1 A cyclist is riding a bicycle along a straight horizontal road AB of length 50 m. The cyclist starts from rest at A and reaches a speed of $6\,\text{m s}^{-1}$ at B. The cyclist produces a constant driving force of magnitude 100 N. There is a resistance force, and the work done against the resistance force from A to B is 3560 J.

Find the total mass of the cyclist and bicycle. [3]

$Work_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{OUT}$

$100 \times 50 + 0 + 0 = \frac{1}{2} \times m \times 6^2 + 0 + 3560$

Work = F×d

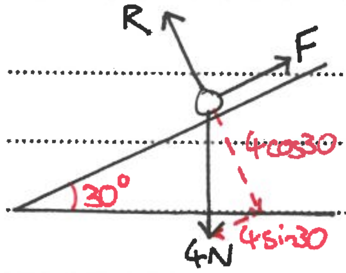
$5000 = 18m + 3560$

$18m = 1440$

$m = \underline{80\,kg}$

2 A particle P of mass 0.4 kg is in limiting equilibrium on a plane inclined at 30° to the horizontal.

(a) Show that the coefficient of friction between the particle and the plane is $\frac{1}{3}\sqrt{3}$. [3]



$R(\nwarrow): R - 4\cos 30 = 0$

$R = 2\sqrt{3}$ N

$R(\swarrow): 4\sin 30 - F = 0$

$2 - \mu R = 0$

$2 - 2\sqrt{3}\mu = 0$

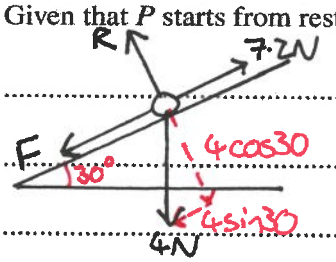
$2\sqrt{3}\mu = 2$

$\mu = \frac{2}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$ QED

A force of magnitude 7.2 N is now applied to P directly up a line of greatest slope of the plane.

(b) Given that P starts from rest, find the time that it takes for P to move 1 m up the plane. [4]



$R(\nearrow): 7.2 - 4\sin 30 - \mu R = ma$

$7.2 - 2 - \frac{\sqrt{3}}{3} \times 2\sqrt{3} = 0.4a$

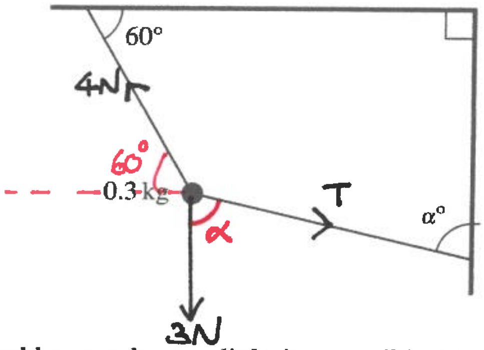
$5.2 - 2 = 0.4a$

$3.2 = 0.4a$

$a = 8\text{ms}^{-2}$

$s = 1$	$s = ut + \frac{1}{2}at^2$
$u = 0$	$1 = 0 + \frac{1}{2}(8) \times t^2$
$v =$	$1 = 4t^2$
$a = 8$	$t^2 = \frac{1}{4}$
$t =$	$t = 0.5$ s

3



A particle of mass 0.3 kg is held at rest by two light inextensible strings. One string is attached at an angle of 60° to a horizontal ceiling. The other string is attached at an angle $\alpha°$ to a vertical wall (see diagram). The tension in the string attached to the ceiling is 4 N.

Find the tension in the string which is attached to the wall and find the value of α. [6]

$R(\uparrow)$: $4\sin 60 - 3 - T\cos\alpha = 0$

$T\cos\alpha = 4\sin 60 - 3$

$T\cos\alpha = 2\sqrt{3} - 3$ 1

$R(\rightarrow)$: $T\sin\alpha - 4\cos 60 = 0$

$T\sin\alpha = 4\cos 60$

$T\sin\alpha = 2$ 2

2 ÷ 1: $\dfrac{T\sin\alpha}{T\cos\alpha} = \dfrac{2}{2\sqrt{3}-3}$

$\tan\alpha = 4.309$

$\alpha = \underline{\underline{76.9°}}$ STO

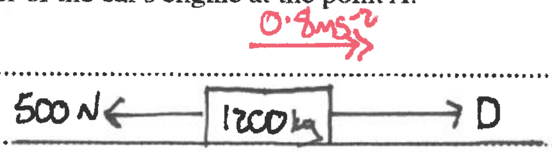
Sub into 2: $T\sin(76.9) = 2$

$T = \dfrac{2}{\sin(76.9)}$

$T = \underline{\underline{2.05\text{N}}}$

4 A car of mass 1200 kg is travelling along a straight horizontal road AB. There is a constant resistance force of magnitude 500 N. When the car passes point A, it has a speed of $15\,\mathrm{m\,s^{-1}}$ and an acceleration of $0.8\,\mathrm{m\,s^{-2}}$.

(a) Find the power of the car's engine at the point A. [3]



$R(\rightarrow)$: $F = ma$

$D - 500 = 1200 \times 0.8$

$D - 500 = 960$

$D = 1460\text{ N}$

Power $= Dv$

$= 1460 \times 15$

$= \underline{21900\text{ W}}$

The car continues to work with this power as it travels from A to B. The car takes 53 seconds to travel from A to B and the speed of the car at B is $32\,\mathrm{m\,s^{-1}}$.

(b) Show that the distance AB is 1362.6 m. [3]

Work done by engine $=$ power $\times$ time

$= 21900 \times 53$

$= 1160700\text{ J}$

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$1160700 + \frac{1}{2} \times 1200 \times 15^2 + 0 = \frac{1}{2} \times 1200 \times 32^2 + 0 + 500d$

↑ Fxd

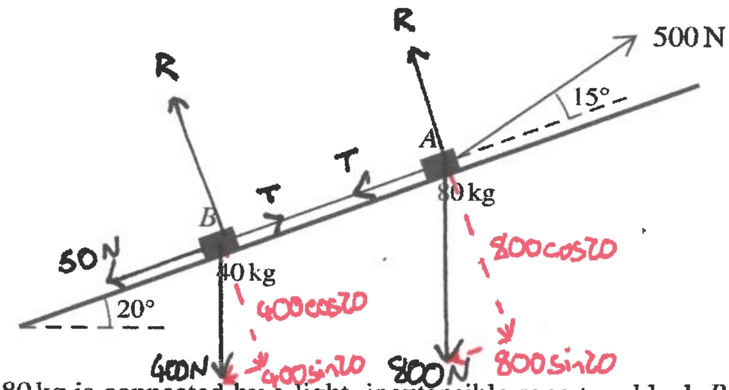
$1160700 + 135000 = 614400 + 500d$

$1295700 = 614400 + 500d$

$500d = 681300$

$d = \underline{1362.6\text{m}}$ QED

5



A block A of mass 80 kg is connected by a light, inextensible rope to a block B of mass 40 kg. The rope joining the two blocks is taut and is parallel to a line of greatest slope of a plane which is inclined at an angle of 20° to the horizontal. A force of magnitude 500 N inclined at an angle of 15° above the same line of greatest slope acts on A (see diagram). The blocks move up the plane and there is a resistance force of 50 N on B, but no resistance force on A.

(a) Find the acceleration of the blocks and the tension in the rope. [5]

A:

$R(\nearrow)$: $500\cos 15 - 800\sin 20 - T = ma$

$500\cos 15 - 800\sin 20 - T = 80a$ (1)

B:

$R(\nearrow)$: $T - 400\sin 20 - 50 = ma$

$T - 400\sin 20 - 50 = 40a$ (2)

(1) + (2):

$500\cos 15 - 800\sin 20 - 400\sin 20 - 50 = 120a$

$22.539 = 120a$

$a = \underline{0.188\text{ ms}^{-2}}$ STO

Sub into (2):

$T - 400\sin 20 - 50 = 40 \times 0.188\ldots$

$T = 7.513 + 400\sin 20 + 50$

$= \underline{194\text{ N}}$

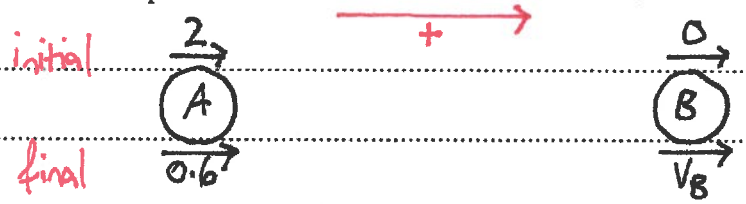
(b) Find the time that it takes for the blocks to reach a speed of $1.2\,\mathrm{m\,s^{-1}}$ from rest. [2]

$s =$
$u = 0$
$v = 1.2$
$a = 0.188$
$t =$

$v = u + at$
$1.2 = 0 + 0.188 \times t$
$1.2 = 0.188t$
$t = \underline{6.39\text{s}}$

6 Three particles A, B and C of masses 0.3 kg, 0.4 kg and m kg respectively lie at rest in a straight line on a smooth horizontal plane. The distance between B and C is 2.1 m. A is projected directly towards B with speed $2\,\mathrm{m\,s^{-1}}$. After A collides with B the speed of A is reduced to $0.6\,\mathrm{m\,s^{-1}}$, still moving in the same direction.

(a) Show that the speed of B after the collision is $1.05\,\mathrm{m\,s^{-1}}$. **[2]**

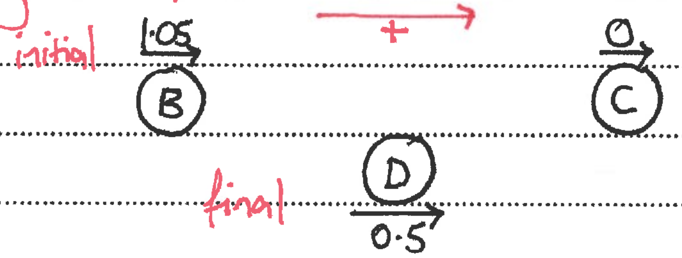


$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$
$$0.3 \times 2 + 0 = 0.3 \times 0.6 + 0.4 V_B$$
$$0.6 = 0.18 + 0.4 V_B$$
$$0.42 = 0.4 V_B \rightarrow V_B = \underline{1.05\,\mathrm{ms^{-1}}} \text{ QED}$$

After the collision between A and B, B moves directly towards C. Particle B now collides with C. After this collision, the two particles coalesce and have a combined speed of $0.5\,\mathrm{m\,s^{-1}}$.

(b) Find m. **[2]**

The plane is smooth and there are no other horizontal forces acting on B, so it remains at $1.05\,\mathrm{ms^{-1}}$ until it hits C:



$$m_B u_B + m_C u_C = m_D v_D$$
$$0.4 \times 1.05 + 0 = (0.4 + m) \times 0.5$$
$$0.42 = 0.2 + 0.5m$$
$$0.22 = 0.5m$$
$$m = \underline{0.44\,\mathrm{kg}}$$

(c) Find the time that it takes, from the instant when B and C collide, until A collides with the combined particle. [5]

How much time passes until B and C collide?

B has travelled 2.1m at a constant speed of $1.05\,ms^{-1}$:

$$\text{time} = \frac{\text{distance}}{\text{Speed}}$$

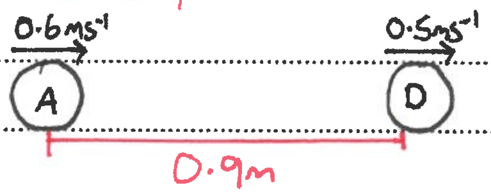
$$= \frac{2.1}{1.05}$$

$$\text{time} = 2s$$

How far has A travelled in this time?

$$\text{distance} = \text{speed} \times \text{time}$$
$$= 0.6 \times 2$$
$$= 1.2m$$

This means that when B and C collide, A is 0.9m away
$(2.1 - 1.2 = 0.9m)$



When A and D collide, A will have travelled 0.9m further than D. So distance travelled by D $= x$, and distance travelled by A $= x + 0.9$.

A: distance = Speed × time

$x + 0.9 = 0.6t$ (1)

D: distance = Speed × time

$x = 0.5t$ (2)

sub. (2) → (1):

$$0.5t + 0.9 = 0.6t$$
$$0.9 = 0.1t$$
$$t = 9s$$

7 A particle P travels in a straight line, starting at rest from a point O. The acceleration of P at time t s after leaving O is denoted by a m s^{-2}, where

$$a = 0.3t^{\frac{1}{2}} \quad \text{for } 0 \leqslant t \leqslant 4,$$
$$a = -kt^{-\frac{3}{2}} \quad \text{for } 4 < t \leqslant T,$$

where k and T are constants.

(a) Find the velocity of P at $t = 4$. [2]

0-4: $V = \int 0.3t^{\frac{1}{2}}\,dt$

$= \frac{2}{3} \times 0.3t^{3/2} + C$

$V = 0.2t^{3/2} + C$

when $t=0$, $V=0$:

$0 = 0 + C$

$\rightarrow V = 0.2t^{3/2}$

$t=4$:

$V = 0.2 \times 4^{3/2}$

$= 0.2 \times 8$

$= \underline{1.6 \text{ ms}^{-1}}$

(b) It is given that there is no change in the velocity of P at $t = 4$ and that the velocity of P at $t = 16$ is 0.3 m s^{-1}.

Show that $k = 2.6$ and find an expression, in terms of t, for the velocity of P for $4 \leqslant t \leqslant T$. [4]

4-T: $V = \int -kt^{-3/2}\,dt$

$V = 2kt^{-\frac{1}{2}} + C$

when $t=4$, $V=1.6$:

$1.6 = 2k(4)^{-\frac{1}{2}} + C$

$1.6 = k + C$ ①

when $t=16$, $V=0.3$:

$0.3 = 2k(16)^{-\frac{1}{2}} + C$

$0.3 = \frac{1}{2}k + C$ ②

① − ②:

$1.3 = \frac{1}{2}k$

$\underline{k = 2.6}$ QED

Sub. into ①:

$1.6 = 2.6 + C$

$\underline{C = -1}$

$\rightarrow V = 2 \times 2.6 \times t^{-\frac{1}{2}} - 1$

$\underline{V = 5.2t^{-\frac{1}{2}} - 1}$

(c) Given that P comes to instantaneous rest at $t = T$, find the exact value of T. [2]

$V = 0$:

$5.2t^{-\frac{1}{2}} - 1 = 0$

$5.2t^{-\frac{1}{2}} = 1$

$\frac{5.2}{\sqrt{t}} = 1$

$5.2 = \sqrt{t}$

$t = \underline{27.04s}$

(d) Find the total distance travelled between $t = 0$ and $t = T$. [4]

Turning points are at $t=0$, $t=4$ and $t=27.04$, so need to find distance between 0-4 and 4-27.04:

0-4s: $s = \int_0^4 0.2t^{\frac{3}{2}}\,dt$

$= \left[0.08t^{\frac{5}{2}}\right]_0^4$

$= \left[0.08 \times 4^{\frac{5}{2}}\right] - [0]$

$= \underline{2.56m}$

4-27.04s: $s = \int_4^{27.04} (5.2t^{-\frac{1}{2}} - 1)\,dt$

$= \left[10.4t^{\frac{1}{2}} - t\right]_4^{27.04}$

$= \left[10.4(27.04)^{\frac{1}{2}} - 27.04\right] - \left[10.4(4)^{\frac{1}{2}} - 4\right]$

$= [27.04] - [16.8]$

$= \underline{10.24m}$

Total distance $= 2.56 + 10.24$

$= \underline{12.8m}$