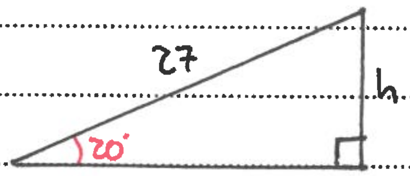
**2** A box of mass 5 kg is pulled at a constant speed of $1.8\,\mathrm{m\,s^{-1}}$ for 15 s up a rough plane inclined at an angle of 20° to the horizontal. The box moves along a line of greatest slope against a frictional force of 40 N. The force pulling the box is parallel to the line of greatest slope.

**(a)** Find the change in gravitational potential energy of the box. [2]

Change in height: Constant speed, so distance = speed × time

$= 1.8 \times 15$

$= 27\,\mathrm{m}$




$\sin 20 = \frac{h}{27}$

$h = \underline{27 \sin 20}$

Change in PE $= mg\Delta h$

$= 5 \times 10 \times 27 \sin 20$

$= \underline{462\,\mathrm{J}}$

STO

**(b)** Find the work done by the pulling force. [2]

$\text{Work}_{\text{in}} + KE_{\text{init}} + PE_{\text{init}} = KE_{\text{fin}} + PE_{\text{fin}} + \text{Work}_{\text{out}}$

$W + \frac{1}{2} \times 5 \times 1.8^2 + 0 = \frac{1}{2} \times 5 \times 1.8^2 + 462 + 40 \times 27$

work = F×d

$W + \cancel{8}.1 = \cancel{8}.1 + 462 + 1080$

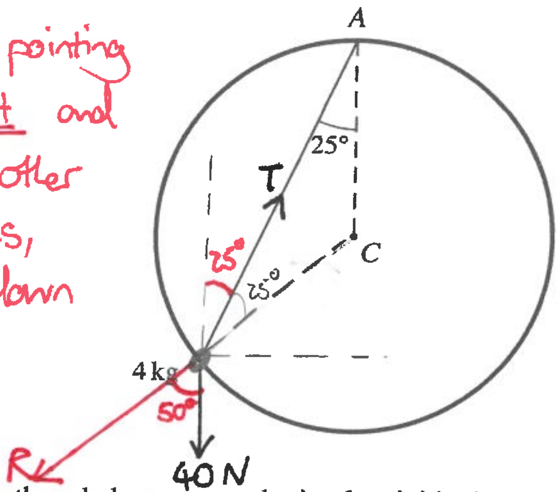
$W = 462 + 1080$

$= \underline{1540\,\mathrm{J}}$ (3sf)

R is perpendicular to surface

3

Since T is pointing up and right and there are no other horizontal forces, R must be down and left.




A ring of mass 4 kg is threaded on a smooth circular rigid wire with centre $C$. The wire is fixed in a vertical plane and the ring is kept at rest by a light string connected to $A$, the highest point of the circle. The string makes an angle of 25° to the vertical (see diagram).

Find the tension in the string and the magnitude of the normal reaction of the wire on the ring. [6]

$R(\rightarrow): \; T\sin 25 - R\sin 50 = 0$

$T\sin 25 = R\sin 50$

$T = \dfrac{R\sin 50}{\sin 25}$ ①

$R(\uparrow): \; T\cos 25 - 40 - R\cos 50 = 0$ ②

Sub ① into ②:

$\dfrac{R\sin 50\cos 25}{\sin 25} - 40 - R\cos 50 = 0$

$R\left(\dfrac{\sin 50\cos 25}{\sin 25} - \cos 50\right) = 40$

$R = 40 \div \left(\dfrac{\sin 50\cos 25}{\sin 25} - \cos 50\right)$

$R = 40\,N$

Sub into ①: $T = \dfrac{40\sin 50}{\sin 25}$

$T = 72.5\,N$

4 A particle $P$ travels in the positive direction along a straight line with constant acceleration. $P$ travels a distance of 52 m during the 2nd second of its motion and a distance of 64 m during the 4th second of its motion.

(a) Find the initial speed and the acceleration of $P$. [5]

$t=1$:

$S=$
$u=u$
$V=$
$a=a$
$t=1$

$S_1 = ut + \frac{1}{2}at^2$
$S_1 = u\times 1 + \frac{1}{2}\times a\times 1^2$
$S_1 = u + \frac{1}{2}a$ (1)

$t=2$:

$S=S_1+52$
$u=u$
$V=$
$a=a$
$t=2$

$S = ut + \frac{1}{2}at^2$
$S_1+52 = 2u + \frac{1}{2}\times a\times 2^2$
$S_1+52 = 2u + 2a$ (2)

Sub. (1) into (2): $u + \frac{1}{2}a + 52 = 2u + 2a$
$\frac{1}{2}a + 52 = u + 2a$
$52 = u + 1.5a$ (A)

$t=3$:

$S=$
$u=u$
$V=$
$a=a$
$t=3$

$S_3 = ut + \frac{1}{2}at^2$
$S_3 = 3u + \frac{1}{2}\times a\times 3^2$
$S_3 = 3u + 4.5a$ (3)

$t=4$:

$S=S_3+64$
$u=u$
$V=$
$a=a$
$t=4$

$S = ut + \frac{1}{2}at^2$
$S_3+64 = 4u + \frac{1}{2}\times a\times 4^2$
$S_3+64 = 4u + 8a$ (4)

Sub. (3) into (4): $3u + 4.5a + 64 = 4u + 8a$
$4.5a + 64 = u + 8a$
$64 = u + 3.5a$ (B)

continued →

B − A: $12 = 2a$

$a = \underline{6\ ms^{-2}}$

sub into A: $52 = u + 1.5 \times 6$

$52 = u + 9$

$43 = u$

$u = \underline{43 ms^{-1}}$

horrible question!

**(b)** Find the distance travelled by $P$ during the first 10 seconds of its motion. [2]

| | |
|---|---|
| $s =$ | $s = ut + \frac{1}{2}at^2$ |
| $u = 43$ | $= 43 \times 10 + \frac{1}{2} \times 6 \times 10^2$ |
| $v =$ | $= 430 + 300$ |
| $a = 6$ | $= \underline{730 m}$ |
| $t = 10$ | |

5 Particles $X$ and $Y$ move in a straight line through points $A$ and $B$. Particle $X$ starts from rest at $A$ and moves towards $B$. At the same instant, $Y$ starts from rest at $B$.

At time $t$ seconds after the particles start moving

- the acceleration of $X$ in the direction $AB$ is given by $(12t + 12)\,\mathrm{m\,s^{-2}}$,
- the acceleration of $Y$ in the direction $AB$ is given by $(24t - 8)\,\mathrm{m\,s^{-2}}$.

**(a)** It is given that the velocities of $X$ and $Y$ are equal when they collide.

Calculate the distance $AB$. [6]

X: $V_x = \int (12t + 12)\,dt$
$= 6t^2 + 12t + C$

$V_x = 0$ when $t = 0$:
$0 = 0 + 0 + C$
$\rightarrow V_x = 6t^2 + 12t$

Y: $V_Y = \int (24t - 8)\,dt$
$= 12t^2 - 8t + C$

$V_Y = 0$ when $t = 0$:
$0 = 0 - 0 + C$
$\rightarrow V_Y = 12t^2 - 8t$

Find time when velocities are equal: $V_x = V_Y$:

$6t^2 + 12t = 12t^2 - 8t$
$-6t^2 + 20t = 0$
$t(-6t + 20) = 0$
$t = 0$ or $-6t + 20 = 0$
$6t = 20$
$t = \frac{10}{3}\,\mathrm{s}$

Find displacement of each particle at $t = \frac{10}{3}$:

X: (from A):
$S_x = \int (6t^2 + 12t)\,dt$
$= 2t^3 + 6t^2 + C$

$S_x = 0$ when $t = 0$:
$0 = 0 + 0 + C$
$\rightarrow S_x = 2t^3 + 6t^2$

Y: (from B):
$S_Y = \int (12t^2 - 8t)\,dt$
$= 4t^3 - 4t^2 + C$

$S_Y = 0$ when $t = 0$:
$0 = 0 - 0 + C$
$\rightarrow S_Y = 4t^3 - 4t^2$

continued..




$t = \frac{10}{3}$:

$S_x = 2\left(\frac{10}{3}\right)^3 + 6\left(\frac{10}{3}\right)^2$

$= \frac{2000}{27} + \frac{200}{3}$

$= \frac{3800}{27}$ m

$t = \frac{10}{3}$:

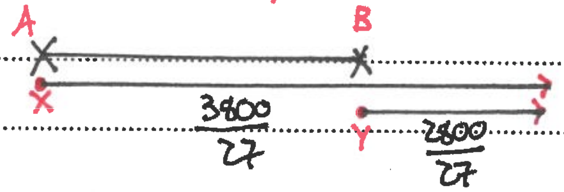
$S_y = 4\left(\frac{10}{3}\right)^3 - 4\left(\frac{10}{3}\right)^2$

$= \frac{4000}{27} - \frac{400}{9}$

$= \frac{2800}{27}$ m

Notice that both particles have moved in the same direction (both positive answers)

So:




So $AB = \frac{3800}{27} - \frac{2800}{27}$

$= \frac{1000}{27}$

$= 37.0$ m

**(b)** It is given instead that $AB = 36$ m.

Verify that $X$ and $Y$ collide after 3 s. [2]

If they collide after 3s, X will have travelled 36m more than Y to have caught it:

$S_x$ when $t = 3$:

$S_x = 2(3)^3 + 6(3)^2$

$= 54 + 54$

$= 108$m

$S_y$ when $t = 3$:

$S_y = 4(3)^3 - 4(3)^2$

$= 108 - 36$

$= 72$m

$S_x - S_y = 108 - 72$

$= 36$m, so they are in the same place after 3s. QED.

**6** A car of mass 1750 kg is pulling a caravan of mass 500 kg. The car and the caravan are connected by a light rigid tow-bar. The resistances to the motion of the car and caravan are 650 N and 150 N respectively.

**(a)** The car and caravan are moving along a straight horizontal road at a constant speed of $24 \text{ m s}^{-1}$.

**(i)** Find the power of the car's engine. [2]

Constant speed, so $a=0$ and Driving Force = Resistive Forces.

Whole system:

$a=0$

$R(\rightarrow): D - 650 - 150 = ma$

$D - 800 = 0$

$D = 800\text{N}$

$\text{Power} = D \times V$

$= 800 \times 24$

$= \underline{19\,200\text{W}}$

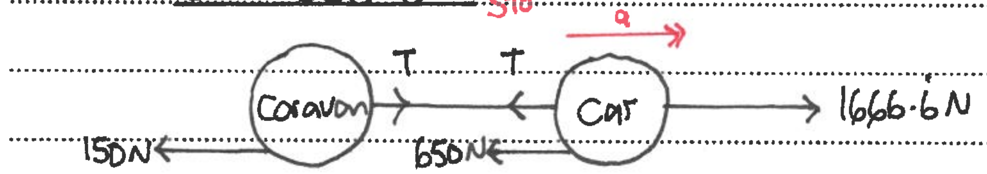
**(ii)** The engine's power is now suddenly increased to 40 kW.

Find the instantaneous acceleration of the car and caravan and find the tension in the tow-bar. [5]

$\text{Power} = D \times V$

$40\,000 = D \times 24$

$\underline{D = 1666.\dot{6}\text{N}}$ STO




Car:

$R(\rightarrow): F = ma$

$1666.\dot{6} - 650 - T = 1750a$

$1016.\dot{6} - T = 1750a$ (1)

Caravan:

$R(\rightarrow): F = ma$

$T - 150 = 500a$ (2)

cont.

①+②: $1016.\dot{6} - 150 = 2250a$

$866.\dot{6} = 2250a$

$\underline{a = 0.385\ ms^{-2}}$

→②:

$T - 150 = 500 \times 0.385$

$T - 150 = 192.6$

$T = 342.6$

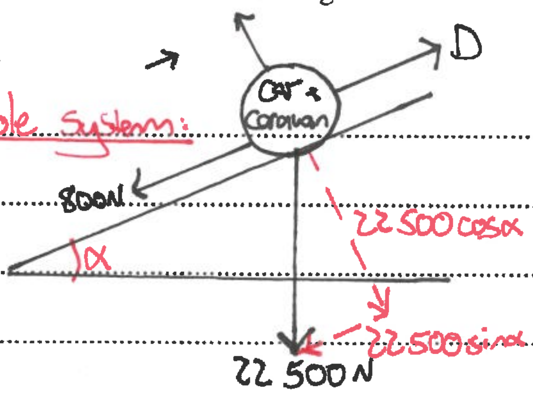
$= \underline{343\ N}$

$\sin\alpha = 0.14$

**(b)** The car and caravan now travel up a straight hill, inclined at an angle $\sin^{-1} 0.14$ to the horizontal, at a constant speed of $v\ \mathrm{m\,s^{-1}}$. The car's engine is working at 31 kW. The resistances to the motion of the car and caravan are unchanged.

Find $v$. [3]

Whole System:




$R(\nearrow)$: $F = ma$ ← $a = 0$ (constant velocity)

$D - 800 - 22500\sin\alpha = 0$

$D - 800 - 22500 \times 0.14 = 0$

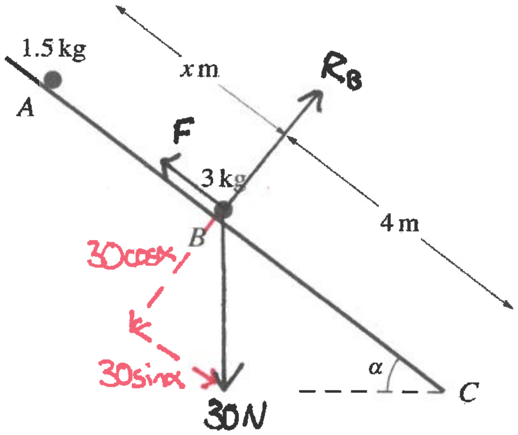
$D = \underline{3950N}$

$\text{Power} = D \times v$

$31\,000 = 3950v$

$\underline{v = 7.85\ ms^{-1}}$

7




Particles of masses 1.5 kg and 3 kg lie on a plane which is inclined at an angle of $\alpha$ to the horizontal, where $\tan\alpha = \frac{3}{4}$. The section of the plane from $A$ to $B$ is smooth and the section of the plane from $B$ to $C$ is rough. The 1.5 kg particle is held at rest at $A$ and the 3 kg particle is in limiting equilibrium at $B$. The distance $AB$ is $x$ m and the distance $BC$ is 4 m (see diagram).

**(a)** Show that the coefficient of friction between the particle at $B$ and the plane is 0.75. [3]

$\tan\alpha = \frac{3}{4}$ : $\sin\alpha = \frac{3}{5}$ $\cos\alpha = \frac{4}{5}$

B: R(↗): $R_B - 30\cos\alpha = 0$

$R_B - 30 \times \frac{4}{5} = 0$

$R_B = 24N$

R(↘): $30\sin\alpha - F = 0$

$30 \times \frac{3}{5} - \mu R = 0$

$18 - \mu \times 24 = 0$

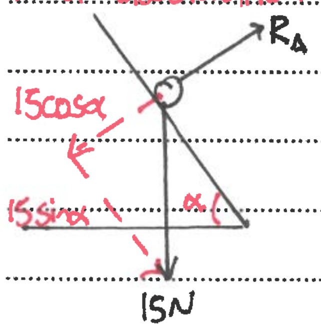
$24\mu = 18$

$\mu = 0.75$

The 1.5 kg particle is released from rest. In the subsequent motion the two particles collide and coalesce. The time taken for the combined particle to travel from $B$ to $C$ is 2 s. The coefficient of friction between the combined particle and the plane is still 0.75.

**(b)** Find $x$. **[6]**

Find acceleration of A using force diagram:




$R(\searrow)$: $15\sin\alpha = ma$

$15 \times \frac{3}{5} = 1.5a$

$9 = 1.5a$

$a = 6\,\text{ms}^{-2}$

Suvat for $A \to B$:

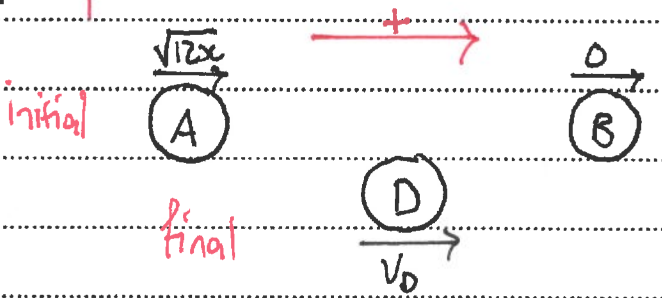
$s = x$ | $v^2 = u^2 + 2as$

$u = 0$ | $= 0^2 + 2 \times 6 \times x$

$v =$ | $v^2 = 12x$

$a = 6$ | $v = \sqrt{12x}$

$t =$




$m_A u_A + m_B u_B = m_D V_D$

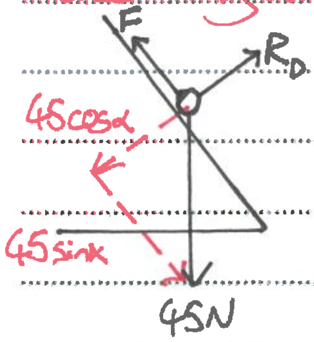
$1.5 \times \sqrt{12x} + 0 = 4.5 V_D$

$4.5 V_D = 1.5 \times \sqrt{12x}$

$V_D = \frac{1}{3}\sqrt{12x}$

Continued..

Force diagram for coalesced particle on surface $B \to C$:




$R(\nearrow)$: $R_D - 45\cos\alpha = 0$

$R_D = 45 \times \frac{4}{5}$

$\underline{R_D = 36N}$

$R(\searrow)$: $45\sin\alpha - F = ma$

$45 \times \frac{3}{5} - \mu R = 4.5a$

$27 - 0.75 \times 36 = 4.5a$

$27 - 27 = 4.5a$

$0 = 4.5a$

$\underline{a = 0}$

Since $a=0$, we can use distance = speed × time:

$4 = \frac{1}{3}\sqrt{12x} \times 2$

$12 = \sqrt{12x} \times 2$

$6 = \sqrt{12x}$

$36 = 12x$

$\underline{x = 3m}$ $\rightarrow$ $V_D = \frac{1}{3}\sqrt{36} = \underline{2ms^{-1}}$

**(c)** Find the total loss of energy of the particles from the time the 1.5 kg particle is released until the combined particle reaches $C$. [3]

Change in potential energy of A:

$\Delta PE = 1.5 \times 10 \times -3\sin\alpha$

$= 1.5 \times 10 \times -1.8$

$= -27J$

Change in potential energy of D:

$\Delta PE = 4.5 \times 10 \times -4\sin\alpha$

$= 4.5 \times 10 \times -2.4$

$= -108J$

Initial KE = 0 because everything starts from rest.

Final KE $= \frac{1}{2} \times 4.5 \times 2^2$

$\underline{= 9J}$

Loss of energy $= -27 - 108 + 9 = -126J$

$\underline{\text{So a } 126J \text{ loss}}$