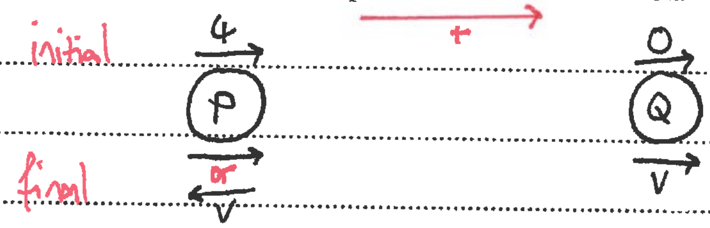
1 Two particles P and Q, of masses 0.1 kg and 0.4 kg respectively, are free to move on a smooth horizontal plane. Particle P is projected with speed $4\,\text{m s}^{-1}$ towards Q which is stationary. After P and Q collide, the speeds of P and Q are equal.

Find the two possible values of the speed of P after the collision. [3]



Scenario 1: P continues moving right after collision:

$$m_P u_P + m_Q u_Q = m_P v_P + m_Q v_Q$$
$$0.1 \times 4 + 0 = 0.1V + 0.4V$$
$$0.4 = 0.5V$$
$$V = \underline{\underline{0.8\,\text{ms}^{-1}}}$$

Scenario 2: P moves left after the collision:

$$m_P u_P + m_Q u_Q = m_P v_P + m_Q v_Q$$
$$0.1 \times 4 + 0 = 0.1 \times -V + 0.4 \times V$$
$$0.4 = -0.1V + 0.4V$$
$$0.4 = 0.3V$$
$$V = \underline{\underline{\tfrac{4}{3}\,\text{ms}^{-1}}}$$

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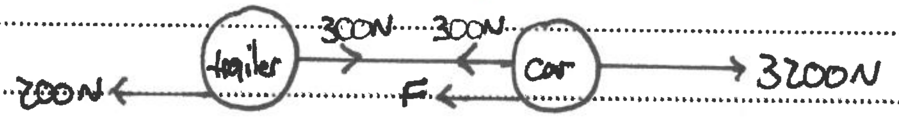
2 A car of mass 1500 kg is towing a trailer of mass m kg along a straight horizontal road. The car and the trailer are connected by a tow-bar which is horizontal, light and rigid. There is a resistance force of F N on the car and a resistance force of 200 N on the trailer. The driving force of the car's engine is 3200 N, the acceleration of the car is $1.25\,\mathrm{m\,s^{-2}}$ and the tension in the tow-bar is 300 N.

Find the value of m and the value of F. $1.25\,\mathrm{ms^{-2}}$ **[4]**



Car:

R(→): $F = ma$

$3200 - 300 - F = 1500 \times 1.25$

$2900 - F = 1875$

$-F = -1025$

$F = 1025\text{ N}$

Trailer:

R(→): $F = ma$

$300 - 200 = m \times 1.25$

$100 = 1.25m$

$m = 80\text{ kg}$

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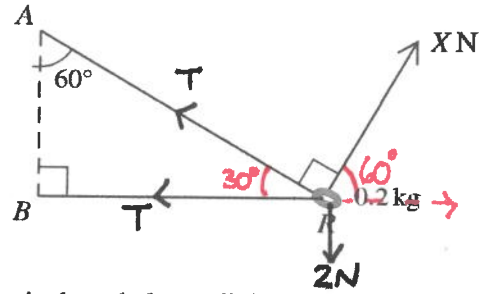
3



A smooth ring R of mass 0.2 kg is threaded on a light string ARB. The ends of the string are attached to fixed points A and B with A vertically above B. The string is taut and angle $ABR = 90°$. The angle between the part AR of the string and the vertical is 60°. The ring is held in equilibrium by a force of magnitude X N, acting on the ring in a direction perpendicular to AR (see diagram).

Calculate the tension in the string and the value of X. [5]

Since ARB is one string, not two, the tension is the same throughout = T.

$R(\uparrow)$: $T\sin 30 + X\sin 60 - 2 = 0$

$\frac{1}{2}T + \frac{\sqrt{3}}{2}X - 2 = 0$

$T + \sqrt{3}X - 4 = 0$

$T = 4 - \sqrt{3}X$ (1)

$R(\rightarrow)$: $X\cos 60 - T - T\cos 30 = 0$

$\frac{1}{2}X - T - \frac{\sqrt{3}}{2}T = 0$

$X - 2T - \sqrt{3}T = 0$ (2)

Sub (1) into (2):

$X - 2(4 - \sqrt{3}X) - \sqrt{3}(4 - \sqrt{3}X) = 0$

$X - 8 + 2\sqrt{3}X - 4\sqrt{3} + 3X = 0$

$4X + 2\sqrt{3}X - 8 - 4\sqrt{3} = 0$

$4X + 2\sqrt{3}X = 8 + 4\sqrt{3}$

$X(4 + 2\sqrt{3}) = 8 + 4\sqrt{3}$

$X = \underline{2N}$

Sub into (1): $T = 4 - \sqrt{3} \times 2$

$= \underline{0.536N}$

4 A lorry of mass 15 000 kg moves on a straight horizontal road in the direction from A to B. It passes A and B with speeds $20\,\mathrm{m\,s^{-1}}$ and $25\,\mathrm{m\,s^{-1}}$ respectively. The power of the lorry's engine is constant and there is a constant resistance to motion of magnitude 6000 N. The acceleration of the lorry at B is 0.5 times the acceleration of the lorry at A.

(a) Show that the power of the lorry's engine is 200 kW, and hence find the acceleration of the lorry when it is travelling at $20\,\mathrm{m\,s^{-1}}$. [5]

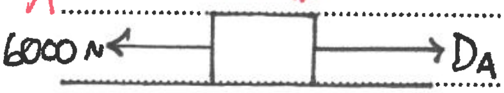
At A: Power $= D_A \times V$ At B: Power $= D_B \times V$

$P = 20D_A$ ① $P = 25D_B$ ②

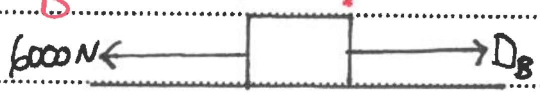
① = ②: $20D_A = 25D_B$

$D_A = 1.25D_B$

A: a



B: $0.5a$



R(→): $D_A - 6000 = ma$ R(→): $D_B - 6000 = m \times 0.5a$

$D_A - 6000 = 15000a$ ③ $D_B - 6000 = 15000 \times 0.5a$

$D_B - 6000 = 7500a$ ×2

$2D_B - 12000 = 15000a$ ④

④ − ③: $2D_B - D_A - 6000 = 0$ → Power $= 25 \times 8000$

$D_A = 1.25D_B$: $2D_B - 1.25D_B = 6000$ $= \underline{200\,000\,W}$ QED

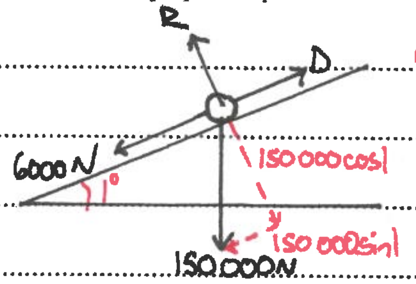
$0.75D_B = 6000$ → ④: $2(8000) - 12000 = 15000a$

$D_B = \underline{8000N}$ $4000 = 15000a$

$\underline{a = 0.267\,\mathrm{ms^{-2}}}$

The lorry begins to ascend a straight hill inclined at 1° to the horizontal. It is given that the power of the lorry's engine and the resistance force do not change.

(b) Find the steady speed up the hill that the lorry could maintain. [2]



Constant speed, so $a = 0$:

R(↗): $D - 6000 - 150000\sin 1 = ma$

$D - 6000 - 2617.86 = 0$

$D = 8617.86N$

Power $= D \times V$

$200\,000 = 8617.86 \times V$

$V = \underline{23.2\,\mathrm{ms^{-1}}}$

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5 A particle starts from rest from a point O and moves in a straight line. The acceleration of the particle at time t s after leaving O is a m s^{-2}, where $a = kt^{\frac{1}{2}}$ for $0 \leqslant t \leqslant 9$ and where k is a constant. The velocity of the particle at $t = 9$ is 1.8 m s^{-1}.

(a) Show that $k = 0.1$. [3]

$V = \int kt^{\frac{1}{2}}\,dt$

$V = \frac{2}{3}kt^{\frac{3}{2}} + C$

$V=0$ at $t=0$:

$0 = 0 + C$

$\rightarrow V = \frac{2}{3}kt^{3/2}$

$V=1.8$ at $t=9$:

$1.8 = \frac{2}{3}k \times 9^{3/2}$

$1.8 = \frac{2}{3}k \times 27$

$1.8 = 18k$

$k = 0.1$ QED $\rightarrow V = \frac{2}{30}t^{3/2}$

For $t > 9$, the velocity v m s^{-1} of the particle is given by $v = 0.2(t-9)^2 + 1.8$. only turning point is at $t=9$, so no problem

(b) Show that the distance travelled in the first 9 seconds is one tenth of the distance travelled between $t = 9$ and $t = 18$. [4]

$0 \rightarrow 9s$: $S = \int_0^9 \frac{2}{30}t^{3/2}\,dt$

$= \left[\frac{2}{5} \times \frac{2}{30}t^{5/2}\right]_0^9 = \left[\frac{2}{75}t^{5/2}\right]_0^9$

$= \left[\frac{2}{75}(9)^{5/2}\right] - [0]$

$= \frac{2}{75} \times 243 = 6.48\text{m}$ 1

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$9 \to 18s$: $S = \int_9^{18} (0.2(t-9)^2 + 1.8)\,dt$

$= \left[\frac{0.2}{3}(t-9)^3 + 1.8t\right]_9^{18} = \left[\frac{1}{15}(t-9)^3 + 1.8t\right]_9^{18}$

$= \left[\frac{1}{15}(9)^3 + 1.8(18)\right] - \left[0 + 1.8(9)\right]$

$= [81] - [16.2]$

$= 64.8\text{m}$ ② $\quad$ ② ① $64.8 = 10 \times 6.48$ QED

(c) Find the greatest acceleration of the particle during the first 10 seconds of its motion. [3]

Acceleration from 0 to 9s:

$a = 0.1t^{\frac{1}{2}}$

max. when $t = 9$:

$a = 0.1 \times 9^{\frac{1}{2}}$

$= 0.1 \times 3$

$= 0.3\text{ms}^{-2}$

Acceleration from 9 to 10s:

$a = \frac{dv}{dt} = 2 \times 0.2(t-9)$

$a = 0.4(t-9)$

max. when $t = 10$:

$a = 0.4(10-9)$

$= 0.4\text{ms}^{-2}$

Therefore max. acceleration $= 0.4\text{ms}^{-2}$

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6 An elevator is pulled vertically upwards by a cable. The elevator accelerates at $0.4\,\mathrm{m\,s^{-2}}$ for 5 s, then travels at constant speed for 25 s. The elevator then decelerates at $0.2\,\mathrm{m\,s^{-2}}$ until it comes to rest.

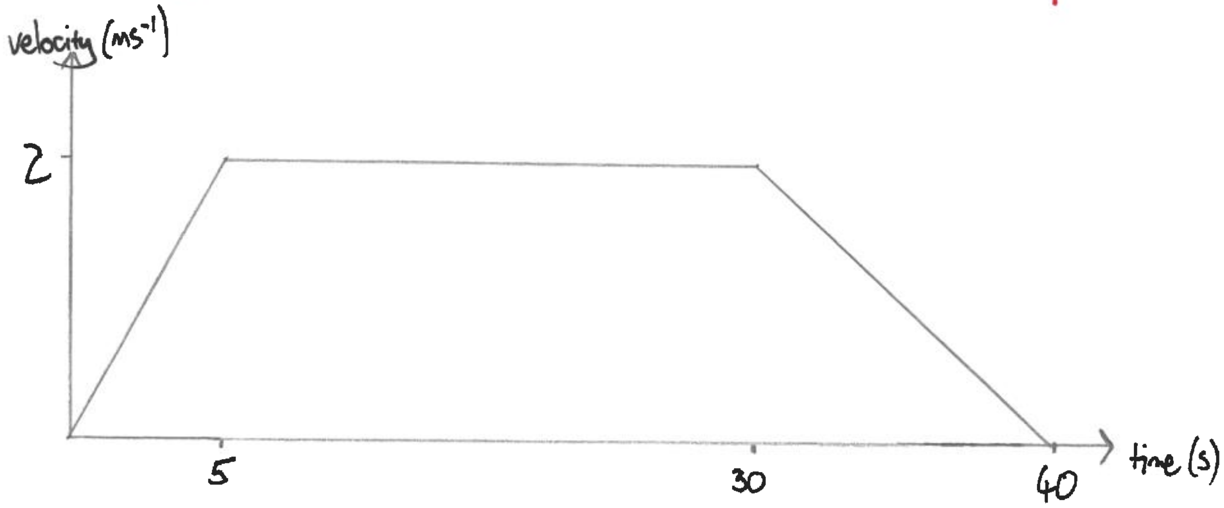
(a) Find the greatest speed of the elevator and hence draw a velocity-time graph for the motion of the elevator. [3]

$s =$
$u = 0$
$v =$
$a = 0.4$
$t = 5$

$v = u + at$
$= 0 + 0.4 \times 5$
$= 2\,\mathrm{ms^{-1}}$

deceleration: $s =$
$u = 2$
$v = 0$
$a = -0.2$
$t =$

$v = u + at$
$0 = 2 + (-0.2)t$
$-2 = -0.2t$
$t = 10\,\mathrm{s}$



(b) Find the total distance travelled by the elevator. [2]

distance = area $= \frac{1}{2}(25 + 40) \times 2$

$= \frac{1}{2} \times 65 \times 2$

$= 65\,\mathrm{m}$

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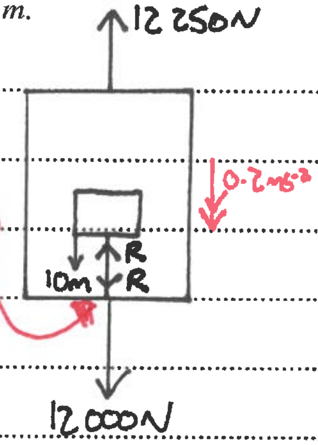
The mass of the elevator is 1200 kg and there is a crate of mass m kg resting on the floor of the elevator.

(c) Given that the tension in the cable when the elevator is decelerating is 12 250 N, find the value of m. [3]

Drawing an imaginary tow-bar helps in questions like these



Consider whole system:

$R(\uparrow): 12250 - 12000 - 10m = (1200 + m)a$

$250 - 10m = (1200+m) \times -0.2$

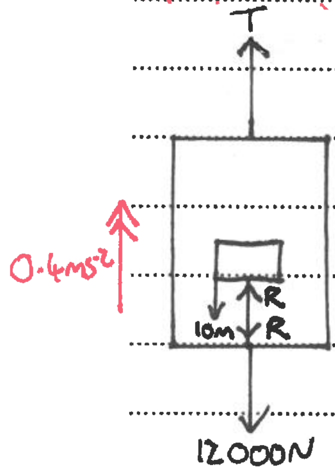
$250 - 10m = -240 - 0.2m$

$490 = 9.8m$

$m = \underline{50 \text{kg}}$

(d) Find the greatest magnitude of the force exerted on the crate by the floor of the elevator, and state its direction. [3]

Greatest magnitude will be when elevator is accelerating (at 0.4ms^{-2})



Consider crate:

$R(\uparrow): R - 10m = ma$

$R - 10 \times 50 = 50 \times 0.4$

$R - 500 = 20$

$R = \underline{520 \text{N upwards}}$

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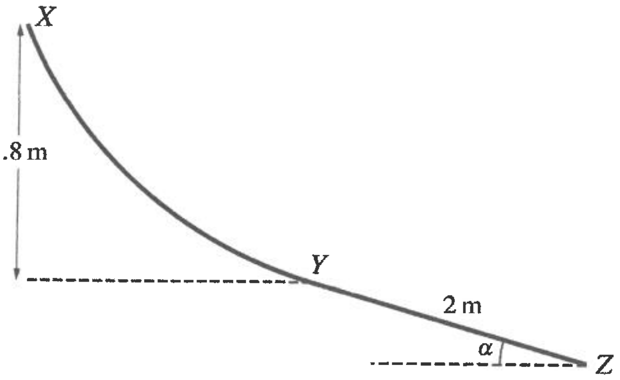
7



The diagram shows the vertical cross-section XYZ of a rough slide. The section YZ is a straight line of length 2 m inclined at an angle of α to the horizontal, where $\sin\alpha = 0.28$. The section YZ is tangential to the curved section XY at Y, and X is 1.8 m above the level of Y. A child of mass 25 kg slides down the slide, starting from rest at X. The work done by the child against the resistance force in moving from X to Y is 50 J.

(a) Find the speed of the child at Y. [4]

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$0 + 0 + 0 = \frac{1}{2} \times 25 \times v^2 + 25 \times 10 \times -1.8 + 50$

$0 = 12.5v^2 - 450 + 50$

$0 = 12.5v^2 - 400$

$12.5v^2 = 400$

$v^2 = 32$

$v = 5.66 \text{ ms}^{-1}$

STO

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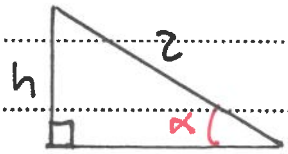
It is given that the child comes to rest at Z.

(b) Use an energy method to find the coefficient of friction between the child and YZ, giving your answer as a fraction in its simplest form. [6]

Change in height:



$\sin\alpha = \frac{h}{2}$

$h = 2\sin\alpha$

$h = 2 \times 0.28 = \underline{0.56\,m}$

$Work_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{OUT}$

$0 + \frac{1}{2} \times 25 \times 5.66^2 + 0 = 0 + 25 \times 10 \times -0.56 + F \times 2$

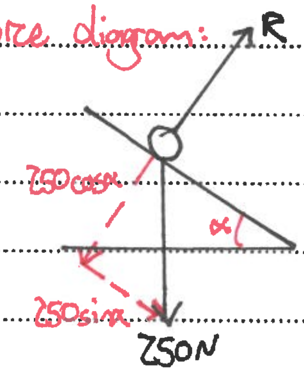
↳ $v=0$ ↳ Work = F×d

$400 = -140 + 2F$

$540 = 2F$

$\underline{F = 270\,N}$

Force diagram:



$\sin\alpha = 0.28$

$\underline{\alpha = 16.26^\circ}$ STO

$R(\uparrow): R - 250\cos(16.26) = 0$

$R = 250\cos(16.26)$

$\underline{R = 240\,N}$

$F = \mu R$

$270 = \mu \times 240$

$\underline{\mu = 1.125}$

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