1 A particle P is projected vertically upwards with speed $v\,\mathrm{m\,s^{-1}}$ from a point on the ground. P reaches its greatest height after 3 s.

(a) Find v. [1]

↑+ $s =$ | $v = u + at$
$u =$ | $0 = u + (-10) \times 3$
$v = 0$ | $0 = u - 30$
$a = -10$ | $u = 30\,\mathrm{ms^{-1}}$
$t = 3$

(b) Find the greatest height of P above the ground. [2]

↑+ $s =$ | $s = \frac{1}{2}(u+v) \times t$
$u = 30$ | $= \frac{1}{2}(30+0) \times 3$
$v = 0$ | $= \frac{1}{2} \times 30 \times 3$
$a = -10$ | $= 45\,\mathrm{m}$
$t = 3$

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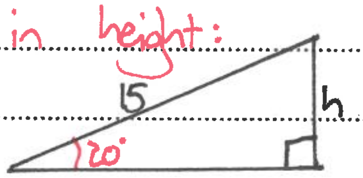
2 A box of mass 5 kg is pulled at a constant speed a distance of 15 m up a rough plane inclined at an angle of 20° to the horizontal. The box moves along a line of greatest slope against a frictional force of 40 N. The force pulling the box is parallel to the line of greatest slope.

(a) Find the work done against friction. [1]

Work done = F × d
= 40 × 15
= 600 J

(b) Find the change in gravitational potential energy of the box. [2]

change in height:



$\sin 20 = \frac{h}{15}$

$h = 15 \sin 20$

change in P.E. = $mg\Delta h$
$= 5 \times 10 \times 15 \sin 20$
$= 257$ J

STO

(c) Find the work done by the pulling force. [1]

$Work_{in} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{out}$

$W + \frac{1}{2} \times 5 \times v^2 + 0 = \frac{1}{2} \times 5 \times v^2 + 257 + 600$

$W + \cancel{2.5v^2} = \cancel{2.5v^2} + 857$

$W = 857$ J

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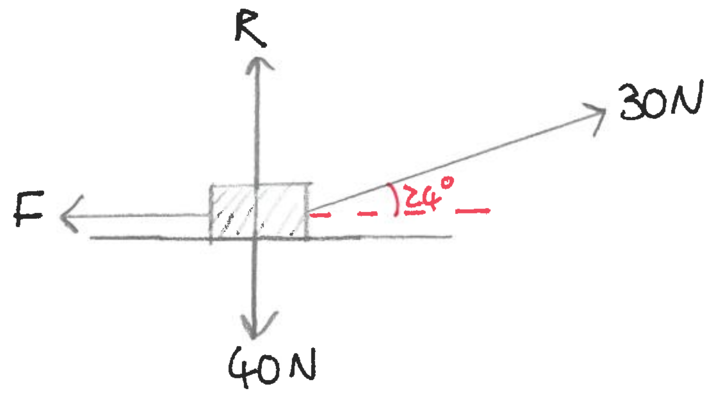
3 A string is attached to a block of mass 4 kg which rests in limiting equilibrium on a rough horizontal table. The string makes an angle of 24° above the horizontal and the tension in the string is 30 N.

(a) Draw a diagram showing all the forces acting on the block. [1]



(b) Find the coefficient of friction between the block and the table. [5]

$R(\uparrow): \quad R + 30\sin 24 - 40 = 0$

$R = 40 - 30\sin 24$

$= \underline{27.798}\ N$

STO

$R(\rightarrow): \quad 30\cos 24 - F = 0$

$30\cos 24 - \mu R = 0$ (limiting eqⁿ so $F = \mu R$)

$30\cos 24 - \mu \times 27.798 = 0$

$27.798\mu = 30\cos 24$

$\mu = \dfrac{30\cos 24}{27.798}$

$\mu = \underline{\underline{0.986}}$

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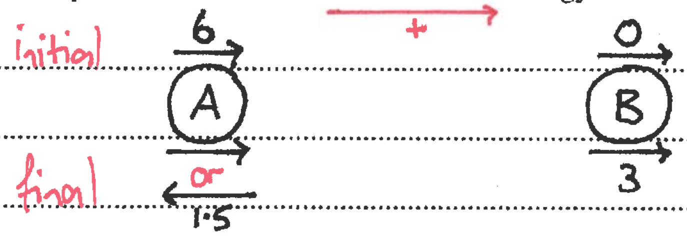
4 Two small smooth spheres A and B, of equal radii and of masses 4 kg and m kg respectively, lie on a smooth horizontal plane. Initially, sphere B is at rest and A is moving towards B with speed $6\,\mathrm{m\,s^{-1}}$. After the collision A moves with speed $1.5\,\mathrm{m\,s^{-1}}$ and B moves with speed $3\,\mathrm{m\,s^{-1}}$.

Find the two possible values of the loss of kinetic energy due to the collision. [6]



Scenario 1: A contines to move to the right after collision:

$m_A u_A + m_B u_B = m_A v_A + m_B v_B$

$4 \times 6 + 0 = 4 \times 1.5 + m \times 3$

$24 = 6 + 3m$

$18 = 3m$

$\underline{m = 6\text{kg}}$

Scenario 2: A rebounds and moves to the left after collision:

$m_A u_A + m_B u_B = m_A v_A + m_B v_B$

$4 \times 6 + 0 = 4 \times -1.5 + m \times 3$

$24 = -6 + 3m$

$30 = 3m$

$\underline{m = 10\text{kg}}$

Scenario 1:

$KE_{init} = \frac{1}{2} \times 4 \times 6^2 = 72\text{J}$ $\qquad KE_{final} = \frac{1}{2} \times 4 \times 1.5^2 + \frac{1}{2} \times 6 \times 3^2 = 31.5\text{J}$

Loss of KE $= 72 - 31.5 = \underline{40.5\text{J}}$

Scenario 2:

$KE_{init} = \frac{1}{2} \times 4 \times 6^2 = 72\text{J}$ $\qquad KE_{final} = \frac{1}{2} \times 4 \times 1.5^2 + \frac{1}{2} \times 10 \times 3^2 = 49.5\text{J}$

Loss of KE $= 72 - 49.5 = \underline{22.5\text{J}}$

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5 A particle P moves in a straight line. It starts at a point O on the line and at time t s after leaving O it has velocity v m s^{-1}, where $v = 4t^2 - 20t + 21$.

(a) Find the values of t for which P is at instantaneous rest. [2]

$V = 0$:

$4t^2 - 20t + 21 = 0$

factorise:

$4t^2 - 6t - 14t + 21 = 0$

$2t(2t-3) - 7(2t-3) = 0$

$(2t-7)(2t-3) = 0$

$2t - 7 = 0$ or $2t - 3 = 0$

$2t = 7$ $\quad$ $2t = 3$

$\underline{t = 3.5\text{s}}$ or $\underline{t = 1.5\text{s}}$

(b) Find the initial acceleration of P. [2]

$a = \frac{dv}{dt} = 8t - 20$

initial acceleration: $t = 0$:

$a = 8(0) - 20$

$= \underline{-20\text{ms}^{-2}}$

(c) Find the minimum velocity of P. [2]

minimum point of V is when $\frac{dv}{dt} = 0$:

$8t - 20 = 0$

$8t = 20$

$t = 2.5$

Sub into V:

$V = 4(2.5)^2 - 20(2.5) + 21$

$= \underline{-4\text{ms}^{-1}}$

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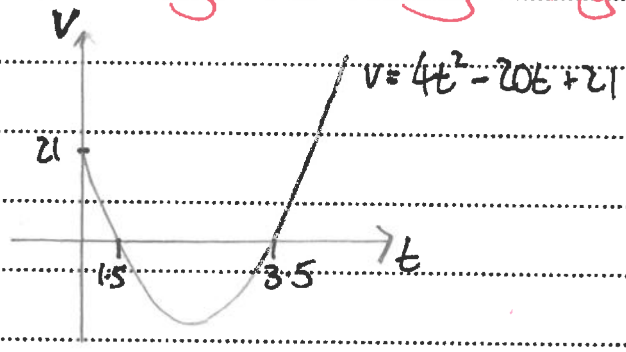
(d) Find the distance travelled by P during the time when its velocity is negative. [4]

It helps to envisage the velocity-time graph:



→ velocity is negative between 1.5s and 3.5s

$S = \int (4t^2 - 20t + 21)\,dt$

$S = \frac{4}{3}t^3 - 10t^2 + 21t + C$

S=0 when t=0:

$0 = 0 - 0 + 0 + C$

$\rightarrow S = \frac{4}{3}t^3 - 10t^2 + 21t$

The turning points of S are at 1.5s and 3.5s, so we don't need to worry about them:

t=1.5:

$S = \frac{4}{3}(1.5)^3 - 10(1.5)^2 + 21(1.5)$

$= 4.5 - 22.5 + 31.5$

$= \underline{13.5m}$

t=3.5:

$S = \frac{4}{3}(3.5)^3 - 10(3.5)^2 + 21(3.5)$

$= \frac{343}{6} - 122.5 + 73.5$

$= \underline{\frac{49}{6} m}$ → distance travelled $= 13.5 - \frac{49}{6}$

$= \underline{5.33m}$

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6 A car of mass 1600 kg is pulling a caravan of mass 800 kg. The car and the caravan are connected by a light rigid tow-bar. The resistances to the motion of the car and caravan are 400 N and 250 N respectively.

(a) The car and caravan are travelling along a straight horizontal road.

(i) Given that the car and caravan have a constant speed of $25\,\mathrm{m\,s^{-1}}$, find the power of the car's engine. [2]

Constant speed, so $a = 0$. Consider whole system:

$R(\rightarrow)$: $D - 400 - 250 = 0$

$D = 650\,\mathrm{N}$

Power $= D \times V$

$= 650 \times 25$

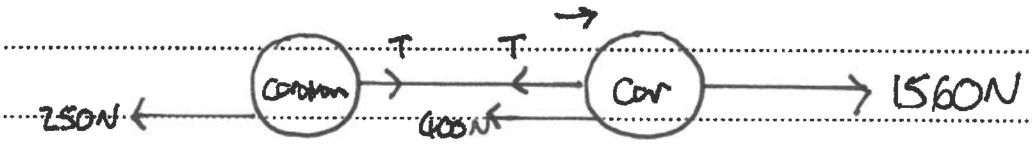
$= 16250\,\mathrm{W}$

(ii) The engine's power is now suddenly increased to 39 kW. Find the instantaneous acceleration of the car and caravan and find the tension in the tow-bar. [5]

Power $= D \times V$

$39000 = D \times 25$

$D = 1560\,\mathrm{N}$



Car:

$R(\rightarrow)$: $F = ma$

$1560 - 400 - T = 1600a$

$1160 - T = 1600a$ (1)

Caravan:

$R(\rightarrow)$: $F = ma$

$T - 250 = 800a$ (2)

cont.

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①+②: $1160 - 250 = 2400a$

$910 = 2400a$

$a = 0.379\,ms^{-2}$ STO

→②: $T - 250 = 800 \times 0.379$

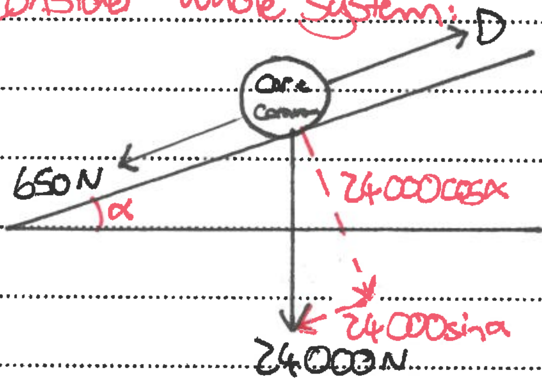
$T - 250 = 303.3$

$T = 553\,N$

(b) The car and caravan now travel up a straight hill, inclined at an angle of $\sin^{-1} 0.05$ to the horizontal, at a constant speed of v m s^{-1}. The car's engine is working at 32.5 kW. ↺ $\sin\alpha = 0.05$

Find v. [3]

Consider whole system:



$a = 0$ (constant speed)

$R(\nearrow)$: $D - 650 - 24000\sin\alpha = ma$

$D - 650 - 24000 \times 0.05 = 0$

$D - 650 - 1200 = 0$

$D = 1850\,N$

Power $= D \times v$

$32500 = 1850v$

$v = 17.6\,ms^{-1}$

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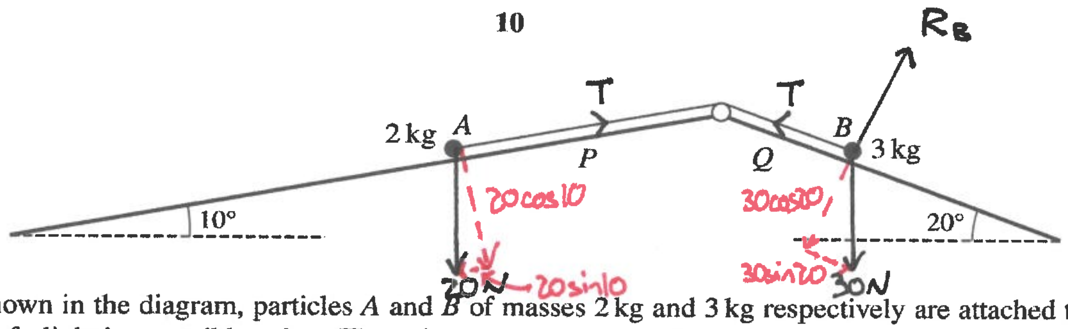

As shown in the diagram, particles A and B of masses 2 kg and 3 kg respectively are attached to the ends of a light inextensible string. The string passes over a small fixed smooth pulley which is attached to the top of two inclined planes. Particle A is on plane P, which is inclined at an angle of 10° to the horizontal. Particle B is on plane Q, which is inclined at an angle of 20° to the horizontal. The string is taut, and the two parts of the string are parallel to lines of greatest slope of their respective planes.

(a) It is given that plane P is smooth, plane Q is rough, and the particles are in limiting equilibrium.

Find the coefficient of friction between particle B and plane Q. [5]

Without friction, B would move down the plane, so friction is acting up the plane.

P:

$R(\rightarrow): T - 20\sin 10 = 0$

$T = 20\sin 10$

Q:

$R(\uparrow): R_B - 30\cos 20 = 0$

$R_B = 30\cos 20$

$R(\searrow): 30\sin 20 - T - F = 0$

$30\sin 20 - 20\sin 10 - \mu R = 0$

$30\sin 20 - 20\sin 10 - \mu \times 30\cos 20 = 0$

$\mu \times 30\cos 20 = 30\sin 20 - 20\sin 10$

$\mu = \dfrac{30\sin 20 - 20\sin 10}{30\cos 20}$

$\mu = 0.241$

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(b) It is given instead that both planes are smooth and that the particles are released from rest at the same horizontal level.

Find the time taken until the difference in the vertical height of the particles is 1 m. [You should assume that this occurs before A reaches the pulley or B reaches the bottom of plane Q.] [6]

P:

$R(\nearrow): T - 20\sin 10 = 2a$ ①

Q:

$R(\searrow): 30\sin 20 - T = 3a$ ②

① + ②:

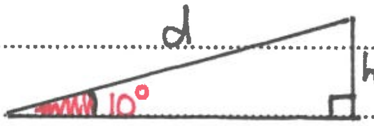
$$30\sin 20 - 20\sin 10 = 5a$$

$$a = \frac{30\sin 20 - 20\sin 10}{5}$$

$$a = 1.3575 \text{ ms}^{-2}$$

STO

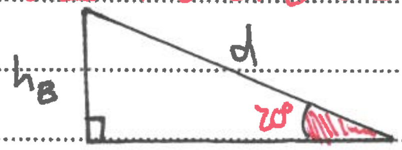
If A moves a distance "d" up the plane:



$\sin 10 = \frac{h_A}{d}$

$\rightarrow h_A = d\sin 10$

If B moves the same distance, "d" down the plane:



$\sin 20 = \frac{h_B}{d}$

$\rightarrow h_B = d\sin 20$

Since A moves up and B moves down, we want $h_A + h_B = 1$:

$d\sin 10 + d\sin 20 = 1$

$d(\sin 10 + \sin 20) = 1$

$d = \frac{1}{\sin 10 + \sin 20}$

$d = 1.939$ m

STO

$s = 1.939$
$u = 0$
$v =$
$a = 1.3575$
$t =$

$s = ut + \frac{1}{2}at^2$

$1.939 = 0 + \frac{1}{2} \times 1.3575\, t^2$

$1.939 = 0.6788t^2$

$t^2 = 2.857$

$t = 1.69$s

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