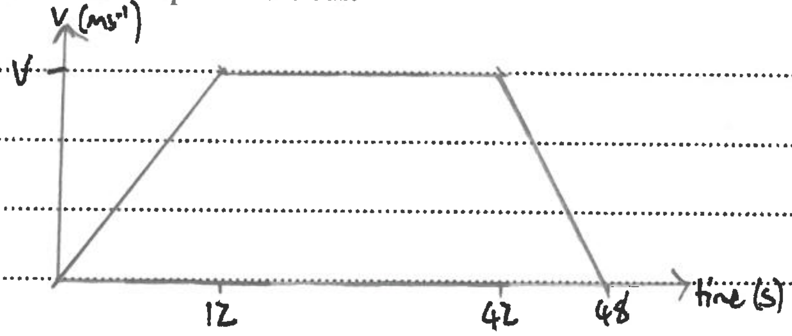
1 A bus moves from rest with constant acceleration for 12 s. It then moves with constant speed for 30 s before decelerating uniformly to rest in a further 6 s. The total distance travelled is 585 m.

(a) Find the constant speed of the bus. [2]



distance = area:

$585 = \frac{1}{2}(30 + 48) \times V$

$585 = \frac{1}{2} \times 78 \times V$

$585 = 39V$

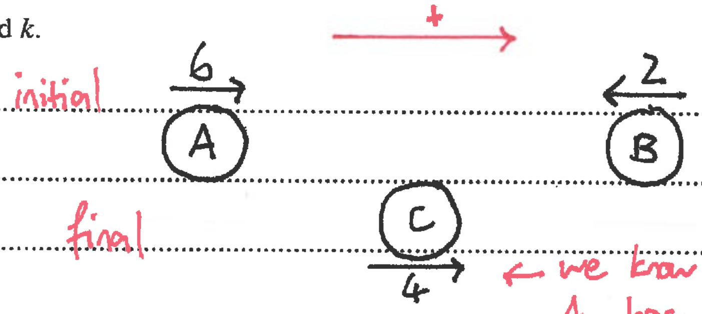
$V = 15 \text{ ms}^{-1}$

(b) Find the magnitude of the deceleration. [1]

$s =$	$v = u + at$
$u = 15$	$0 = 15 + 6a$
$v = 0$	$6a = -15$
$a =$	$a = -2.5$
$t = 6$	magnitude of $a = 2.5 \text{ ms}^{-2}$

2 Two small smooth spheres A and B, of equal radii and of masses km kg and m kg respectively, where $k > 1$, are free to move on a smooth horizontal plane. A is moving towards B with speed $6\,\mathrm{m\,s^{-1}}$ and B is moving towards A with speed $2\,\mathrm{m\,s^{-1}}$. After the collision A and B coalesce and move with speed $4\,\mathrm{m\,s^{-1}}$.

(a) Find k. [3]



← we know C is moving right because A has more momentum than B (6 > 2 and km > m)

$$m_A u_A + m_B u_B = m_C v_C$$
$$km \times 6 + m \times -2 = (km + m) \times 4$$
$$6km - 2m = 4km + 4m \quad \div m$$
$$6k - 2 = 4k + 4$$
$$2k = 6$$
$$\underline{k = 3}$$

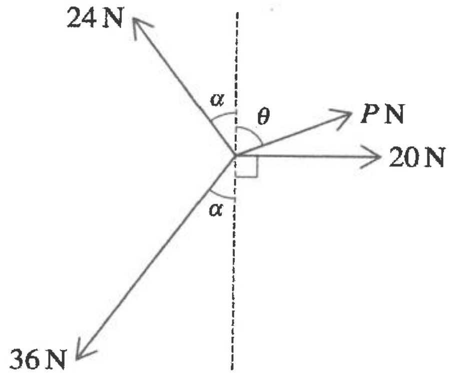
(b) Find, in terms of m, the loss of kinetic energy due to the collision. [2]

$$KE_{initial} = \frac{1}{2} \times 3m \times 6^2 + \frac{1}{2} \times m \times 2^2$$
$$= 54m + 2m$$
$$= \underline{56m\ J}$$

$$KE_{final} = \frac{1}{2} \times 4m \times 4^2$$
$$= \underline{32m\ J}$$

$$\text{Loss in KE} = 56m - 32m = \underline{24m\ J}$$

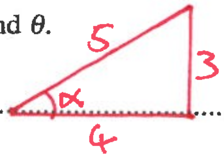
3



Coplanar forces of magnitudes 24 N, P N, 20 N and 36 N act at a point in the directions shown in the diagram. The system is in equilibrium.

Given that $\sin\alpha = \frac{3}{5}$, find the values of P and θ. [6]

$\sin\alpha = \frac{3}{5} \rightarrow$



$\cos\alpha = \frac{4}{5}$

$\tan\alpha = \frac{3}{4}$

$R(\uparrow)$: $24\cos\alpha + P\cos\theta - 36\cos\alpha = 0$

$24 \times \frac{4}{5} + P\cos\theta - 36 \times \frac{4}{5} = 0$

$19.2 + P\cos\theta - 28.8 = 0$

$P\cos\theta - 9.6 = 0$

$P\cos\theta = 9.6$ ①

$R(\rightarrow)$: $20 + P\sin\theta - 24\sin\alpha - 36\sin\alpha = 0$

$20 + P\sin\theta - 24 \times \frac{3}{5} - 36 \times \frac{3}{5} = 0$

$20 + P\sin\theta - 14.4 - 21.6 = 0$

$P\sin\theta - 16 = 0$

$P\sin\theta = 16$ ②

② ÷ ①: $\frac{P\sin\theta}{P\cos\theta} = \frac{16}{9.6}$

$\tan\theta = \frac{5}{3}$

$\theta = 59.0°$

STO

Sub into ①:

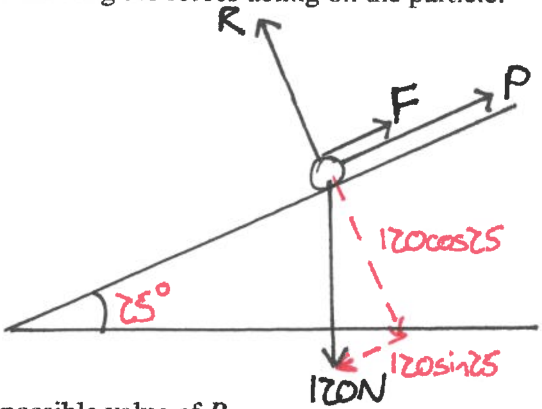
$P\cos(59.0) = 9.6$

$P = \frac{9.6}{\cos(59.0)}$

$= 18.7\text{N}$

4 A particle of mass 12 kg is stationary on a rough plane inclined at an angle of 25° to the horizontal. A force of magnitude P N acting parallel to a line of greatest slope of the plane is used to prevent the particle sliding down the plane. The coefficient of friction between the particle and the plane is 0.35.

(a) Draw a sketch showing the forces acting on the particle. [1]



(b) Find the least possible value of P. [5]

Least possible value of P is when particle is in limiting equilibrium, about to slide down the plane, so friction is up the plane.

R(↖): $R - 120\cos 25 = 0$

$\underline{R = 120\cos 25 \text{ N}}$

R(↙): $120\sin 25 - F - P = 0$

$120\sin 25 - \mu R - P = 0$

$120\sin 25 - 0.35 \times 120\cos 25 - P = 0$

$120\sin 25 - 42\cos 25 - P = 0$

$P = 120\sin 25 - 42\cos 25$

$= \underline{12.6 \text{N}}$

$\sin\alpha = 0.12$

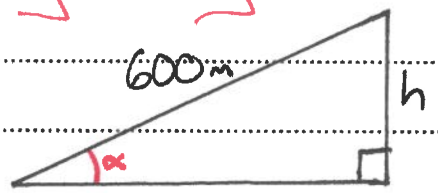
5 A car of mass 1600 kg travels at constant speed $20\,\mathrm{m\,s^{-1}}$ up a straight road inclined at an angle of $\sin^{-1} 0.12$ to the horizontal.

(a) Find the change in potential energy of the car in 30 s. [3]

Constant speed, so distance = speed × time: $d = 20 \times 30$

$= 600\,\mathrm{m}$

Change in height when travelling 600m:



$\sin\alpha = \frac{h}{600}$

$h = 600\sin\alpha$

$= 600 \times 0.12$

$= 72\,\mathrm{m}$

Change in PE $= mg\Delta h$

$= 1600 \times 10 \times 72$

$= \underline{1\,152\,000\ \mathrm{J}}$ (increase)

(b) Given that the total work done by the engine of the car in this time is 1960 kJ, find the constant force resisting the motion. [3]

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$1960000 + \frac{1}{2}\times1600\times20^2 + 0 = \frac{1}{2}\times1600\times20^2 + 1152000 + \underbrace{600F}_{F\times d}$

$1\,960\,000 + \cancel{320\,000} = \cancel{320\,000} + 1\,152\,000 + 600F$

$1\,960\,000 = 1\,152\,000 + 600F$

$808\,000 = 600F$

$F = \underline{1350\,\mathrm{N}}$ (3sf)

STO

(c) Calculate, in kW, the power developed by the engine of the car. [2]

$$\text{Power} = \frac{\text{Work done}}{\text{time}}$$

$$= \frac{1\,960\,000}{30}$$

$$= 65333 \text{ W}$$

STO

$$= 65.3 \text{ kW (3sf)}$$

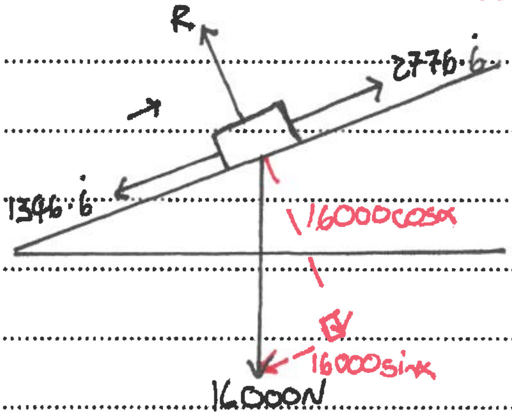
(d) Given that this power is suddenly decreased by 15%, find the instantaneous deceleration of the car. [3]

$65333 \times 0.85 = 55533$

$\text{Power} = Dv$

$55533 = D \times 20$

$D = 2776.\dot{6}$ STO



$R(\nearrow)$: $F = ma$

$2776.\dot{6} - 1346.\dot{6} - 16000\sin\alpha = 1600a$

$1430 - 16000\sin\alpha = 1600a$

$1430 - 16000 \times 0.12 = 1600a$

$1430 - 1920 = 1600a$

$-490 = 1600a$

$a = -0.306 \text{ ms}^{-2}$

$\rightarrow$ deceleration $= 0.306 \text{ ms}^{-2}$

6 A particle P moves in a straight line starting from a point O and comes to rest 14 s later. At time t s after leaving O, the velocity $v\ \mathrm{m\,s^{-1}}$ of P is given by

$$\text{A} \quad v = pt^2 - qt \qquad 0 \leqslant t \leqslant 6,$$
$$\text{B} \quad v = 63 - 4.5t \qquad 6 \leqslant t \leqslant 14,$$

where p and q are positive constants.

The acceleration of P is zero when $t = 2$.

(a) Given that there are no instantaneous changes in velocity, find p and q. [3]

$a = \frac{dv}{dt} = 2pt - q$

$a = 0$ when $t = 2$:

$0 = 2p \times 2 - q$

$0 = 4p - q$ ①

No instantaneous change in velocity so velocity at $t = 6$s must be same in both equations:

A: $v = p \times 6^2 - q \times 6$

$v = 36p - 6q$

B: $v = 63 - 4.5 \times 6$

$v = 36$

$36p - 6q = 36 \quad \div 6$

$6p - q = 6$ ②

② − ①: $2p = 6$

$\underline{p = 3}$

①: $0 = 4(3) - q$

$0 = 12 - q$

$\underline{q = 12}$

(b) Sketch the velocity-time graph. [3]

A: $V = 3t^2 - 12t$

quadratic with roots at: $3t^2 - 12t = 0$

$t^2 - 4t = 0$

$t(t-4) = 0$

$\underline{t = 0},\ \underline{t = 4}$

turning point: $V = 3(t^2 - 4t)$

$= 3[(t-2)^2 - 4]$

$= 3(t-2)^2 - 12$

$\underline{(2, -12)}$

B: $V = 63 - 4.5t$

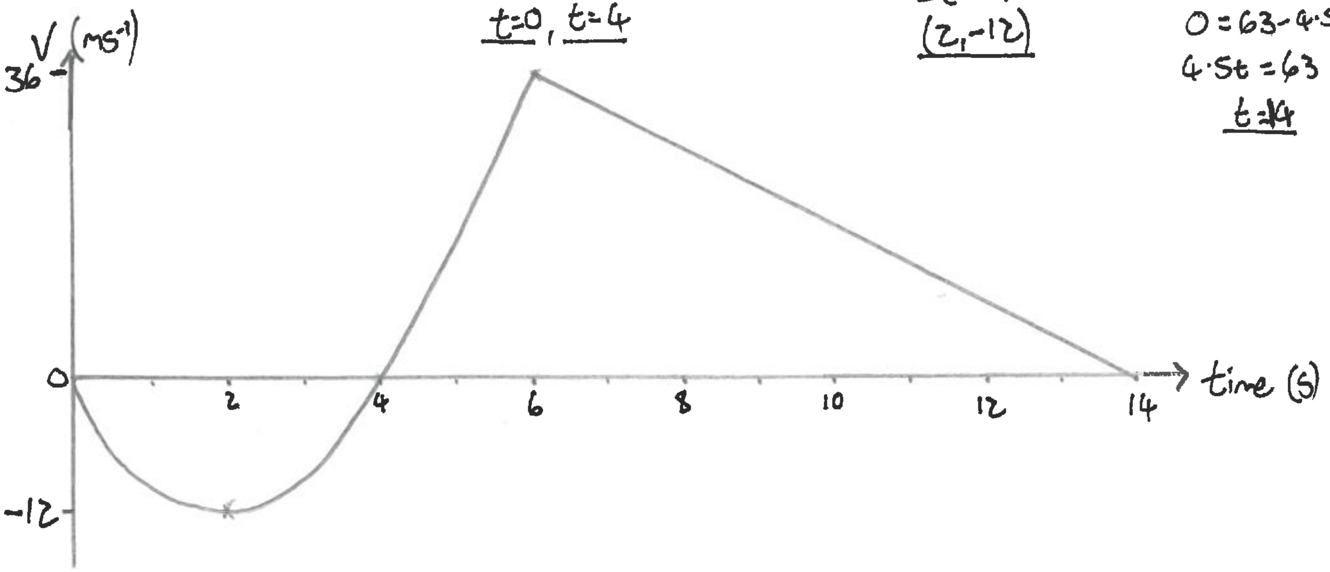
Straight line

$V = 0$:

$0 = 63 - 4.5t$

$4.5t = 63$

$\underline{t = 14}$



(c) Find the total distance travelled by P during the 14 s. [5]

$0 \to 6$s: Need to find displacements between turning points of s. These occur when $v=0$, so at $t=0$, $t=4$, $t=6$:

$0 \to 4$s: $s = \int_0^4 (3t^2 - 12t)\,dt$

$= \left[t^3 - 6t^2\right]_0^4$

$= \left[4^3 - 6(4)^2\right] - \left[0\right] = -32\text{m}$

$4 \to 6$s: $s = \int_4^6 (3t^2 - 12t)\,dt$

$= \left[t^3 - 6t^2\right]_4^6$

$= \left[6^3 - 6(6)^2\right] - \left[4^3 - 6(4)^2\right]$

$= \left[0\right] - \left[-32\right] = 32\text{m}$

distance travelled between 0 and 6s: $d = 32 + 32 = 64\text{m}$

$6 \to 14$s: Find area of triangle on v-t graph:

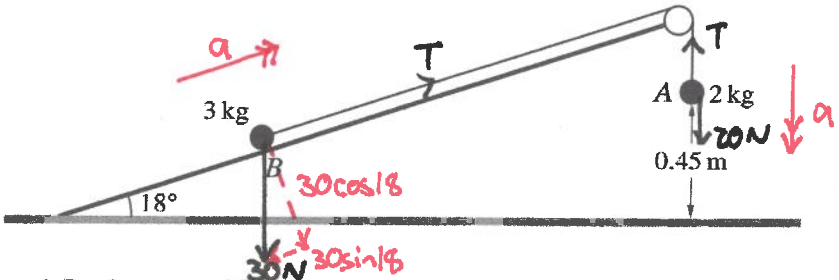
Area $= \frac{1}{2} \times 8 \times 36$

$= 144\text{m}$

Total distance $= 64 + 144$

$= 208\text{m}$

7



Two particles A and B of masses 2 kg and 3 kg respectively are connected by a light inextensible string. Particle B is on a smooth fixed plane which is at an angle of 18° to horizontal ground. The string passes over a fixed smooth pulley at the top of the plane. Particle A hangs vertically below the pulley and is 0.45 m above the ground (see diagram). The system is released from rest with the string taut. When A reaches the ground, the string breaks.

Find the total distance travelled by B before coming to instantaneous rest. You may assume that B does not reach the pulley. [8]

A:

$R(\downarrow): 20 - T = ma$

$20 - T = 2a$ ①

B:

$R(\nearrow): T - 30\sin 18 = ma$

$T - 30\sin 18 = 3a$ ②

① + ②:

$20 - 30\sin 18 = 5a$

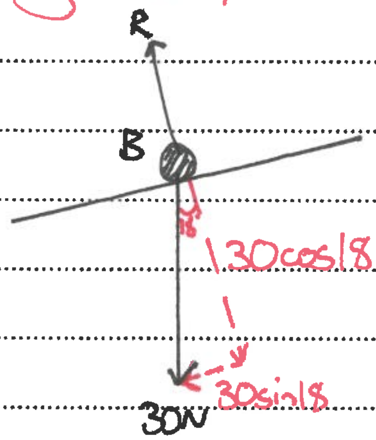
$a = \dfrac{20 - 30\sin 18}{5}$

$a = 2.146\ \text{ms}^{-2}$ STO

Find speed of B when A hits ground:

$s = 0.45$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2(2.146) \times 0.45$
$v =$	$v^2 = 1.931$
$a = 2.146$	$v = 1.39\ \text{ms}^{-1}$
$t =$	STO

String breaks, so no more tension. Re-draw force diagram:



$R(\nearrow): -30\sin 18 = ma$

$-30\sin 18 = 3a$

$a = \underline{-3.09\,ms^{-2}}$ STO

Find distance travelled by B after string has snapped:

$\overset{+}{\rightarrow}$ s =	$v^2 = u^2 + 2as$
u = 1.39	$0^2 = 1.39^2 + 2(-3.09) \times s$
v = 0	$0 = 1.931 - 6.18s$
a = -3.09	$6.18s = 1.931$
t =	$\underline{s = 0.312\,m}$

when attached — after string broke

Total distance travelled by B = 0.45 + 0.312

= $\underline{0.762\,m}$