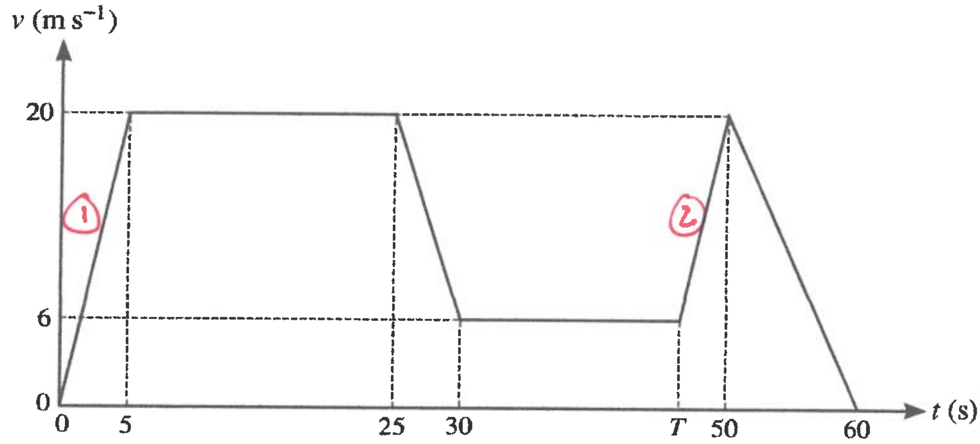
1



The diagram shows a velocity-time graph which models the motion of a car. The graph consists of six straight line segments. The car accelerates from rest to a speed of $20\,\mathrm{m\,s^{-1}}$ over a period of 5 s, and then travels at this speed for a further 20 s. The car then decelerates to a speed of $6\,\mathrm{m\,s^{-1}}$ over a period of 5 s. This speed is maintained for a further $(T-30)$ s. The car then accelerates again to a speed of $20\,\mathrm{m\,s^{-1}}$ over a period of $(50-T)$ s, before decelerating to rest over a period of 10 s.

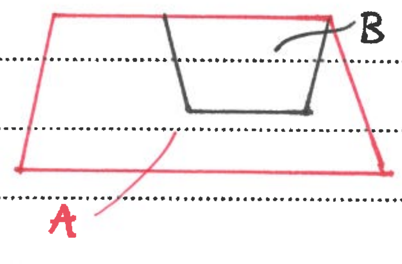
(a) Given that during the two stages of the motion when the car is accelerating, the accelerations are equal, find the value of T. [2]

1: $s=$	$v=u+at$	2: $s=$	$v=u+at$
$u=0$	$20=0+5a$	$u=6$	$20=6+4t$
$v=20$	$5a=20$	$v=20$	$14=4t$
$a=$	$a=4$	$a=4$	$t=3.5$
$t=5$		$t=$	

$T=50-3.5$

$=46.5\,s$

(b) Find the total distance travelled by the car during the motion. [2]



Area = Trapezium A − Trapezium B

Area A : $A=\frac{1}{2}(60+45)\times 20$

$=\frac{1}{2}\times 105\times 20$

$=1050$

Area B: $B=\frac{1}{2}(25+16.5)\times 14$

$=\frac{1}{2}\times 41.5\times 14$

$=290.5$

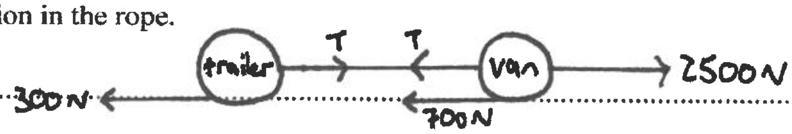
$A-B=1050-290.5$

$=759.5\,m$

2 A van of mass 3600 kg is towing a trailer of mass 1200 kg along a straight horizontal road using a light horizontal rope. There are resistance forces of 700 N on the van and 300 N on the trailer.

(a) The driving force exerted by the van is 2500 N.

Find the tension in the rope. [4]



Trailer:

$R(\rightarrow): T - 300 = ma$

$T - 300 = 1200a$ ①

Van:

$R(\rightarrow): 2500 - 700 - T = ma$

$1800 - T = 3600a$ ②

① + ②: $1500 = 4800a$

$a = \underline{0.3125 \text{ ms}^{-2}}$

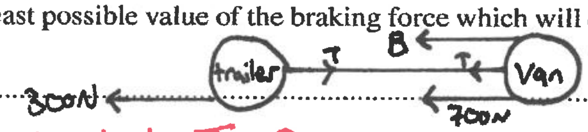
Sub into ①: $T - 300 = 1200 \times 0.3125$

$T - 300 = 375$

$T = \underline{675 \text{ N}}$

The driving force is now removed and the van driver applies a braking force which acts only on the van. The resistance forces remain unchanged.

(b) Find the least possible value of the braking force which will cause the rope to become slack. [2]



If rope is slack, $T = 0$:

Trailer:

$R(\rightarrow): -300 = ma$

$-300 = 1200a$

$a = \underline{-0.25 \text{ms}^{-2}}$

Van:

$R(\rightarrow): -B - 700 = ma$

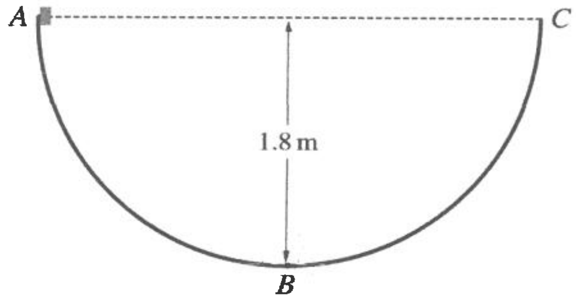
$-B - 700 = 3600 \times -0.25$

$-B - 700 = -900$

$-B = -200$

$B = \underline{200 \text{ N}}$

3



The diagram shows a semi-circular track ABC of radius 1.8 m which is fixed in a vertical plane. The points A and C are at the same horizontal level and the point B is at the bottom of the track. The section AB is smooth and the section BC is rough. A small block is released from rest at A.

(a) Show that the speed of the block at B is $6\,\text{m s}^{-1}$. [2]

$A \to B$ $\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$0 + 0 + 0 = \frac{1}{2} \times m \times v^2 + m \times 10 \times -1.8 + 0$

$0 = \frac{1}{2}mv^2 - 18m$

$\frac{1}{2}mv^2 = 18m$

$\cancel{m}v^2 = 36\cancel{m}$

$v^2 = 36 \rightarrow V = 6\text{ms}^{-1}$

The block comes to instantaneous rest for the first time at a height of 1.2 m above the level of B. The work done against the resistance force during the motion of the block from B to this point is 4.5 J.

(b) Find the mass of the block. [3]

$B \to C$: $\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$0 + \frac{1}{2}m \times 6^2 + 0 = 0 + m \times 10 \times 1.2 + 4.5$

$18m = 12m + 4.5$

$6m = 4.5$

$m = 0.75\text{ kg}$

4 A cyclist starts from rest at a point A and travels along a straight road AB, coming to rest at B. The displacement of the cyclist from A at time t s after the start is s m, where

$$s = 0.004(75t^2 - t^3).$$

(a) Show that the distance AB is 250 m. $\hookrightarrow S = 0.3t^2 - 0.004t^3$ [4]

find when cyclist is at rest:

$V = \frac{ds}{dt} = 0.6t - 0.012t^2$

$V=0$: $0.6t - 0.012t^2 = 0$ $\div 0.012$

$50t - t^2 = 0$

$t(50 - t) = 0$

$\underline{t=0}$, $\underline{t=50}$

Sub. $t=50$ into s:

$S = 0.3(50)^2 - 0.004(50)^3$

$= 750 - 500$

$= \underline{250\text{m}}$ QED

(b) Find the maximum velocity of the cyclist. [3]

$V = 0.6t - 0.012t^2$ is a negative quadratic, so max. V is at $\frac{dv}{dt} = 0$:

$\frac{dv}{dt} = 0.6 - 0.024t$

$0.6 - 0.024t = 0$

$0.024t = 0.6$

$t = 25\text{s}$

Sub. $t = 25$ into v:

$V = 0.6(25) - 0.012(25)^2$

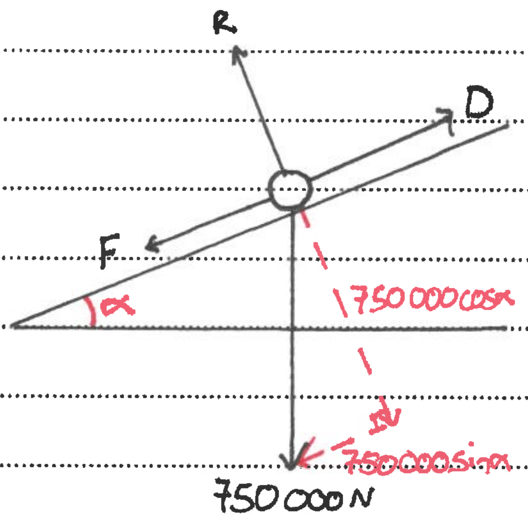
$= 15 - 7.5$

$= \underline{7.5\text{ms}^{-1}}$

5 A railway engine of mass 75 000 kg is moving up a straight hill inclined at an angle α to the horizontal, where $\sin\alpha = 0.01$. The engine is travelling at a constant speed of $30\,\mathrm{m\,s^{-1}}$. The engine is working at 960 kW. There is a constant force resisting the motion of the engine.

(a) Find the resistance force. [3]

Constant speed, so acceleration = 0:



Power = Dv

960 000 = D×30

D = 32000N

R(↗) : F = ma

D − F − 750000sinα = 0 ← a=0

32000 − F − 750000×0.01 = 0

32000 − F − 7500 = 0

24500 − F = 0

F = 24500N

The engine comes to a section of track which is horizontal. At the start of the section the engine is travelling at $30\,\mathrm{m\,s^{-1}}$ and the power of the engine is now reduced to 900 kW. The resistance to motion is no longer constant, but in the next 60 s the work done against the resistance force is 46 500 kJ.

(b) Find the speed of the engine at the end of the 60 s. [4]

Work done = Power × time

= 900 000 × 60

= 54 000 000 J

$Work_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{OUT}$

$54\,000\,000 + \frac{1}{2} \times 75000 \times 30^2 + 0 = \frac{1}{2} \times 75000 \times v^2 + 0 + 46\,500\,000$

$54\,000\,000 + 33\,750\,000 = 37500v^2 + 46\,500\,000$

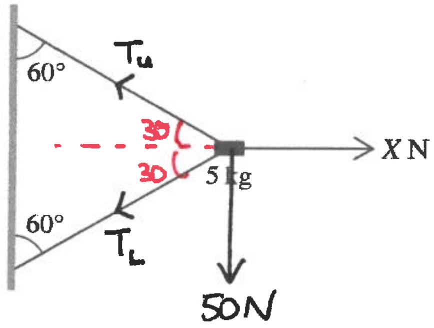
$87\,750\,000 = 37500v^2 + 46\,500\,000$

$41\,250\,000 = 37500v^2$

$v^2 = 1100$

$\underline{v = 33.2\,\mathrm{ms^{-1}}}$

6



A block of mass 5 kg is held in equilibrium near a vertical wall by two light strings and a horizontal force of magnitude X N, as shown in the diagram. The two strings are both inclined at 60° to the vertical.

(a) Given that $X = 100$, find the tension in the lower string. [4]

$R(\uparrow)$: $T_u \sin 30 - T_L \sin 30 - 50 = 0$

$\frac{1}{2} T_u - \frac{1}{2} T_L = 50$

$T_u - T_L = 100$ ①

$R(\rightarrow)$: $100 - T_u \cos 30 - T_L \cos 30 = 0$

$100 - \frac{\sqrt{3}}{2} T_u - \frac{\sqrt{3}}{2} T_L = 0$

$200 - \sqrt{3} T_u - \sqrt{3} T_L = 0$

$\sqrt{3} T_u + \sqrt{3} T_L = 200$

$T_u + T_L = \frac{200}{\sqrt{3}}$ ②

① + ②: $2T_u = 100 + \frac{200}{\sqrt{3}}$

$2T_u = 215.470$

$T_u = 107.735$

Sub into ①: $T_u - T_L = 100$

$107.735 - T_L = 100$

$-T_L = -7.735$

$T_L = \underline{7.74 N}$

(b) Find the least value of X for which the block remains in equilibrium in the position shown. [4]

As X decreases, so will T_u and T_L

T_u cannot be zero or there would be nothing to hold the block up, so set $T_L = 0$:

$R(\uparrow)$: $T_u \sin 30 - 50 = 0$

$T_u \sin 30 = 50$

$T_u = \underline{100\ N}$

$R(\rightarrow)$: $X - T_u \cos 30 = 0$

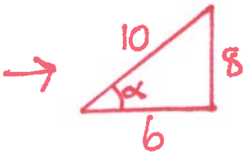
$X - 100 \cos 30 = 0$

$X = 100 \cos 30$

$= \underline{86.6 N}$

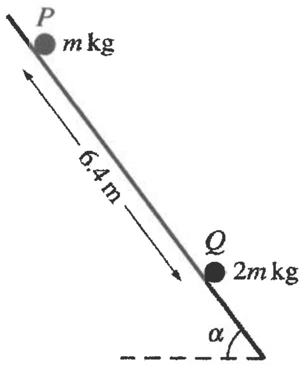
7

$\sin\alpha = 0.8$
$= \frac{8}{10}$



$\cos\alpha = \frac{6}{10} = 0.6$

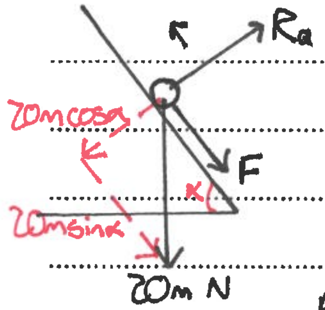
$\tan\alpha = \frac{8}{6}$



Particles P and Q have masses m kg and $2m$ kg respectively. The particles are initially held at rest 6.4 m apart on the same line of greatest slope of a rough plane inclined at an angle α to the horizontal, where $\sin\alpha = 0.8$ (see diagram). Particle P is released from rest and slides down the line of greatest slope. Simultaneously, particle Q is projected up the same line of greatest slope at a speed of $10\,\text{m s}^{-1}$. The coefficient of friction between each particle and the plane is 0.6.

(a) Show that the acceleration of Q up the plane is $-11.6\,\text{m s}^{-2}$. [4]

Q:



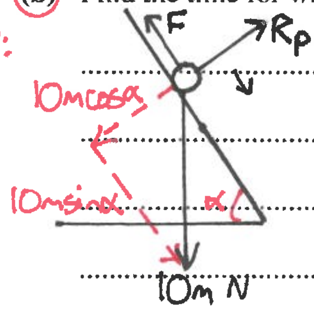
$R(\nearrow)$: $R_Q - 20m\cos\alpha = 0$
$R_Q - 20m \times 0.6 = 0$
$R_Q - 12m = 0$
$R_Q = 12m$

$R(\nwarrow)$: $-20m\sin\alpha - F = 2ma$
$-20m \times 0.8 - \mu R = 2ma$
$-16m - 0.6 \times 12m = 2ma$
$-16m - 7.2m = 2ma$
$-23.2m = 2ma$ $\div 2m$
$a = -11.6\,\text{ms}^{-2}$ QED

incredibly difficult (recommend skipping)

(b) Find the time for which the particles are in motion before they collide. [5]

P:



$R(\nearrow)$: $R_P - 10m\cos\alpha = 0$
$R_P - 10m \times 0.6 = 0$
$R_P - 6m = 0$
$R_P = 6m$

continued

$R(\searrow)$: $10m\sin\alpha - F = ma$

$10m \times 0.8 - \mu R = ma$

$8m - 0.6 \times 6m = ma$

$4.4m = ma$

$a = 4.4\,ms^{-2}$

* At this point we are expected to realise that Q goes up, stops, and begins to fall back down before P and Q collide. This makes the question incredibly difficult, especially for 5 marks. I can only assume a mistake was made when setting the Q. *

Note that friction changes direction when Q is falling down, so the acceleration changes.

Q: On the way up:

$\nwarrow +$		
$s =$	$v = u + at$	$v^2 = u^2 + 2as$
$u = 10$	$0 = 10 - 11.6t$	$0^2 = 10^2 + 2(-11.6) \times s$
$v = 0$	$11.6t = 10$	$0 = 100 - 23.2s$
$a = -11.6$	$t = 0.862s$ STO	$23.2s = 100$
$t =$		$s = 4.31\,m$ STO

So after $0.862s$, Q is $4.31m$ up the plane.

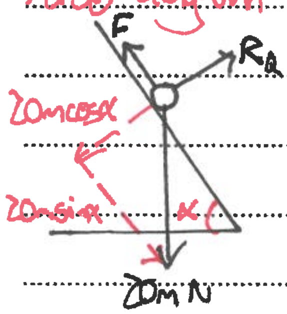
Where is P after $0.862s$?

$\searrow +$		
$s =$	$s = ut + \frac{1}{2}at^2$	$v = u + at$
$u = 0$	$= 0 + \frac{1}{2} \times 4.4 \times 0.862^2$	$= 0 + 4.4 \times 0.862$
$v =$	$= 1.635\,m$ down the plane. STO	$= 3.79\,ms^{-1}$ STO
$a = 4.4$		
$t = 0.862$		

→ Distance between P and Q: $d = 6.4 - 4.31 - 1.635$

$= 0.455\,m$ STO

Force diagram for Q on the way down:



$R(\searrow)$: $20m\sin\alpha - \mu R = 2ma$

$16m - 7.2m = 2ma$

$8.8m = 2ma$

$\underline{a = 4.4\text{ms}^{-2}}$

$\searrow^{+}$ Q:

$S = x$ | $S = ut + \frac{1}{2}at^2$

$u = 0$ | $x = 0 + \frac{1}{2} \times 4.4t^2$

$V =$ | $\underline{x = 2.2t^2}$ (1)

$a = 4.4$

$t =$

$\searrow^{+}$ P:

$S = x + 0.455$ | $S = ut + \frac{1}{2}at^2$

$u = 3.79$ | $\underline{x + 0.455 = 3.79t + 2.2t^2}$ (2)

$V =$ | Sub. (1) → (2):

$a = 4.4$ | $\cancel{2.2t^2} + 0.455 = 3.79t + \cancel{2.2t^2}$

$t =$ | $3.79t = 0.455$

$t = 0.120$ STO

(c) The particles coalesce on impact.

Find the speed of the combined particle immediately after the impact. [4]

Time elapsed $= 0.862 + 0.120$

$= \underline{0.982\text{s}}$

Find velocities of P and Q at moment of impact:

$\searrow^{+}$ P:

$S =$ | $V = u + at$

$u = 0$ | $= 0 + 4.4(0.982)$

$V =$ | $= \underline{4.32\text{ ms}^{-1}}$ STO

$a = 4.4$

$t = 0.982$

$\searrow^{+}$ Q:

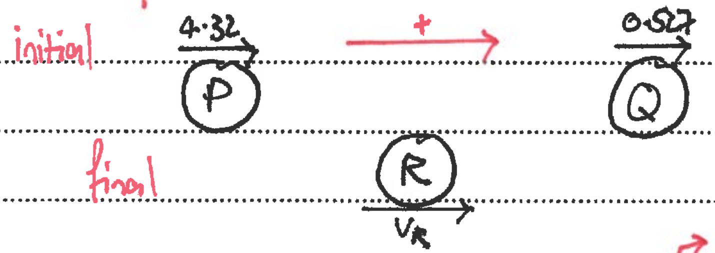
$S =$ | $V = u + at$

$u = 0$ | $= 0 + 4.4(1.20)$

$V =$ | $= \underline{0.527\text{ms}^{-1}}$ STO

$a = 4.4$

$t = 0.120$



$m_P u_P + m_Q u_Q = m_R V_R$

$\cancel{m} \times 4.32 + 2\cancel{m} \times 0.527 = 3\cancel{m} V_Q$

$5.375 = 3V_Q$

$\underline{V_Q = 1.79\text{ms}^{-1}}$