1 A summary of 50 values of $x$ gives

$$\Sigma(x - q) = 700, \qquad \Sigma(x - q)^2 = 14\,235,$$

where $q$ is a constant.

(a) Find the standard deviation of these values of $x$. [2]

Standard deviation of $x - q$ = Standard deviation of $x$

$$\text{s.d.} = \sqrt{\frac{\Sigma(x-q)^2}{n} - \left(\frac{\Sigma(x-q)}{n}\right)^2}$$

← mean of $x - q$

$$= \sqrt{\frac{14\,235}{50} - \left(\frac{700}{50}\right)^2}$$

$$= \sqrt{284.7 - 14^2}$$

$$= 9.42$$

(1)

(b) Given that $\Sigma x = 2865$, find the value of $q$. [2]

$\Sigma(x - q) = 700$

$\Sigma x = 700 + 50q$ (2)

(1) = (2)

$700 + 50q = 2865$

$50q = 2165$

$q = 43.3$

2 (a) Find the number of ways in which a committee of 6 people can be chosen from 6 men and 8 women if it must include 3 men and 3 women. [2]

$${}^{6}C_{3} \times {}^{8}C_{3} = \underline{1120}$$

3 men from 6

3 women from 8

A different committee of 6 people is to be chosen from 6 men and 8 women. Three of the 6 men are brothers.

(b) Find the number of ways in which this committee can be chosen if there are no restrictions on the numbers of men and women, but it must include no more than two of the brothers. [3]

No Brothers:

$${}^{3}C_{0} \times {}^{11}C_{6} = 462$$

0 brothers from 3

6 from remaining 11 people

One Brother:

$${}^{3}C_{1} \times {}^{11}C_{5} = 1386$$

Two Brothers:

$${}^{3}C_{2} \times {}^{11}C_{4} = 990$$

$$462 + 1386 + 990 = \underline{2838}$$

3 (a) Find the number of different arrangements of the 8 letters in the word COCOONED. [1]

CCOOONED

$$\frac{8!}{2! \times 3!} = \underline{3360}$$

2 Cs → 2! × 3! ← 3 Os

(b) Find the number of different arrangements of the 8 letters in the word COCOONED in which the first letter is O and the last letter is N. [2]

O __ __ __ __ __ __ N

fixed fixed

$$\frac{6!}{2! \times 2!} = \underline{180}$$

2 Cs → 2! × 2! ← 2 Os in the middle

**(c)** Find the probability that a randomly chosen arrangement of the 8 letters in the word COCOONED has all three Os together given that the two Cs are next to each other. [3]

$$P(O \cap C) = P(O|C) \times P(C)$$

$$P(O|C) = \frac{P(O \cap C)}{P(C)}$$

Probability of Cs together:

one object → CC

$$\frac{7!}{3!} = 840$$

3Os → 3!

Probability of Os together and Cs together:

one object → OOO    CC one object

$$5! = 120$$

Without restrictions = 3360 (from part (a))

$$P(C) = \frac{840}{3360} \qquad P(O \cap C) = \frac{120}{3360}$$

$$P(O|C) = \frac{\frac{120}{3360}}{\frac{840}{3360}}$$

$$= \frac{1}{7}$$

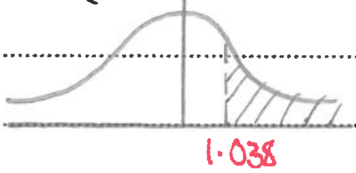
4 A mathematical puzzle is given to a large number of students. The times taken to complete the puzzle are normally distributed with mean 14.6 minutes and standard deviation 5.2 minutes.

(a) In a random sample of 250 of the students, how many would you expect to have taken more than 20 minutes to complete the puzzle? [4]

$P(T > 20)$

$P\left(Z > \dfrac{20 - 14.6}{5.2}\right)$

$P(Z > 1.038)$




$= 1 - \Phi(1.038)$

$= 1 - 0.8504$

$= \underline{0.1496}$

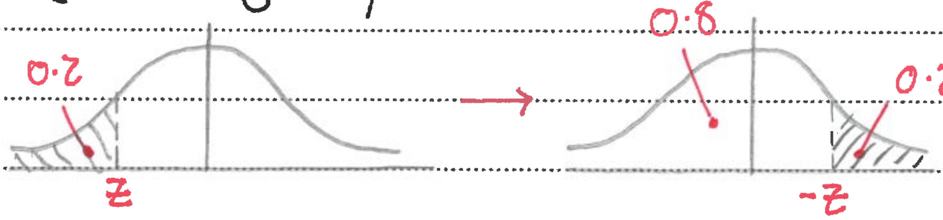
$250 \times 0.1496 = \underline{37 \text{ students}}$

All the students are given a second puzzle to complete. Their times, in minutes, are normally distributed with mean $\mu$ and standard deviation $\sigma$. It is found that 20% of the students have times less than 14.5 minutes and 67% of the students have times greater than 18.5 minutes.

**(b)** Find the value of $\mu$ and the value of $\sigma$. [5]

$P(T < 14.5) = 0.2$

$P\left(Z < \frac{14.5-\mu}{\sigma}\right) = 0.2$




$0.8 = \Phi(0.842)$

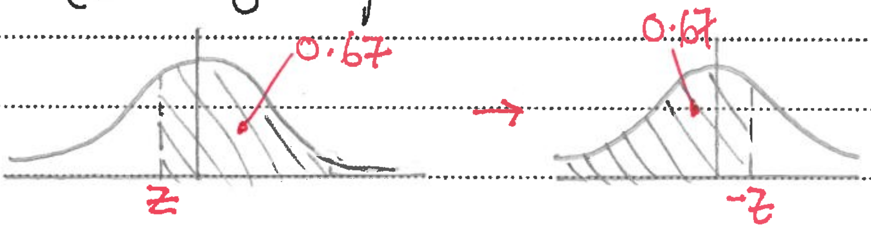
$z = -0.842$

$\frac{14.5-\mu}{\sigma} = -0.842$

$14.5-\mu = -0.842\sigma$ ①

$P(T > 18.5) = 0.67$

$P\left(Z > \frac{18.5-\mu}{\sigma}\right) = 0.67$




$0.67 = \Phi(0.44)$

$z = -0.44$

$\frac{18.5-\mu}{\sigma} = -0.44$

$18.5-\mu = -0.44\sigma$ ②

② − ①: $4 = 0.402\sigma$

$\sigma = 9.95$ STO

→ ②: $18.5-\mu = -0.44 \times 9.95$

$18.5-\mu = -4.378$

$-\mu = -22.88$

$\mu = 22.9$

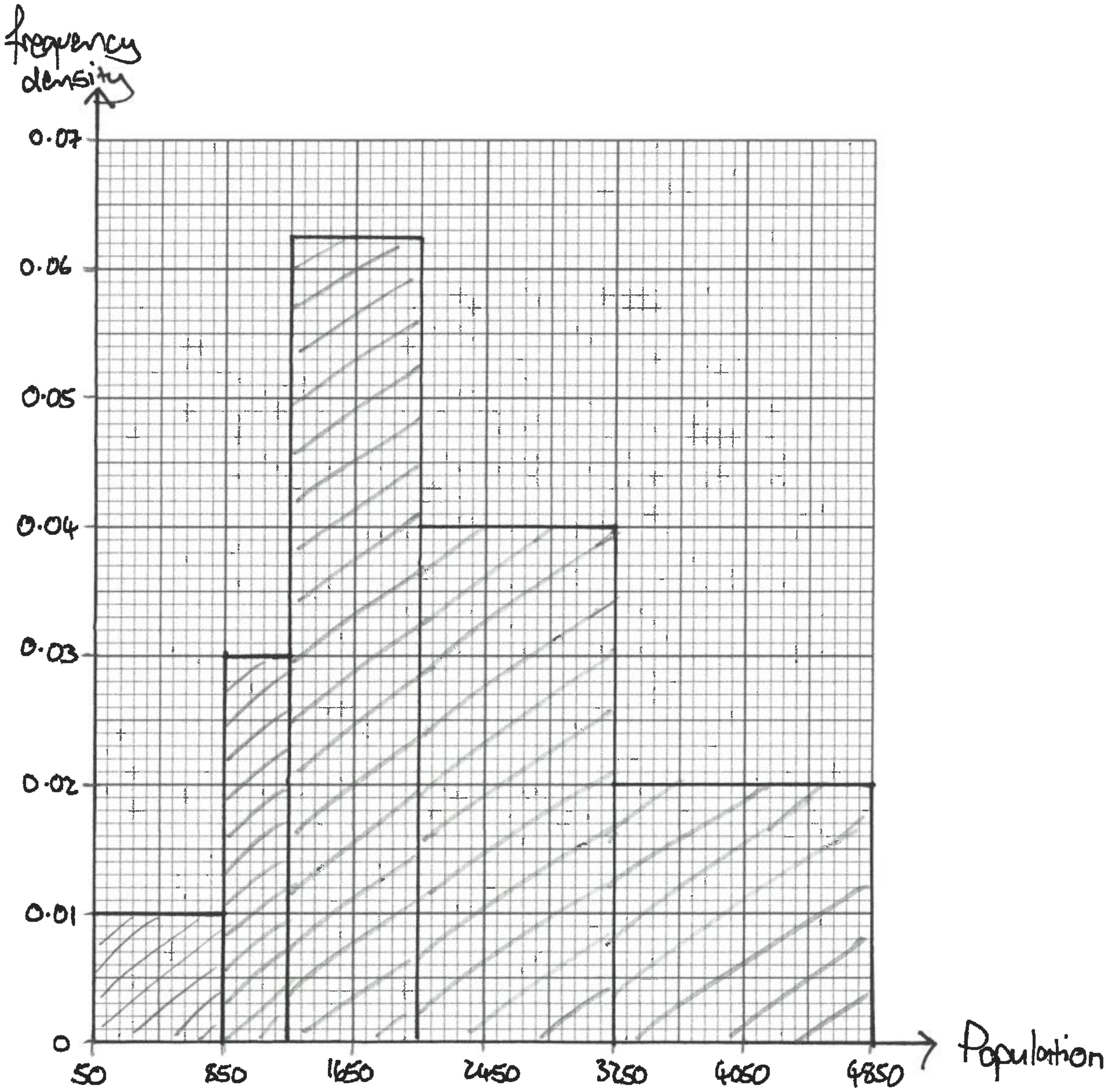
5 The populations of 150 villages in the UK, to the nearest hundred, are summarised in the table.

50 850 1250 2050 3250 4850

| Population | 100 – 800 | 900 – 1200 | 1300 – 2000 | 2100 – 3200 | 3300 – 4800 |
|---|---|---|---|---|---|
| Number of villages | 8 | 12 | 50 | 48 | 32 |

| | | | | | |
|---|---|---|---|---|---|
| class width | 800 | 400 | 800 | 1200 | 1600 |
| f.d. | 0.01 | 0.03 | 0.0625 | 0.04 | 0.02 |

**(a)** Draw a histogram to represent this information. [4]



**(b)** Write down the class interval which contains the median for this information. [1]

| | 50 | 850 | 1250 | 2050 | 3250 | 4850 |
|---|---|---|---|---|---|---|
| Population | 100 - 800 | 900 - 1200 | 1300 - 2000 | 2100 - 3200 | 3300 - 4800 | |
| Cumulative freq | 8 | 20 | 70 | 118 | 150 | |

$Q_2$

$Q_2: \frac{150}{2} = 75^{th}$ value

$Q_2$ lies in 2100 - 3200

**(c)** Find the greatest possible value of the interquartile range for the populations of the 150 villages. [2]

$Q_1: \frac{150}{4} = 37.5^{th}$ value (1300 - 2000 interval)

$Q_3: \frac{3(150)}{4} = 112.5^{th}$ value (2100 - 3200 interval)

Greatest IQR = $\max(Q_3) - \min(Q_1)$

$= 3249 - 1250$

$= 1999$

6 Eli has four fair 4-sided dice with sides labelled 1, 2, 3, 4. He throws all four dice at the same time. The random variable $X$ denotes the number of 2s obtained.

**(a)** Show that $P(X = 3) = \frac{3}{64}$. [2]

$X \sim B(4, \frac{1}{4})$

$P(X=3) = {}^4C_3 \times \left(\frac{1}{4}\right)^3 \times \frac{3}{4}$

$= 4 \times \frac{1}{64} \times \frac{3}{4}$

$= \frac{3}{64}$ QED

**(b)** Complete the following probability distribution table for $X$. [2]

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{81}{256}$ | $\frac{27}{64}$ | $\frac{27}{128}$ | $\frac{3}{64}$ | $\frac{1}{256}$ |

$P(X=1) = {}^4C_1 \times \left(\frac{1}{4}\right)^1 \times \left(\frac{3}{4}\right)^3$

$= 4 \times \frac{1}{4} \times \frac{27}{64}$

$= \frac{27}{64}$

$P(X=2) = {}^4C_2 \times \left(\frac{1}{4}\right)^2 \times \left(\frac{3}{4}\right)^2$

$= 6 \times \frac{1}{16} \times \frac{9}{16}$

$= \frac{27}{128}$

**(c)** Find E($X$). [2]

$$E(X) = 0 \times \frac{81}{256} + 1 \times \frac{27}{64} + 2 \times \frac{27}{128} + 3 \times \frac{3}{64} + 4 \times \frac{1}{256}$$

$$= 0 + \frac{27}{64} + \frac{9}{28} + \frac{27}{64} + \frac{1}{64}$$

$$= \underline{1}$$

Eli throws the four dice at the same time on 96 occasions.

**(d)** Use an approximation to find the probability that he obtains at least two 2s on fewer than 20 of these occasions. [5]

$$P(\text{at least two 2s}) = \frac{27}{128} + \frac{3}{64} + \frac{1}{256} = \frac{67}{256}$$

$$X \sim B\left(96, \frac{67}{256}\right)$$

$$\mu = 96 \times \frac{67}{256} \qquad \sigma^2 = 25.125\left(\frac{189}{256}\right)$$

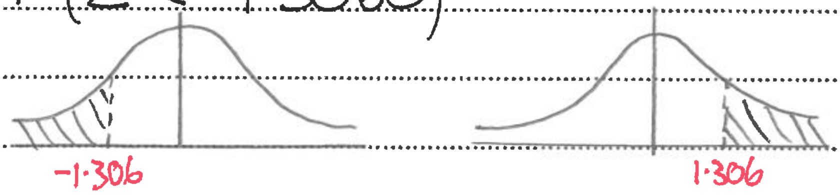
$$= 25.125 \qquad = 18.549 \text{ STO}$$

$$X \sim N(25.125, 18.549)$$

$$P(X < 20) \rightarrow P(X < 19.5) \quad \text{(continuity correction)}$$

$$P\left(Z < \frac{19.5 - 25.125}{\sqrt{18.549}}\right)$$

$$= P(Z < -1.3060)$$




$$= 1 - \Phi(1.306)$$

$$= 1 - 0.9042$$

$$= \underline{0.0958}$$

7 A children's wildlife magazine is published every Monday. For the next 12 weeks it will include a model animal as a free gift. There are five different models: tiger, leopard, rhinoceros, elephant and buffalo, each with the same probability of being included in the magazine.

Sahim buys one copy of the magazine every Monday.

E

**(a)** Find the probability that the first time that the free gift is an elephant is before the 6th Monday. [2]

$P(E) = 0.2$

$E \sim Geo(0.2)$

$P(E < 6) = P(E \leq 5)$

$= 1 - q^5$ probability of 5 failures

$= 1 - 0.8^5$

$= 0.67232$

**(b)** Find the probability that Sahim will get more than two leopards in the 12 magazines. [3]

$P(L) = 0.2$

$L \sim B(12, 0.2)$

$P(L > 2) = 1 - (P(0) + P(1) + P(2))$

$= 1 - ({}^{12}C_0 \times 0.2^0 \times 0.8^{12} + {}^{12}C_1 \times 0.2^1 \times 0.8^{11} + {}^{12}C_2 \times 0.2^2 \times 0.8^{10})$

$= 1 - (0.5583)$

$= 0.442$

(c) Find the probability that after 5 weeks Sahim has exactly one of each animal. [3]

Probability of getting all five animals in order:
Tiger, Leopard, Rhino, Elephant, Buffalo:

$0.2 \times 0.2 \times 0.2 \times 0.2 \times 0.2 = 0.00032$

But order doesn't matter, so permutate these five objects by multiplying by 5! :

$0.00032 \times 5! = \underline{0.0384}$