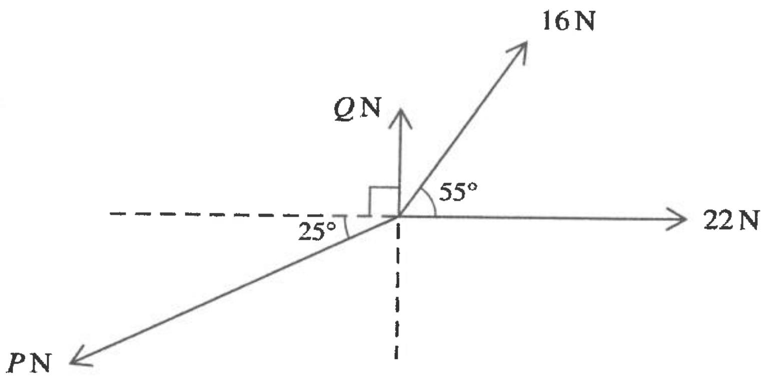
1



Coplanar forces of magnitudes P N, Q N, 16 N and 22 N act at a point in the directions shown in the diagram. The forces are in equilibrium.

Find the values of P and Q. [5]

$R(\rightarrow)$: $22 + 16\cos 55 - P\cos 25 = 0$

$P\cos 25 = 22 + 16\cos 55$

$P = \dfrac{22 + 16\cos 55}{\cos 25}$

$= \underline{34.4\,\text{N}}$ STO

$R(\uparrow)$: $Q + 16\sin 55 - P\sin 25 = 0$

$Q + 16\sin 55 - 34.4\sin 25 = 0$

$Q = 34.4\sin 25 - 16\sin 55$

$= \underline{1.43\,\text{N}}$

2 Small smooth spheres A and B, of equal radii and of masses 6 kg and 2 kg respectively, lie on a smooth horizontal plane. Initially A is moving towards B with speed $5\,\mathrm{m\,s^{-1}}$ and B is moving towards A with speed $3\,\mathrm{m\,s^{-1}}$. After the spheres collide, both A and B move in the same direction and the difference in the speeds of the spheres is $2\,\mathrm{m\,s^{-1}}$.

Find the loss of kinetic energy of the system due to the collision. [5]

Spheres must be moving right because the momentum of A is greater than B. B must be $2\,\mathrm{ms^{-1}}$ faster than A - if it was the other way around they would coalesce.

$$m_A u_A + m_B u_B = m_A V_A + m_B V_B$$
$$6\times 5 + 2\times -3 = 6\times V + 2(V+2)$$
$$30 - 6 = 6V + 2V + 4$$
$$24 = 8V + 4$$
$$20 = 8V$$
$$V = 2.5\,\mathrm{ms^{-1}}$$

$\rightarrow V_A = 2.5\,\mathrm{ms^{-1}}$, $V_B = 4.5\,\mathrm{ms^{-1}}$

$$KE_{initial} = \tfrac{1}{2}\times 6\times 5^2 + \tfrac{1}{2}\times 2\times 3^2$$
$$= 75 + 9$$
$$= 84\,\mathrm{J}$$

$$KE_{final} = \tfrac{1}{2}\times 6\times 2.5^2 + \tfrac{1}{2}\times 2\times 4.5^2$$
$$= 18.75 + 20.25$$
$$= 39\,\mathrm{J}$$

Loss of KE $= 84 - 39 = 45\,\mathrm{J}$

3 A constant resistance of magnitude 1400 N acts on a car of mass 1250 kg.

(a) The car is moving along a straight level road at a constant speed of 28 m s^{-1}.

Find, in kW, the rate at which the engine of the car is working. [2]

Constant speed, so $a = 0$ and driving force = resistance = 1400N

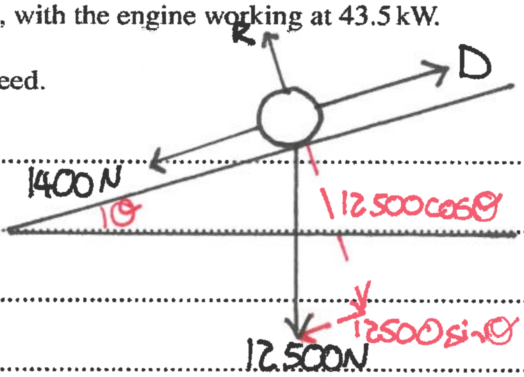
Power = D × V

= 1400 × 28

= 39 200 W

(b) The car now travels at a constant speed up a hill inclined at an angle of θ to the horizontal, where $\sin\theta = 0.12$, with the engine working at 43.5 kW.

Find this speed. [3]



a = 0 (constant speed)

R(↗): D − 1400 − 12500sinθ = ma

D − 1400 − 12500 × 0.12 = 0

D − 1400 − 1500 = 0

D = 2900N

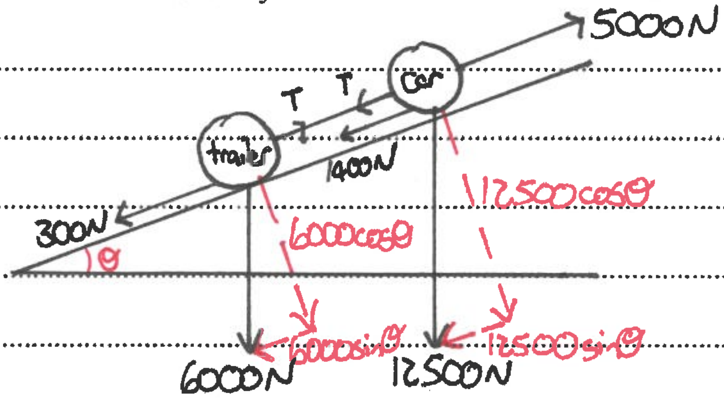
Power = D × V

43 500 = 2900v

v = 15 ms^{-1}

(c) On another occasion, the car pulls a trailer of mass 600 kg up the same hill. The system of the car and the trailer is modelled as particles connected by a light inextensible cable. The car's engine produces a driving force of 5000 N and the resistance to the motion of the trailer is 300 N. The resistance to the motion of the car remains 1400 N.

Find the acceleration of the system and the tension in the cable. [4]



Car:

$R(\nearrow)$: $5000 - 1400 - 12500\sin\theta - T = 1250a$

$3600 - 12500 \times 0.12 - T = 1250a$

$2100 - T = 1250a$ ①

Trailer:

$R(\nearrow)$: $T - 300 - 6000\sin\theta = 600a$

$T - 300 - 6000 \times 0.12 = 600a$

$T - 1020 = 600a$ ②

① + ②: $2100 - 1020 = 1850a$

$1080 = 1850a$

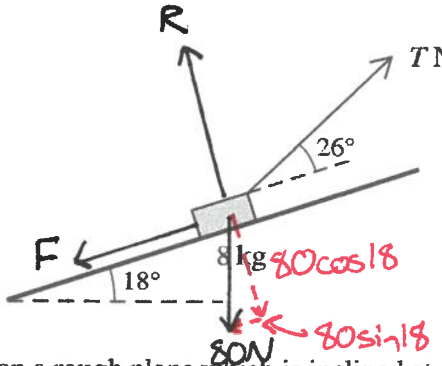
$a = 0.584\ \text{ms}^{-2}$ STO

→ ②: $T - 1020 = 600 \times 0.584$

$T = 350.27 + 1020$

$= 1370\text{N}$

4



A block of mass 8 kg is placed on a rough plane which is inclined at an angle of 18° to the horizontal. The block is pulled up the plane by a light string that makes an angle of 26° above a line of greatest slope. The tension in the string is T N (see diagram). The coefficient of friction between the block and plane is 0.65.

(a) The acceleration of the block is $0.2\,\mathrm{m\,s^{-2}}$.

Find T. [7]

$R(\nwarrow)$: $R + T\sin 26 - 80\cos 18 = 0$

$R = 80\cos 18 - T\sin 26$

$R(\nearrow)$: $T\cos 26 - 80\sin 18 - F = ma$

$T\cos 26 - 80\sin 18 - \mu R = 8 \times 0.2$

$T\cos 26 - 80\sin 18 - 0.65(80\cos 18 - T\sin 26) = 1.6$

$T\cos 26 - 80\sin 18 - 52\cos 18 + 0.65T\sin 26 = 1.6$

$T\cos 26 + 0.65T\sin 26 = 1.6 + 80\sin 18 + 52\cos 18$

$T(\cos 26 + 0.65\sin 26) = 1.6 + 80\sin 18 + 52\cos 18$

$T = \dfrac{1.6 + 80\sin 18 + 52\cos 18}{\cos 26 + 0.65\sin 26}$

$T = \underline{64.0\text{N}}$

(b) The block is initially at rest.

Find the distance travelled by the block during the fourth second of motion. [2]

$t=3$:

$s=$	$s=ut+\frac{1}{2}at^2$
$u=0$	$=0+\frac{1}{2}(0.2)\times3^2$
$v=$	$s=\underline{0.9\text{ m}}$
$a=0.2$	
$t=3$	

$t=4$:

$s=$	$s=ut+\frac{1}{2}at^2$
$u=0$	$=0+\frac{1}{2}(0.2)\times4^2$
$v=$	$s=\underline{1.6\text{ m}}$
$a=0.2$	
$t=4$	

so distance travelled in fourth second $=1.6-0.9$

$=\underline{\underline{0.7\text{ m}}}$

5 A particle P moves on the x-axis from the origin O with an initial velocity of $-20\,\mathrm{m\,s^{-1}}$. The acceleration $a\,\mathrm{m\,s^{-2}}$ at time t s after leaving O is given by $a = 12 - 2t$.

(a) Sketch a velocity-time graph for $0 \leqslant t \leqslant 12$, indicating the times when P is at rest. [5]

$V = \int (12 - 2t)\,dt$

$= 12t - t^2 + C$

$V = -20$ when $t = 0$:

$-20 = 0 - 0 + C$

$\rightarrow V = 12t - t^2 - 20$

$V = -t^2 + 12t - 20$

find roots: $V = 0$:

$-t^2 + 12t - 20 = 0$

$t^2 - 12t + 20 = 0$

$(t-2)(t-10) = 0$

$t = 2 \quad t = 10$

turning point:

$V = -t^2 + 12t - 20$

$= -[t^2 - 12t] - 20$

$= -[(t-6)^2 - 36] - 20$

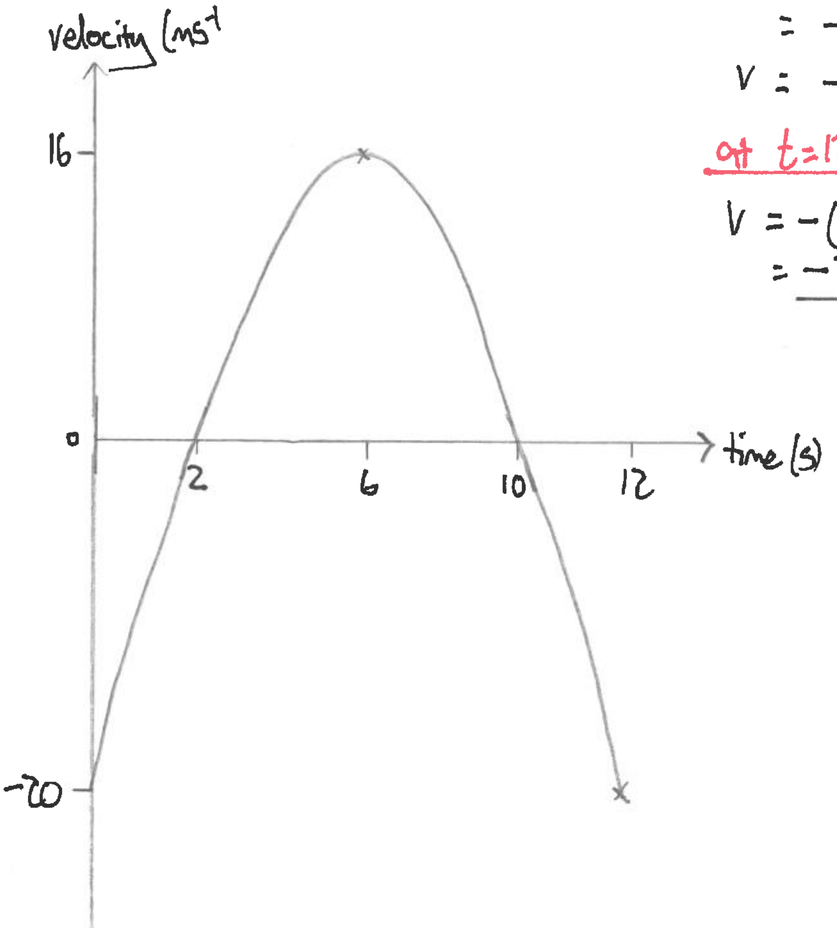
$= -(t-6)^2 + 36 - 20$

$V = -(t-6)^2 + 16$ turning point at $(6, 16)$

at $t = 12$:

$V = -(12)^2 + 12(12) - 20$

$= \underline{-20\ \mathrm{ms^{-1}}}$



(b) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 12$. [5]

Turning points of s are when $v=0$, so at $t=2$ and $t=10$, so need to find s between 0 and 2s, 2s and 10s, then 10s and 12s.

0-2s: $s = \int_0^2 (-t^2 + 12t - 20)\,dt$

$= \left[-\frac{1}{3}t^3 + 6t^2 - 20t\right]_0^2$

$= \left[-\frac{1}{3}\times 2^3 + 6\times 2^2 - 20\times 2\right] - [0] = \frac{-56}{3}$ m

2-10s: $s = \int_2^{10} (-t^2 + 12t - 20)\,dt$

$= \left[-\frac{1}{3}t^3 + 6t^2 - 20t\right]_2^{10}$

$= \left[-\frac{1}{3}\times 10^3 + 6\times 10^2 - 20\times 10\right] - \left[-\frac{1}{3}\times 2^3 + 6\times 2^2 - 20\times 2\right]$

$= \left[\frac{200}{3}\right] - \left[\frac{-56}{3}\right] = \frac{256}{3}$ m

10-12s: Equal to 0-2s by symmetry so $= \frac{-56}{3}$ m

Total distance $= \frac{56}{3} + \frac{256}{3} + \frac{56}{3}$

$= 123$m (3sf)

6

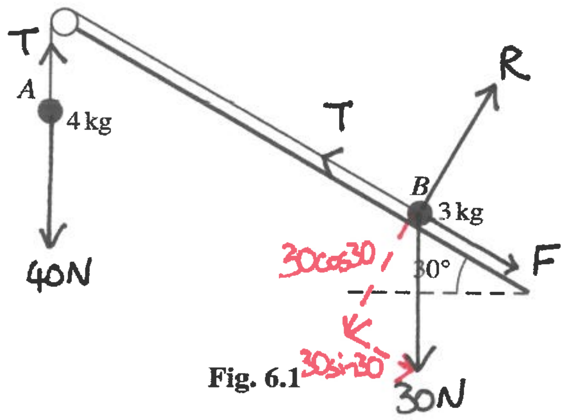


Fig. 6.1

Fig. 6.1 shows particles A and B, of masses 4 kg and 3 kg respectively, attached to the ends of a light inextensible string that passes over a small smooth pulley. The pulley is fixed at the top of a plane which is inclined at an angle of 30° to the horizontal. A hangs freely below the pulley and B is on the inclined plane. The string is taut and the section of the string between B and the pulley is parallel to a line of greatest slope of the plane.

(a) It is given that the plane is rough and the particles are in limiting equilibrium.

Find the coefficient of friction between B and the plane. [6]

A: R(↓): $40 - T = 0$

$T = 40N$

B: R(↗): $R - 30\cos 30 = 0$

$R = 30\cos 30$

$R = 15\sqrt{3}\,N$

R(↖): $T - 30\sin 30 - F = 0$

$40 - 15 - F = 0$

$25 - F = 0$

$F = 25$

$\mu R = 25$

$\mu \times 15\sqrt{3} = 25$

$\mu = 0.962$

(b)

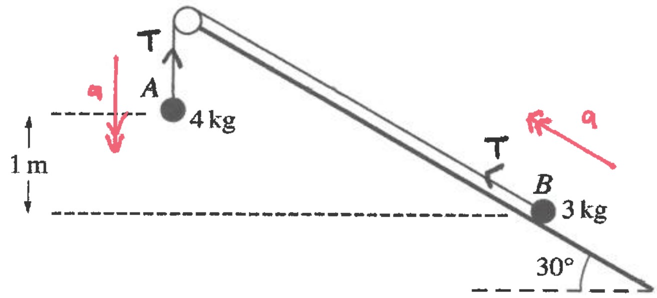
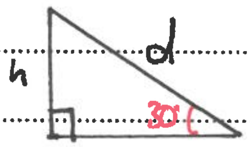


Fig. 6.2

It is given instead that the plane is smooth and the particles are released from rest when the difference in the vertical heights of the particles is 1 m (see Fig. 6.2).

Use an energy method to find the speed of the particles at the instant when the particles are at the same horizontal level. [6]

If A travels downwards a distance of d, find increase in height of B:



$\sin 30 = \frac{h}{d}$

$\underline{h = d\sin 30}$

If they're at the same level, the vertical distances must sum to 1:

$d + d\sin 30 = 1$

$d + \frac{d}{2} = 1$

$\frac{3d}{2} = 1$

$3d = 2$

$\underline{d = \frac{2}{3}\,m}$

$\rightarrow h = \frac{2}{3} \times \sin 30$

$\underline{= \frac{1}{3}\,m}$

continued...

A: $\text{Work}_{in} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{out}$

$0 + 0 + 0 = \frac{1}{2} \times 4 \times v^2 + 4 \times 10 \times -\frac{2}{3} + T \times \frac{2}{3}$

$0 = 2v^2 - \frac{80}{3} + \frac{2}{3}T$ ×3

$0 = 6v^2 - 80 + 2T$

$2T + 6v^2 = 80$

$T + 3v^2 = 40$ (1)

B: $\text{Work}_{in} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{out}$

$T \times \frac{2}{3} + 0 + 0 = \frac{1}{2} \times 3 \times v^2 + 3 \times 10 \times \frac{1}{3} + 0$

$\frac{2}{3}T = 1.5v^2 + 10$ ×3

$2T = 4.5v^2 + 30$

$2T - 4.5v^2 = 30$ ÷2

$T - 2.25v^2 = 15$ (2)

(1) − (2): $3v^2 - -2.25v^2 = 40 - 15$

$5.25v^2 = 25$

$v^2 = \frac{100}{21}$

$v = 2.18 \text{ ms}^{-1}$