1 Two particles P and Q, of masses 0.2 kg and 0.5 kg respectively, are at rest on a smooth horizontal plane. P is projected towards Q with speed $2\,\mathrm{m\,s^{-1}}$.

(a) Write down the momentum of P. [1]

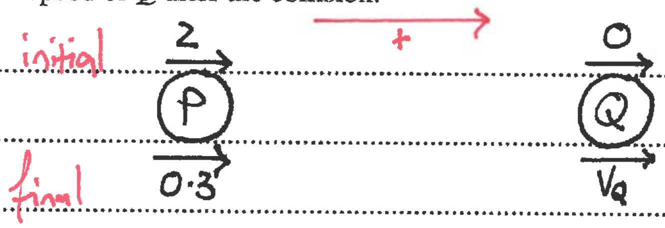
Momentum $= mv$

$= 0.2 \times 2$

$= 0.4\ \mathrm{kgms^{-1}}$

(b) After the collision P continues to move in the same direction with speed $0.3\,\mathrm{m\,s^{-1}}$.

Find the speed of Q after the collision. [2]



$m_P u_P + m_Q u_Q = m_P V_P + m_Q V_Q$

$0.2 \times 2 + 0 = 0.2 \times 0.3 + 0.5 \times V_Q$

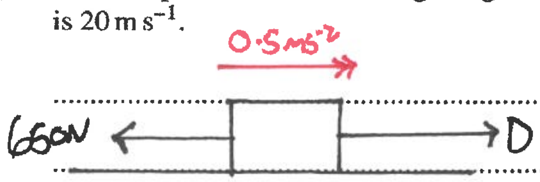
$0.4 = 0.06 + 0.5 V_Q$

$0.34 = 0.5 V_Q$

$V_Q = 0.68\ \mathrm{ms^{-1}}$

2 A car of mass 1800 kg is travelling along a straight horizontal road. The power of the car's engine is constant. There is a constant resistance to motion of 650 N.

(a) Find the power of the car's engine, given that the car's acceleration is $0.5\,\mathrm{m\,s^{-2}}$ when its speed is $20\,\mathrm{m\,s^{-1}}$. [3]



$R(\rightarrow)$: $F = ma$

$D - 650 = 1800 \times 0.5$

$D - 650 = 900$

$D = 1550\,N$

Power $= D \times V$

$= 1550 \times 20$

$= 31000\,W$

(b) Find the steady speed which the car can maintain with the engine working at this power. [2]

At steady speed, $a = 0$, so Driving force = resistance:

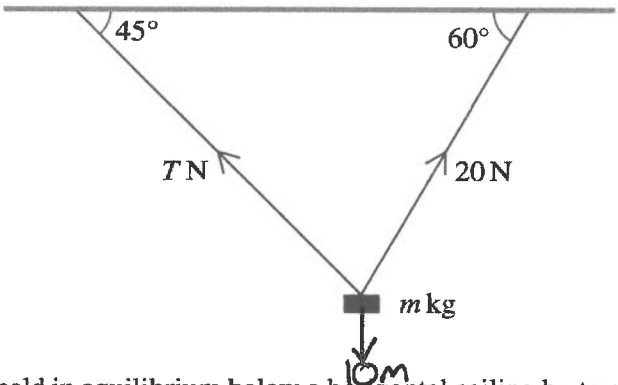
Driving force $= 650\,N$

Power $= D \times V$

$31000 = 650V$

$V = 47.7\,\mathrm{ms^{-1}}$

3



A block of mass m kg is held in equilibrium below a horizontal ceiling by two strings, as shown in the diagram. One of the strings is inclined at 45° to the horizontal and the tension in this string is T N. The other string is inclined at 60° to the horizontal and the tension in this string is 20 N.

Find T and m. [5]

$R(\rightarrow): 20\cos 60 - T\cos 45 = 0$

$10 - T\cos 45 = 0$

$T\cos 45 = 10$

$T = 10\sqrt{2}$ N

$R(\uparrow): T\sin 45 + 20\sin 60 - 10m = 0$

$10\sqrt{2}\sin 45 + 10\sqrt{3} - 10m = 0$

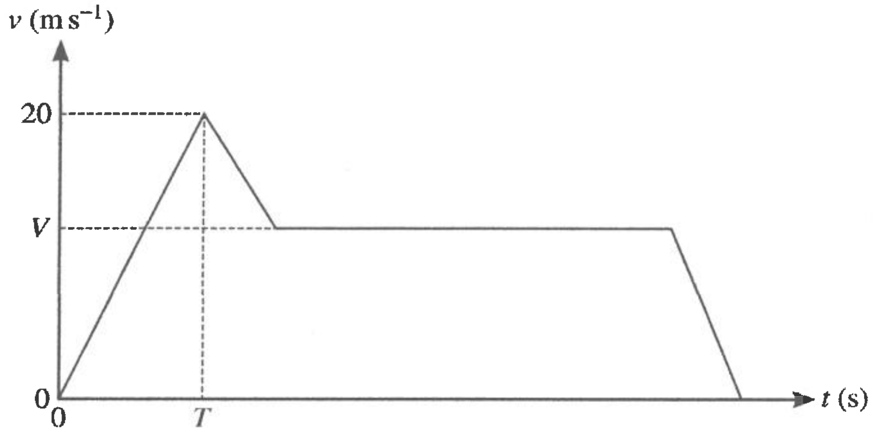
$10 + 10\sqrt{3} - 10m = 0$

$10m = 10 + 10\sqrt{3}$

$m = 1 + \sqrt{3}$

$= 2.73$ kg

4

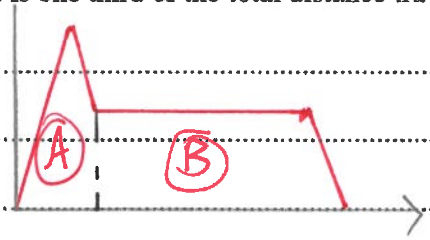


The diagram shows a velocity-time graph which models the motion of a car. The graph consists of four straight line segments. The car accelerates at a constant rate of $2\,\text{m}\,\text{s}^{-2}$ from rest to a speed of $20\,\text{m}\,\text{s}^{-1}$ over a period of T s. It then decelerates at a constant rate for 5 seconds before travelling at a constant speed of $V\,\text{m}\,\text{s}^{-1}$ for 27.5 s. The car then decelerates to rest at a constant rate over a period of 5 s.

(a) Find T. **[1]**

$s =$	$v = u + at$
$u = 0$	$20 = 0 + 2t$
$v = 20$	$20 = 2t$
$a = 2$	$t = 10$
$t =$	$\rightarrow$ $T = 10\,s$

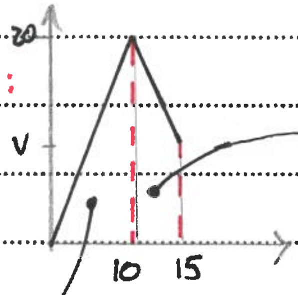
(b) Given that the distance travelled up to the point at which the car begins to move with constant speed is one third of the total distance travelled, find V. [4]



If $A = \frac{1}{3}$ Total and $B = \frac{2}{3}$ Total

Then $B = 2 \times A$

Area A:



$\text{area} = \frac{1}{2}(a+b) \times h$

$= \frac{1}{2}(20+V) \times 5$

$= 2.5(20+V)$

$= 50 + 2.5V$

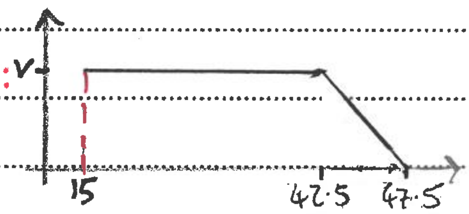
$\text{area} = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 10 \times 20$

$= 100\text{ m}$

$\rightarrow A = 150 + 2.5V$

Area B:



$\text{Area} = \frac{1}{2}(a+b) \times h$

$= \frac{1}{2}(27.5 + 32.5) \times V$

$= \frac{1}{2} \times 60 \times V$

$= 30V$

$B = 2 \times A$:

$30V = 2(150 + 2.5V)$

$30V = 300 + 5V$

$25V = 300$

$V = 12\text{ ms}^{-1}$

5 A particle is projected vertically upwards with speed $40\,\text{m}\,\text{s}^{-1}$ alongside a building of height h m.

(a) Given that the particle is above the level of the top of the building for 4 s, find h. [4]

Find maximum height of particle:

$\uparrow +$ $s =$	$v^2 = u^2 + 2as$
$u = 40$	$0^2 = 40^2 + 2(-10) \times s$
$v = 0$	$0 = 1600 - 20s$
$a = -10$	$20s = 1600$
$t =$	$s = 80\,\text{m}$

Find height 2s later, on the way down:
$\leftarrow 4 \div 2$

$\downarrow +$ $s =$	$s = ut + \frac{1}{2}at^2$
$u = 0$	$s = 0 + \frac{1}{2}(10) \times 2^2$
$v =$	$s = 5 \times 4$
$a = 10$	$= 20$
$t = 2$	

height above ground $= 80 - 20$
$= \underline{\underline{60\,\text{m}}}$

(b) One second after the first particle is projected, a second particle is projected vertically upwards from the top of the building with speed $20\,\text{m}\,\text{s}^{-1}$.

Denoting the time after projection of the first particle by t s, find the value of t for which the two particles are at the same height above the ground. [4]

First particle (from ground):

↑+ $s =$ | $s = ut + \frac{1}{2}at^2$
$u = 40$ | $= 40t + \frac{1}{2}(-10) \times t^2$
$v =$ | $= 40t - 5t^2$
$a = -10$
$t =$ | $\underline{s_1 = 40t - 5t^2}$

Second particle (from top of building):

↑+ $s =$ | $s = ut + \frac{1}{2}at^2$
$u = 20$ | $= 20(t-1) + \frac{1}{2}(-10)(t-1)^2$
$v =$ | $= 20t - 20 - 5(t^2 - 2t + 1)$
$a = -10$ | $= 20t - 20 - 5t^2 + 10t - 5$
$t = t-1$ | $\underline{s = -5t^2 + 30t - 25}$

has been travelling for 1s less than first particle

height of particle 2 above ground = $s + 60$:

$\underline{s_2 = -5t^2 + 30t + 35}$

$s_1 = s_2$:

$40t - 5t^2 = -5t^2 + 30t + 35$

$40t = 30t + 35$

$10t = 35$

$\underline{\underline{t = 3.5\text{s}}}$

6 A block of mass 5 kg is placed on a plane inclined at 30° to the horizontal. The coefficient of friction between the block and the plane is μ.

(a)

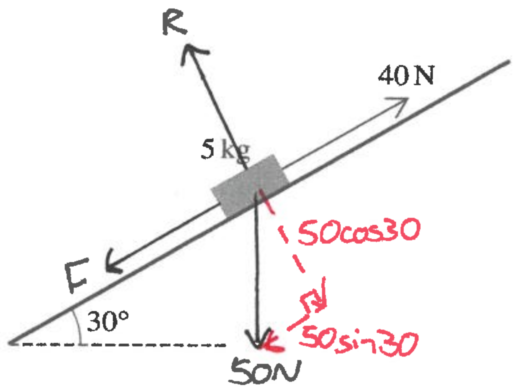


Fig. 6.1

When a force of magnitude 40 N is applied to the block, acting up the plane parallel to a line of greatest slope, the block begins to slide up the plane (see Fig. 6.1).

Show that $\mu < \frac{1}{5}\sqrt{3}$. [4]

$R(\nwarrow): \quad R - 50\cos 30 = 0$

$R = 50\cos 30$

$\underline{R = 25\sqrt{3}\ N}$

$R(\nearrow): \quad 40 - 50\sin 30 - F > 0$

$40 - 25 - \mu R > 0$ because particle is accelerating up the plane

$15 - \mu \times 25\sqrt{3} > 0$

$15 > 25\sqrt{3}\mu$

$25\sqrt{3}\mu < 15$

$\mu < \frac{15}{25\sqrt{3}}$

$\mu < \frac{3}{5\sqrt{3}} \quad \frac{\times\sqrt{3}}{\times\sqrt{3}}$

$\mu < \frac{3\sqrt{3}}{15}$

$\mu < \frac{\sqrt{3}}{5}$ QED

(b)

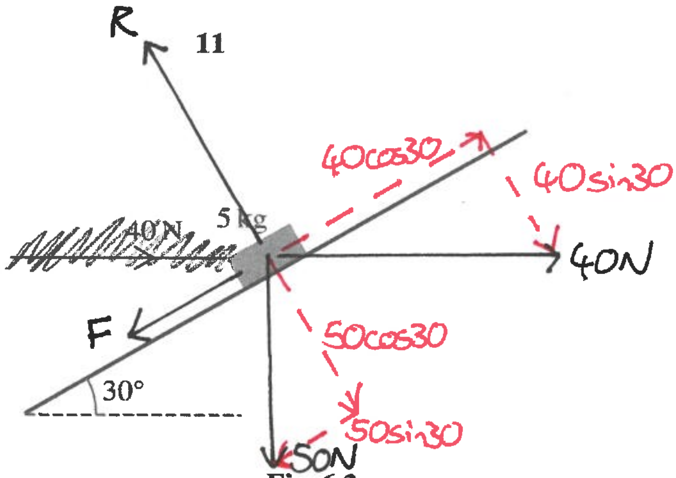


Fig. 6.2

When a force of magnitude 40 N is applied horizontally, in a vertical plane containing a line of greatest slope, the block does not move (see Fig. 6.2).

Show that, correct to 3 decimal places, the least possible value of μ is 0.152. [4]

$40\cos 30 > 50\sin 30$, so without friction it would move up the plane, so friction is down the plane.

Least possible value of μ is when block is in limiting equilibrium ($F = \mu R$)

$R(\nwarrow)$: $R - 40\sin 30 - 50\cos 30 = 0$

$R = 40\sin 30 + 50\cos 30$

$R = \underline{20 + 25\sqrt{3}}$

$R(\nearrow)$: $40\cos 30 - 50\sin 30 - F = 0$

$20\sqrt{3} - 25 - \mu R = 0$

$20\sqrt{3} - 25 - \mu(20 + 25\sqrt{3}) = 0$

$\mu(20 + 25\sqrt{3}) = 20\sqrt{3} - 25$

$$\mu = \frac{20\sqrt{3} - 25}{20 + 25\sqrt{3}}$$

$\mu = \underline{0.152}$ QED

7 A particle P moves in a straight line, starting from a point O with velocity $1.72\,\mathrm{m\,s^{-1}}$. The acceleration $a\,\mathrm{m\,s^{-2}}$ of the particle, t s after leaving O, is given by $a = 0.1t^{\frac{3}{2}}$.

(a) Find the value of t when the velocity of P is $3\,\mathrm{m\,s^{-1}}$. [4]

$V = \int 0.1t^{3/2}\,dt$

$= \frac{2}{5} \times 0.1t^{5/2} + C$

$V = \frac{1}{25}t^{5/2} + C$

when $t=0$, $V=1.72$:

$1.72 = 0 + C$

$\rightarrow V = \frac{1}{25}t^{5/2} + 1.72$

$V=3$:

$3 = \frac{1}{25}t^{5/2} + 1.72$

$1.28 = \frac{1}{25}t^{5/2}$

$t^{5/2} = 32$

$t^5 = 1024$

$\underline{t = 4\,\mathrm{s}}$

(b) Find the displacement of P from O when $t = 2$, giving your answer correct to 2 decimal places. [3]

$S = \int\left(\frac{1}{25}t^{\frac{5}{2}} + 1.72\right)dt$

$S = \frac{2}{7} \times \frac{1}{25}t^{\frac{7}{2}} + 1.72t + C$

when $t = 0$, $S = 0$:

$0 = 0 + 0 + C$

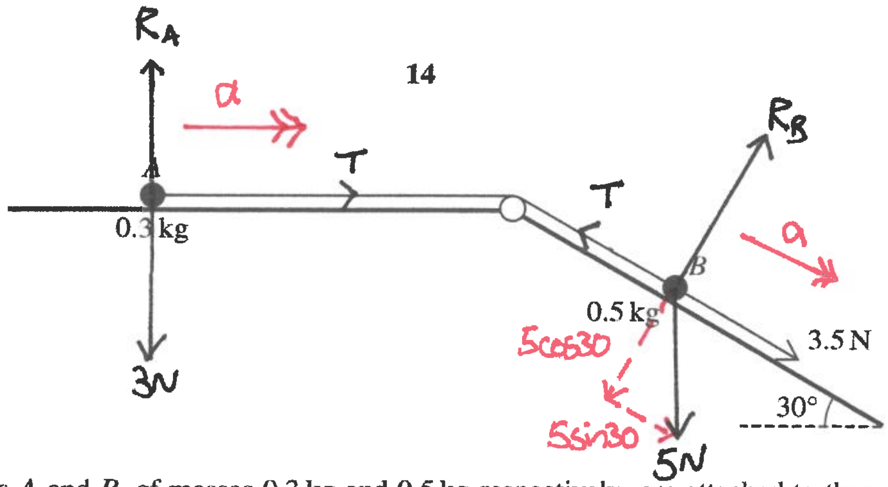
$\rightarrow S = \frac{2}{175}t^{\frac{7}{2}} + 1.72t$

$t = 2$:

$S = \frac{2}{175}(2)^{\frac{7}{2}} + 1.72(2)$

$= \underline{3.57\text{m}}$

8



Two particles A and B, of masses 0.3 kg and 0.5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to a horizontal plane and to the top of an inclined plane. The particles are initially at rest with A on the horizontal plane and B on the inclined plane, which makes an angle of 30° with the horizontal. The string is taut and B can move on a line of greatest slope of the inclined plane. A force of magnitude 3.5 N is applied to B acting down the plane (see diagram).

(a) Given that both planes are smooth, find the tension in the string and the acceleration of B. [5]

A: $R(\rightarrow)$: $T = ma$

$T = 0.3a$ ①

B: $R(\searrow)$: $3.5 + 5\sin 30 - T = ma$

$3.5 + 2.5 - T = 0.5a$

$6 - T = 0.5a$ ②

Sub ① into ②:

$6 - 0.3a = 0.5a$

$6 = 0.8a$

$a = \underline{7.5\text{ms}^{-2}}$

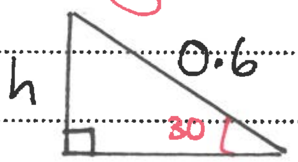
Sub into ①: $T = 0.3 \times 7.5$

$= \underline{2.25\text{ N}}$

(b) It is given instead that the two planes are rough. When each particle has moved a distance of 0.6 m from rest, the total amount of work done against friction is 1.1 J.

Use an energy method to find the speed of B when it has moved this distance down the plane. [You should assume that the string is sufficiently long so that A does not hit the pulley when it moves 0.6 m.] [4]

If both particles have moved a distance of 0.6m, let's find change in height of B:



$\sin 30 = \frac{h}{0.6}$

$h = 0.6\sin 30 = \underline{0.3m}$

Consider the whole system:

$\text{Work}_{IN}(A) + \text{Work}_{IN}(B) + KE_{init}(A) + KE_{init}(B) + PE_{init}(A) + PE_{init}(B)$

$T \times 0.6 + 3.5 \times 0.6 + 0 + 0 + 0 + 0$

$= KE_{fin}(A) + KE_{fin}(B) + PE_{fin}(A) + PE_{fin}(B) + \text{Work}_{OUT}(A) + \text{Work}_{OUT}(B)$

$\frac{1}{2} \times 0.3 \times v^2 + \frac{1}{2} \times 0.5 \times v^2 + 0 + 0.5 \times 10 \times -0.3 + 1.1 + T \times 0.6$

$\rightarrow 0.6T + 2.1 = 0.15v^2 + 0.25v^2 - 1.5 + 1.1 + 0.6T$

$\cancel{0.6T} + 2.1 = 0.4v^2 - 0.4 + \cancel{0.6T}$

$2.5 = 0.4v^2$

$v^2 = 6.25$

$\underline{v = 2.5ms^{-1}}$