1 The random variable X takes the values -2, 2 and 3. It is given that

$$P(X = x) = k(x^2 - 1),$$

where k is a constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

$x = -2: \quad k((-2)^2 - 1) = 3k$

$x = 2: \quad k(2^2 - 1) = 3k$

$x = 3: \quad k(3^2 - 1) = 8k$

$\Sigma P = 1: \quad 3k + 3k + 8k = 1$

$14k = 1$

$k = \frac{1}{14}$

x	-2	2	3
$P(X=x)$	$3/14$	$3/14$	$8/14$

(b) Find $E(X)$ and $Var(X)$. [3]

$E(x) = -2 \times \frac{3}{14} + 2 \times \frac{3}{14} + 3 \times \frac{8}{14}$

$= \frac{-3}{7} + \frac{3}{7} + \frac{12}{7}$

$= \frac{12}{7}$

$Var(x) = (-2)^2 \times \frac{3}{14} + 2^2 \times \frac{3}{14} + 3^2 \times \frac{8}{14} - (E(x))^2$

$= \frac{6}{7} + \frac{6}{7} + \frac{36}{7} - \left(\frac{12}{7}\right)^2$

$= \frac{48}{7} - \frac{144}{49} = \frac{192}{49}$

2 A sports event is taking place for 4 days, beginning on Sunday. The probability that it will rain on Sunday is 0.4. On any subsequent day, the probability that it will rain is 0.7 if it rained on the previous day and 0.2 if it did not rain on the previous day.

(a) Find the probability that it does **not** rain on any of the 4 days of the event. [1]

$P(R'R'R'R') = 0.6 \times 0.8 \times 0.8 \times 0.8$

$= 0.3072$

Helpful table for this Q:

		Yesterday	
		Rained	Didn't Rain
Today	$P(R)$	0.7	0.2
	$P(R')$	0.3	0.8

(b) Find the probability that the first day on which it rains during the event is Tuesday. [2]

$P(R'R'R) = 0.6 \times 0.8 \times 0.2$

$= 0.096$

(c) Find the probability that it rains on exactly one of the 4 days of the event. [3]

$P(RR'R'R') = 0.4 \times 0.3 \times 0.8 \times 0.8$
$= 0.0768$

$P(R'RR'R') = 0.6 \times 0.2 \times 0.3 \times 0.8$
$= 0.0288$

$P(R'R'RR') = 0.6 \times 0.8 \times 0.2 \times 0.3$
$= 0.0288$

$P(R'R'R'R) = 0.6 \times 0.8 \times 0.8 \times 0.2$
$= 0.0768$

$0.0768 + 0.0288 + 0.0288 + 0.0768 = \underline{0.2112}$

3 The following back-to-back stem-and-leaf diagram represents the monthly salaries, in dollars, of 27 employees at each of two companies, A and B.

Company A								Company B							
		5	4	1	1	0	25	4	4	5	6	6	7		
9	9	8	7	2	1	0	26	0	1	3	5	5	7	9	9
	8	6	4	2	1	0	27	1	3	4	6	6	8	8	
		6	5	4	2	0	28	0	1	2	2	2			
				9	8	5	29								
						1	30	9							

Key: 1 | 27 | 6 means \$2710 for company A and \$2760 for company B

(a) Find the median and the interquartile range of the monthly salaries of employees in company A. [3]

Q_1: $\frac{27+1}{4} = 7^{th}$ Q_2: $\frac{27+1}{2} = 14^{th}$ Q_3: $\frac{3(27+1)}{4} = 21^{st}$

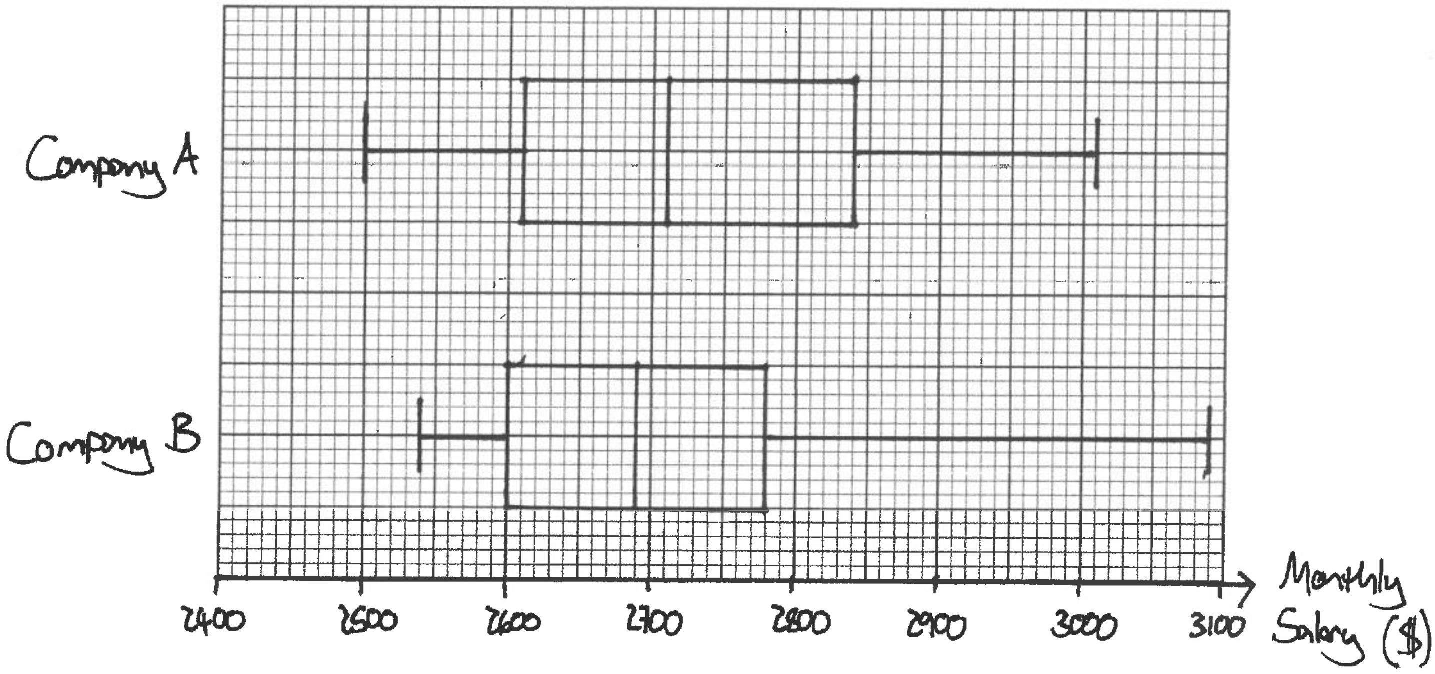
$Q_1 = 2610$ $Q_2 = 2710$ $Q_3 = 2840$

Median = \$2710 IQR = 2840 − 2610

= \$230

The lower quartile, median and upper quartile for company B are \$2600, \$2690 and \$2780 respectively.

(b) Draw two box-and-whisker plots in a single diagram to represent the information for the salaries of employees at companies A and B. [3]



(c) Comment on whether the mean would be a more appropriate measure than the median for comparing the given information for the two companies. [1]

No, the median would be more appropriate because the mean is affected by extreme values like \$3090 for company B.

4 A fair 5-sided spinner has sides labelled 1, 2, 3, 4, 5. The spinner is spun repeatedly until a 2 is obtained on the side on which the spinner lands. The random variable X denotes the number of spins required.

(a) Find $P(X = 4)$. [1]

$X \sim Geo(0.2)$

$P(X=4) = 0.2 \times 0.8^3$

$= \underline{0.1024}$

(b) Find $P(X < 6)$. [2]

$P(X<6) = P(X \leq 5)$

$= 1 - q^5$

probability of 5 failures

$= 1 - 0.8^5$

$= \underline{0.67232}$

Two fair 5-sided spinners, each with sides labelled 1, 2, 3, 4, 5, are spun at the same time. If the numbers obtained are equal, the score is 0. Otherwise, the score is the higher number minus the lower number.

(c) Find the probability that the score is greater than 0 given that the score is **not** equal to 2. [3]

	1	2	3	4	5
1	0	1	2	3	4
2	1	0	1	2	3
3	2	1	0	1	2
4	3	2	1	0	1
5	4	3	2	1	0

$P(>0 \cap 2') = P(>0 | 2') \times P(2')$

$P(>0|2') = \dfrac{P(>0 \cap 2')}{P(2')}$

$P(2') = \frac{19}{25}$ $\quad P(>0 \cap 2') = \frac{14}{25}$

$P(>0 \mid 2') = \dfrac{\frac{14}{25}}{\frac{19}{25}}$

$= \frac{14}{19}$

The two spinners are spun at the same time repeatedly .

(d) For 9 randomly chosen spins of the two spinners, find the probability that the score is greater than 2 on at least 3 occasions. [3]

$P(>2) = \frac{6}{25}$

$S \sim B(9, \frac{6}{25})$

$P(S \geqslant 3) = 1 - (P(0) + P(1) + P(2))$

$= 1 - \left({}^9C_0 \times \left(\frac{6}{25}\right)^0 \left(\frac{19}{25}\right)^9 + {}^9C_1 \times \left(\frac{6}{25}\right)^1 \left(\frac{19}{25}\right)^8 + {}^9C_2 \times \left(\frac{6}{25}\right)^2 \left(\frac{19}{25}\right)^7\right)$

$= 1 - 0.6287$

$= 0.371$

5 The lengths of Western bluebirds are normally distributed with mean 16.5 cm and standard deviation 0.6 cm.

A random sample of 150 of these birds is selected.

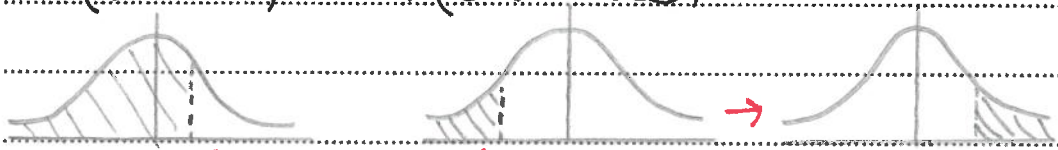
(a) How many of these 150 birds would you expect to have length between 15.4 cm and 16.8 cm? [4]

$P(15.4 < L < 16.8)$

$P\left(\frac{15.4 - 16.5}{0.6} < Z < \frac{16.8 - 16.5}{0.6}\right)$

$P(-1.833 < Z < 0.5)$

$= P(Z < 0.5) - P(Z < -1.833)$



$= \Phi(0.5) - (1 - \Phi(1.833))$

$= 0.6915 - (1 - 0.9668)$

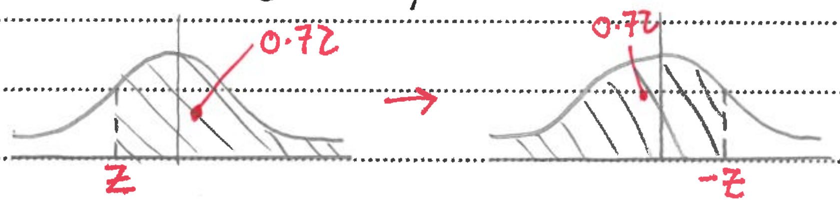
$= 0.6583 \rightarrow 150 \times 0.6583 = 99$ birds

The lengths of Eastern bluebirds are normally distributed with mean 18.4 cm and standard deviation σ cm. It is known that 72% of Eastern bluebirds have length greater than 17.1 cm.

(b) Find the value of σ. [3]

$P(L > 17.1) = 0.72$

$P\left(Z > \frac{17.1 - 18.4}{\sigma}\right) = 0.72$



$0.72 = \Phi(0.583)$ from table

$z = -0.583$

$$\frac{17.1 - 18.4}{\sigma} = -0.583$$

$$-1.3 = -0.583\sigma$$

$$\sigma = 2.23$$

A random sample of 120 Eastern bluebirds is chosen.

(c) Use an approximation to find the probability that fewer than 80 of these 120 bluebirds have length greater than 17.1 cm. [5]

$$L \sim B(120, 0.72)$$

$\mu = 120 \times 0.72$
$= 86.4$

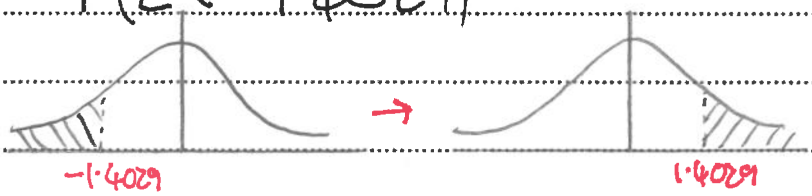
$\sigma^2 = 86.4(0.28)$
$= 24.192$

$$L \sim N(86.4, 24.192)$$

$P(L < 80) \rightarrow P(L < 79.5)$ (continuity correction)

$$P\left(Z < \frac{79.5 - 86.4}{\sqrt{24.192}}\right)$$

$$= P(Z < -1.4029)$$



$$= 1 - \Phi(1.403)$$

$$= 1 - 0.9196$$

$$= 0.0804$$

6 In a group of 25 people there are 6 swimmers, 8 cyclists and 11 runners. Each person competes in only one of these sports. A team of 7 people is selected from these 25 people to take part in a competition.

(a) Find the number of different ways in which the team of 7 can be selected if it consists of exactly 1 swimmer, at least 4 cyclists and at most 2 runners. [4]

pick 1 swimmer from 6

S	C	R	
1	4	2	${}^6C_1 \times {}^8C_4 \times {}^{11}C_2 = 23100$
1	5	1	${}^6C_1 \times {}^8C_5 \times {}^{11}C_1 = 3696$
1	6	0	${}^6C_1 \times {}^8C_6 \times {}^{11}C_0 = 168$

$23100 + 3696 + 168 = \underline{26964}$

For another competition, a team of 9 people consists of 2 swimmers, 3 cyclists and 4 runners. The team members stand in a line for a photograph.

(b) How many different arrangements are there of the 9 people if the swimmers stand together, the cyclists stand together and the runners stand together? [2]

swimmers cyclists runners

$2! \times 3! \times 4!$

but the swimmers could be in the middle, or the end etc. so groups can be in any order, so $\times 3!$

$\rightarrow 2! \times 3! \times 4! \times 3! = \underline{1728}$

(c) How many different arrangements are there of the 9 people if none of the cyclists stand next to each other? [4]

Arrange the swimmers and runners:

$6!$

Arrange the cyclists in gaps:

7 spaces $\rightarrow$ $^7P_3$ $\leftarrow$ 3 cyclists

$6! \times {}^7P_3 = \underline{\underline{151\,200}}$