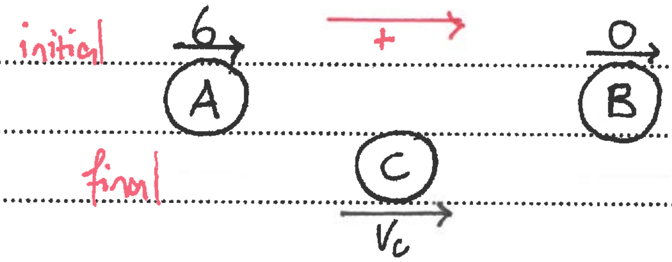
1 A particle B of mass 5 kg is at rest on a smooth horizontal table. A particle A of mass 2.5 kg moves on the table with a speed of $6\,\mathrm{m\,s^{-1}}$ and collides directly with B. In the collision the two particles coalesce.

(a) Find the speed of the combined particle after the collision. [2]



$$m_A u_A + m_B u_B = m_C V_C$$

$$2.5 \times 6 + 0 = 7.5 \times V_c$$

$$15 = 7.5 V_c$$

$$V_c = \underline{2\,\mathrm{ms^{-1}}}$$

(b) Find the loss of kinetic energy of the system due to the collision. [3]

Initial Kinetic Energy (only A moving):

$$KE_{init} = \frac{1}{2} \times 2.5 \times 6^2$$

$$= \underline{45\,\mathrm{J}}$$

Final Kinetic Energy (only C moving):

$$KE_{fin} = \frac{1}{2} \times 7.5 \times 2^2$$

$$= \underline{15\,\mathrm{J}}$$

$$\text{Loss in KE} = 45 - 15 = \underline{30\,\mathrm{J}}$$

2 A car of mass 1400 kg is moving along a straight horizontal road against a resistance of magnitude 350 N.

(a) Find, in kW, the rate at which the engine of the car is working when it is travelling at a constant speed of $20\,\mathrm{m\,s^{-1}}$. [2]

At constant speed, $a = 0$, so driving force = resistance :

D = resistance = 350N

Power = $D \times V$

$= 350 \times 20$

$= 7000\,W$

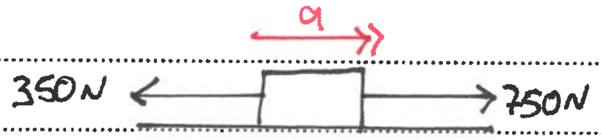
$= 7\,kW$

(b) Find the acceleration of the car when its speed is $20\,\mathrm{m\,s^{-1}}$ and the engine is working at 15 kW. [3]

Power = $D \times V$

$15000 = D \times 20$

$D = 750N$



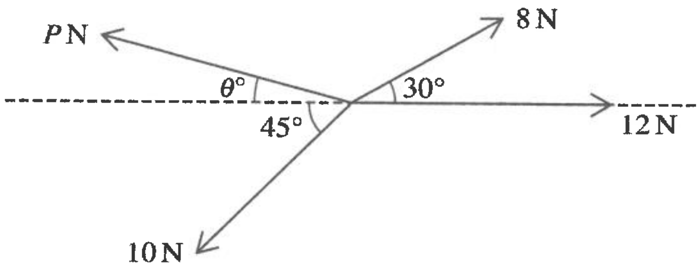
$R(\rightarrow)$: $F = ma$

$750 - 350 = 1400a$

$400 = 1400a$

$a = \frac{2}{7}\,ms^{-2}$

3



Coplanar forces of magnitudes 8 N, 12 N, 10 N and P N act at a point in the directions shown in the diagram. The system is in equilibrium.

Find P and θ. [6]

$R(\uparrow)$: $P\sin\theta + 8\sin 30 - 10\sin 45 = 0$

$P\sin\theta + 4 - 5\sqrt{2} = 0$

$P\sin\theta = 5\sqrt{2} - 4$ (1)

$R(\rightarrow)$: $12 + 8\cos 30 - P\cos\theta - 10\cos 45 = 0$

$12 + 4\sqrt{3} - P\cos\theta - 5\sqrt{2} = 0$

$P\cos\theta = 12 + 4\sqrt{3} - 5\sqrt{2}$ (2)

(1) ÷ (2):

$$\frac{P\sin\theta}{P\cos\theta} = \frac{5\sqrt{2} - 4}{12 + 4\sqrt{3} - 5\sqrt{2}}$$

$\tan\theta = 0.259$

$\theta = 14.5°$ STO

Sub into (1): $P\sin(14.5) = 5\sqrt{2} - 4$

$$P = \frac{5\sqrt{2} - 4}{\sin(14.5)}$$

$= 12.2\text{N}$

4 A particle P moves in a straight line. It starts from rest at a point O on the line and at time t s after leaving O it has acceleration a m s^{-2}, where $a = 6t - 18$.

Find the distance P moves before it comes to instantaneous rest. [6]

$V = \int (6t - 18)\,dt$

$= 3t^2 - 18t + C$

$V = 0$ when $t = 0$:

$0 = 0 - 0 + C$

$\rightarrow V = 3t^2 - 18t$

$S = \int (3t^2 - 18t)\,dt$

$= t^3 - 9t^2 + C$

$S = 0$ when $t = 0$:

$0 = 0 - 0 + C$

$\rightarrow S = t^3 - 9t^2$

Find when P comes to an instantaneous rest by setting $V = 0$:

$3t^2 - 18t = 0$

$t^2 - 6t = 0$

$t(t - 6) = 0$

$t = 0$, $t = 6$

Find distance moved from $t = 0$ to $t = 6$:

$S = t^3 - 9t^2$

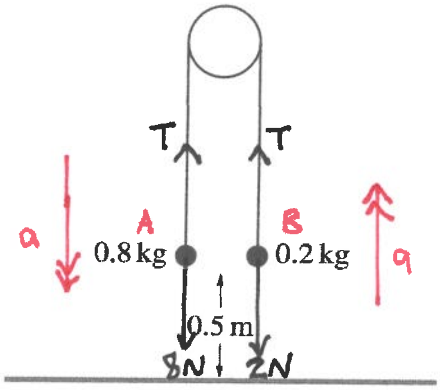
$= 6^3 - 9 \times 6^2$

$= 216 - 324$

$= -108$

So distance $=$ 108m

5



Two particles of masses 0.8 kg and 0.2 kg are connected by a light inextensible string that passes over a fixed smooth pulley. The system is released from rest with both particles 0.5 m above a horizontal floor (see diagram). In the subsequent motion the 0.2 kg particle does not reach the pulley.

(a) Show that the magnitude of the acceleration of the particles is $6\,\text{m}\,\text{s}^{-2}$ and find the tension in the string. [4]

A:

$R(\downarrow)$: $8 - T = ma$

$8 - T = 0.8a$ ①

B:

$R(\uparrow)$: $T - 2 = ma$

$T - 2 = 0.2a$ ②

① + ②:

$8 - 2 = 1a$

$6\,\text{ms}^{-2} = a$ QED

Sub into ②:

$T - 2 = 0.2 \times 6$

$T - 2 = 1.2$

$T = 3.2\text{N}$

(b) When the 0.8 kg particle reaches the floor it comes to rest.

Find the greatest height of the 0.2 kg particle above the floor. [3]

Find speed of B when A hits floor:

$S = 0.5$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2(6) \times 0.5$
$v =$	$v^2 = 6$
$a = 6$	$v = \sqrt{6}\ ms^{-1}$
$t =$	

Once A hits the floor, B moves freely under gravity:

↑+ $S =$	$v^2 = u^2 + 2as$
$u = \sqrt{6}$	$0^2 = (\sqrt{6})^2 + 2(-10) \times S$
$v = 0$	$0 = 6 - 20S$
$a = -10$	$20s = 6$
$t =$	$s = 0.3m$

initial height   speed while connected   under freefall

Greatest height reached $= 0.5 + 0.5 + 0.3$

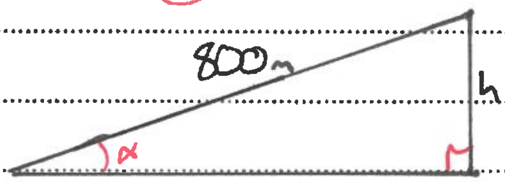
$= 1.3m$

$\sin\alpha = 0.08$

6 A car of mass 1500 kg is pulling a trailer of mass 750 kg up a straight hill of length 800 m inclined at an angle of $\sin^{-1} 0.08$ to the horizontal. The resistances to the motion of the car and trailer are 400 N and 200 N respectively. The car and trailer are connected by a light rigid tow-bar. The car and trailer have speed $30\,\text{m s}^{-1}$ at the bottom of the hill and $20\,\text{m s}^{-1}$ at the top of the hill.

(a) Use an energy method to find the constant driving force as the car and trailer travel up the hill. [5]

Change in height when travelling 800m:



$\sin\alpha = \frac{h}{800}$

$h = 800\sin\alpha$

$= 800 \times 0.08$

$= \underline{64\text{ m}}$

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

Work = F×d

$800D + \frac{1}{2}\times 2250 \times 30^2 + 0 = \frac{1}{2}\times 2250 \times 20^2 + 2250 \times 10 \times 64 + 600 \times 800$

F×d

$800D + 1012500 = 450000 + 1440000 + 480000$

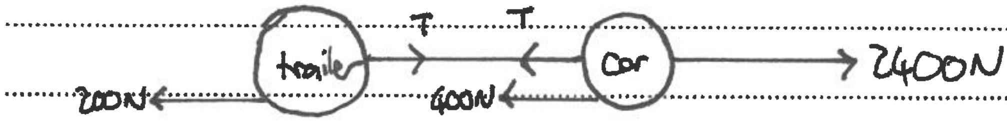
$800D + 1012500 = 2370000$

$800D = 1357500$

$\underline{D = 1696.875\text{ N}}$

After reaching the top of the hill the system consisting of the car and trailer travels along a straight level road. The driving force of the car's engine is 2400 N and the resistances to motion are unchanged.

(b) Find the acceleration of the system and the tension in the tow-bar. [4]



Car:

R(→): $F = ma$

$2400 - 400 - T = 1500a$

$2000 - T = 1500a$ ①

Trailer:

R(→): $F = ma$

$T - 200 = 750a$ ②

① + ②: $2000 - 200 = 2250a$

$1800 = 2250a$

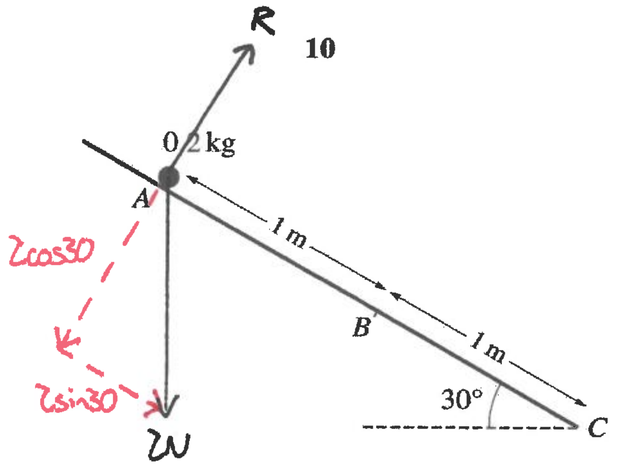
$\underline{a = 0.8 \text{ ms}^{-2}}$

→ ②: $T - 200 = 750 \times 0.8$

$T - 200 = 600$

$\underline{T = 800 \text{ N}}$

7



Three points A, B and C lie on a line of greatest slope of a plane inclined at an angle of 30° to the horizontal, with $AB = 1$ m and $BC = 1$ m, as shown in the diagram. A particle of mass 0.2 kg is released from rest at A and slides down the plane. The part of the plane from A to B is smooth. The part of the plane from B to C is rough, with coefficient of friction μ between the plane and the particle.

(a) Given that $\mu = \frac{1}{2}\sqrt{3}$, find the speed of the particle at C. [8]

$R(\nearrow)$: $R - 2\cos 30 = 0$

$R = 2\cos 30$

$\underline{R = \sqrt{3}\text{ N}}$

Smooth section:

$R(\searrow)$: $2\sin 30 = ma$

$1 = 0.2a$

$\underline{a = 5\text{ ms}^{-2}}$

$s = 1$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2 \times 5 \times 1$
$v =$	$v^2 = 10$
$a = 5$	$\underline{v = \sqrt{10}\text{ ms}^{-1}}$
$t =$	

Rough section:

$R(\searrow)$: $2\sin 30 - \mu R = ma$

$1 - \frac{1}{2}\sqrt{3} \times \sqrt{3} = 0.2a$

$1 - 1.5 = 0.2a$

$-0.5 = 0.2a$

$\underline{a = -2.5\,ms^{-2}}$

$s = 1$	$v^2 = u^2 + 2as$
$u = \sqrt{10}$	$= (\sqrt{10})^2 + 2(-2.5) \times 1$
$v =$	$= 10 - 5$
$a = -2.5$	$v^2 = 5$
$t =$	$v = \sqrt{5} \longrightarrow \underline{\underline{v = 2.24\,ms^{-1}}}$

(b) Given instead that the particle comes to rest at C, find the exact value of μ. **[4]**

Find required acceleration for $v=0$:

$s = 1$	$v^2 = u^2 + 2as$
$u = \sqrt{10}$	$0^2 = (\sqrt{10})^2 + 2 \times a \times 1$
$v = 0$	$0 = 10 + 2a$
$a =$	$2a = -10$
$t =$	$\underline{a = -5\,ms^{-2}}$

Now resolve forces for rough section again:

$R(\searrow):\ 2\sin 30 - \mu R = ma$

$1 - \mu \times \sqrt{3} = 0.2 \times -5$

$1 - \sqrt{3}\mu = -1$

$-\sqrt{3}\mu = -2$

$\sqrt{3}\mu = 2$

$\mu = \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$\underline{\underline{\mu = \frac{2\sqrt{3}}{3}}}$