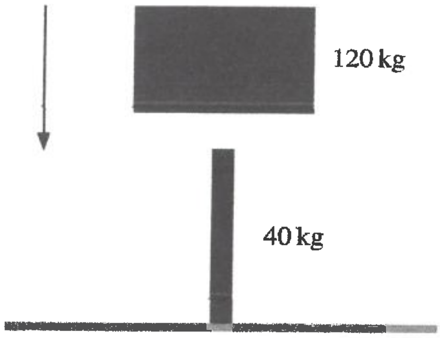
1




A metal post is driven vertically into the ground by dropping a heavy object onto it from above. The mass of the object is 120 kg and the mass of the post is 40 kg (see diagram). The object hits the post with speed 8 m s$^{-1}$ and remains in contact with it after the impact.

**(a)** Calculate the speed with which the combined post and object moves immediately after the impact. [2]

initial

8 ↓ O

final

Q ↓ $V_Q$

0 ↓ P

+ ↓

$$m_O u_O + m_P u_P = m_Q V_Q$$

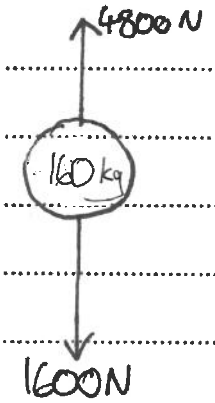
$$120 \times 8 + 0 = 160 V_Q$$

$$960 = 160 V_Q$$

$$V_Q = \underline{6\ \text{ms}^{-1}}$$

**(b)** There is a constant force resisting the motion of magnitude 4800 N.

Calculate the distance the post is driven into the ground. [3]




$R(\downarrow)$: $F = ma$

$$1600 - 4800 = 160a$$

$$-3200 = 160a$$

$$a = \underline{-20\ \text{ms}^{-2}}$$

+ ↓

| | |
|---|---|
| $s =$ | $v^2 = u^2 + 2as$ |
| $u = 6$ | $0^2 = 6^2 + 2 \times -20 \times s$ |
| $v = 0$ | $0 = 36 - 40s$ |
| $a = -20$ | $40s = 36$ |
| $t =$ | $s = \underline{0.9\ \text{m}}$ |

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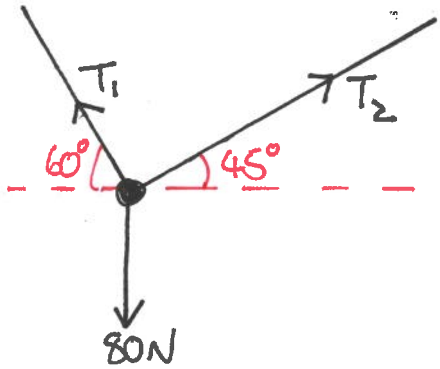
2 A particle of mass 8 kg is suspended in equilibrium by two light inextensible strings which make angles of 60° and 45° above the horizontal.

(a) Draw a diagram showing the forces acting on the particle. [1]




(b) Find the tensions in the strings. [6]

$R(\rightarrow)$: $T_2 \cos 45 - T_1 \cos 60 = 0$

$T_2 \cos 45 = T_1 \cos 60$

$\frac{\sqrt{2}}{2} T_2 = \frac{1}{2} T_1$

$\sqrt{2} T_2 = T_1$

$T_1 = \sqrt{2} T_2$ ①

$R(\uparrow)$: $T_1 \sin 60 + T_2 \sin 45 - 80 = 0$

$\frac{\sqrt{3}}{2} T_1 + \frac{\sqrt{2}}{2} T_2 = 80$

$\sqrt{3} T_1 + \sqrt{2} T_2 = 160$ ②

sub ① → ②: $\sqrt{3}(\sqrt{2} T_2) + \sqrt{2} T_2 = 160$

$\sqrt{6} T_2 + \sqrt{2} T_2 = 160$

$T_2(\sqrt{6} + \sqrt{2}) = 160$

$T_2 = \frac{160}{\sqrt{6}+\sqrt{2}}$

$= \underline{41.4N}$ STO

sub into ①: $T_1 = \sqrt{2} \times 41.4$

$= \underline{58.6N}$

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**3** A ball of mass 1.6 kg is released from rest at a point 5 m above horizontal ground. When the ball hits the ground it instantaneously loses 8 J of kinetic energy and starts to move upwards.

**(a)** Use an energy method to find the greatest height that the ball reaches after hitting the ground. [3]

On the way down: $Work_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{OUT}$

$0 + 0 + 0 = KE_{fin} + 1.6 \times 10 \times -5 + 0$

$0 = KE_{fin} - 80$

$KE_{fin} = 80J$

KE after bounce = 72J

after bounce: $Work_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{OUT}$

$0 + 72 + 0 = 0 + 1.6 \times 10 \times h + 0$

v=0

$72 = 16h$

$h = 4.5m$

**(b)** Find the total time taken, from the initial release of the ball until it reaches this greatest height. [3]

On the way down:

$\downarrow +$

| | |
|---|---|
| $S = 5$ | $s = ut + \frac{1}{2}at^2$ |
| $u = 0$ | $5 = 0 + \frac{1}{2} \times 10 \times t^2$ |
| $v =$ | $5 = 5t^2$ |
| $a = 10$ | $t^2 = 1$ |
| $t =$ | $t = 1s$ |

On the way up:

$\uparrow +$

| | |
|---|---|
| $S = 4.5$ | $S = vt - \frac{1}{2}at^2$ |
| $u =$ | $4.5 = 0 - \frac{1}{2} \times -10 \times t^2$ |
| $v = 0$ | $4.5 = 5t^2$ |
| $a = -10$ | $t^2 = 0.9$ |
| $t =$ | $t = 0.949s$ |

Total time $= 1 + 0.949$

$= 1.95s$



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**4** A car of mass 1400 kg is moving on a straight road against a constant force of 1250 N resisting the motion.

**(a)** The car moves along a horizontal section of the road at a constant speed of $36\,\mathrm{m\,s^{-1}}$.

**(i)** Calculate the work done against the resisting force during the first 8 seconds. [2]

Constant speed, so distance = speed × time:

$d = 36 \times 8$

$= 288\,\mathrm{m}$

Work done = Force × distance

$= 1250 \times 288$

$= 360\,000\,\mathrm{J}$

**(ii)** Calculate, in kW, the power developed by the engine of the car. [2]

Acceleration = 0, so driving force = resistive force = 1250N

Power = $D \times v$

$= 1250 \times 36$

$= 45\,000\,\mathrm{W}$

$= 45\,\mathrm{kW}$

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**(iii)** Given that this power is suddenly increased by 12 kW, find the instantaneous acceleration of the car. [3]

Power $= 45000 + 12000$

$= 57000$ W

Power $= D \times v$

$57000 = D \times 36$

$D = \underline{1583.3\,N}$ STO


$R(\rightarrow)$: $F = ma$

$1583.3 - 1250 = 1400a$

$333.3 = 1400a$

$a = \underline{\underline{0.238\,ms^{-2}}}$

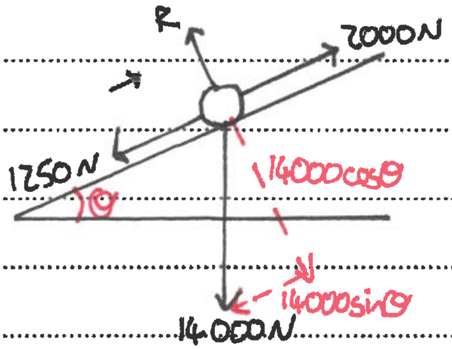
**(b)** The car now travels at a constant speed of $32\,\mathrm{m\,s^{-1}}$ up a section of the road inclined at $\theta°$ to the horizontal, with the engine working at 64 kW.

Find the value of $\theta$. [2]

Power $= Dv$

$64000 = D \times 32$

$D = 2000N$




a=0 (constant speed)

$R(\nearrow)$: $F = ma$

$2000 - 1250 - 14000\sin\theta = 0$

$750 - 14000\sin\theta = 0$

$14000\sin\theta = 750$

$\sin\theta = \frac{750}{14000}$

$\theta = \underline{3.1°}$ (1dp)



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5 A particle $P$ moves in a straight line, starting from rest at a point $O$ on the line. At time $t$ s after leaving $O$ the acceleration of $P$ is $k(16 - t^2)$ m s$^{-2}$, where $k$ is a positive constant, and the displacement from $O$ is $s$ m. The velocity of $P$ is 8 m s$^{-1}$ when $t = 4$.

**(a)** Show that $s = \frac{1}{64}t^2(96 - t^2)$. [5]

$a = 16k - kt^2$

$v = \int(16k - kt^2)\,dt$

$v = 16kt - \frac{1}{3}kt^3 + c$

$v = 0$ when $t = 0$:

$0 = 0 - 0 + c$

$\rightarrow v = 16kt - \frac{1}{3}kt^3$

$v = 8$ when $t = 4$:

$8 = 16k \times 4 - \frac{1}{3}k \times 4^3$

$8 = 64k - \frac{64}{3}k$ $\times 3$

$24 = 192k - 64k$

$24 = 128k$

$k = \frac{3}{16}$

$\rightarrow v = 16 \times \frac{3}{16}t - \frac{1}{3} \times \frac{3}{16}t^3$

$v = 3t - \frac{1}{16}t^3$

$s = \int\left(3t - \frac{1}{16}t^3\right)dt$

$s = \frac{3}{2}t^2 - \frac{1}{64}t^4 + c$

$s = 0$ when $t = 0$:

$0 = 0 - 0 + c$

$s = \frac{3}{2}t^2 - \frac{1}{64}t^4$

$s = \frac{1}{64}t^2(96 - t^2)$ QED



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**(b)** Find the speed of $P$ at the instant that it returns to $O$. [3]

Returns to O when S=0:

$$\frac{1}{64}t^2(96-t^2)=0$$

$$t^2(96-t^2)=0$$

$t^2=0$ or $96-t^2=0$

$t=0$ $\qquad t^2=96$

$t=4\sqrt{6}$ ✓ or $=-4\sqrt{6}$ ✗

Sub. $t=4\sqrt{6}$ into V:

$$V=3(4\sqrt{6})-\frac{1}{16}(4\sqrt{6})^3$$

$$=12\sqrt{6}-24\sqrt{6}$$

$$=-29.4\ ms^{-1}$$

speed $=\underline{29.4\ ms^{-1}}$

**(c)** Find the maximum displacement of the particle from $O$. [3]

$S=\frac{3}{2}t^2-\frac{1}{64}t^4$ is a negative quadratic, so need to find maximum point by doing $\frac{ds}{dt}=0$:

$$\frac{ds}{dt}=V=3t-\frac{1}{16}t^3$$

$$3t-\frac{1}{16}t^3=0$$

$$48t-t^3=0$$

$$t(48-t^2)=0$$

$t=0$ or $48-t^2=0$

$t^2=48$

$t=4\sqrt{3}$ ✓ or $-4\sqrt{3}$ ✗

Sub. $t=4\sqrt{3}$ into S:

$$S=\frac{1}{64}(4\sqrt{3})^2(96-(4\sqrt{3})^2)$$

$$=\frac{1}{64}(48)(96-48)$$

$$=\frac{1}{64}\times 48\times 48$$

$$=\underline{36m}$$

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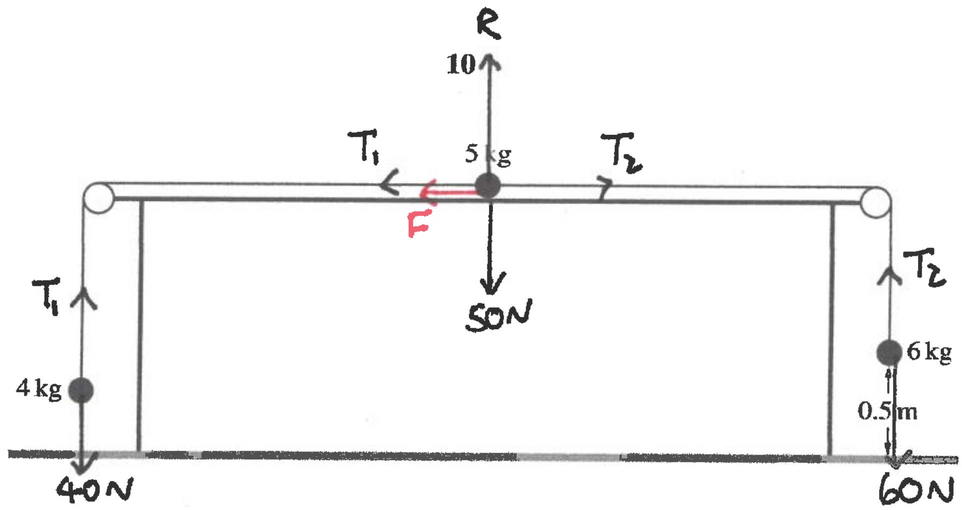
6




The diagram shows a particle of mass 5 kg on a rough horizontal table, and two light inextensible strings attached to it passing over smooth pulleys fixed at the edges of the table. Particles of masses 4 kg and 6 kg hang freely at the ends of the strings. The particle of mass 6 kg is 0.5 m above the ground. The system is in limiting equilibrium.

**(a)** Show that the coefficient of friction between the 5 kg particle and the table is 0.4. [2]

4kg:

$R(\uparrow): T_1 - 40 = 0$

$T_1 = 40N$

6kg:

$R(\downarrow): 60 - T_2 = 0$

$T_2 = 60N$

So Friction must be acting to the left and must be 20N.

$F = \mu R$

$20 = \mu \times 50$

$\mu = \frac{20}{50}$

$\mu = 0.4$ QED

The 6 kg particle is now replaced by a particle of mass 8 kg and the system is released from rest.

**(b)** Find the acceleration of the 4 kg particle and the tensions in the strings. [5]

4kg will move up, 5kg will move right, 8kg will move down.

4kg:

$R(\uparrow): T_1 - 40 = 4a$

$T_1 = 40 + 4a$ ①

8kg:

$R(\downarrow): 80 - T_2 = 8a$

$T_2 = 80 - 8a$ ②

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5kg:

R(→): $T_2 - T_1 - F = 5a$

$T_2 - T_1 - \mu R = 5a$

$T_2 - T_1 - 0.4 \times 50 = 5a$

Sub. ① and ② into this ⑤

$(80 - 8a) - (40 + 4a) - 20 = 5a$

$80 - 8a - 40 - 4a - 20 = 5a$

$20 - 12a = 5a$

$20 = 17a$

$a = 1.18 ms^{-2}$ STO

Sub into ①:

$T_1 = 40 + 4(1.18)$

$= 44.7N$

Sub into ②:

$T_2 = 80 - 8(1.18)$

$= 70.6N$

**(c)** In the subsequent motion the 8 kg particle hits the ground and does not rebound.

Find the time that elapses after the 8 kg particle hits the ground before the other two particles come to instantaneous rest. (You may assume this occurs before either particle reaches a pulley.) **[5]**

Find speed of 4kg and 5kg particles when 8kg hits ground:

| | |
|---|---|
| $S = 0.5$ | $V^2 = u^2 + 2as$ |
| $u = 0$ | $V^2 = 0^2 + 2(1.18) \times 0.5$ |
| $V =$ | $V^2 = 1.18$ |
| $a = 1.18$ | $V = 1.085 ms^{-1}$ STO |
| $t =$ | |

Resolve forces for 4kg and 5kg now string has gone slack:

4kg:

R(↑): $T_1 - 40 = 4a$ ①

5kg:

R(→): $-T_1 - F = 5a$

$-T_1 - \mu R = 5a$

$-T_1 - 0.4 \times 50 = 5a$

continued →



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$-T_1 - 20 = 5a$ (2)

(1) + (2):

$-40 - 20 = 9a$

$-60 = 9a$

$a = \frac{-20}{3} \text{ ms}^{-2}$

Find time that elapses before 4kg + 5kg come to rest:

| | |
|---|---|
| $s =$ | $v = u + at$ |
| $u = 1.085$ | $0 = 1.085 + \left(\frac{-20}{3}\right)t$ |
| $v = 0$ | $\frac{20}{3}t = 1.085$ |
| $a = \frac{-20}{3}$ | |
| $t =$ | $t = 0.163\text{s}$ |

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