1 A particle of mass 1.6 kg is dropped from a height of 9 m above horizontal ground. The speed of the particle at the instant before hitting the ground is $12\,\mathrm{m\,s^{-1}}$.

Find the work done against air resistance. [3]

$Work_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{OUT}$

$0 + 0 + 0 = \frac{1}{2} \times 1.6 \times 12^2 + 1.6 \times 10 \times -9 + W$

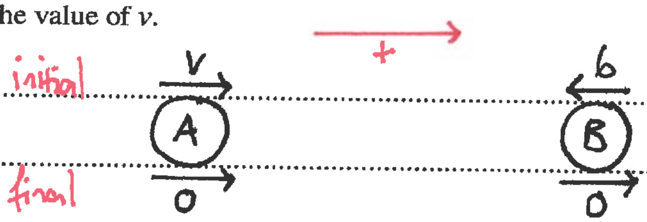
$0 = 115.2 - 144 + W$

$0 = -28.8 + W$

$W = 28.8\,J$

2 Two particles A and B, of masses 3.2 kg and 2.4 kg respectively, lie on a smooth horizontal table. A moves towards B with a speed of $v\,\mathrm{m\,s^{-1}}$ and collides with B, which is moving towards A with a speed of $6\,\mathrm{m\,s^{-1}}$. In the collision the two particles come to rest.

(a) Find the value of v. [2]



$m_A u_A + m_B u_B = m_A v_A + m_B v_B$

$3.2 \times V + 2.4 \times -6 = 0 + 0$

$3.2V - 14.4 = 0$

$3.2V = 14.4$

$V = \underline{4.5\,\mathrm{ms^{-1}}}$

(b) Find the loss of kinetic energy of the system due to the collision. [2]

Initial Kinetic Energy (both A and B moving):

$KE_{init} = \frac{1}{2} \times 3.2 \times 4.5^2 + \frac{1}{2} \times 2.4 \times 6^2$

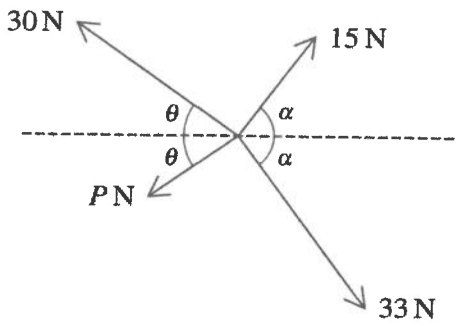
$= 32.4 + 43.2$

$= 75.6\,\mathrm{J}$

Final KE = 0 (both particles stationary)

So loss of KE = $\underline{75.6\,\mathrm{J}}$

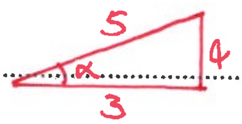
3



Coplanar forces of magnitudes 30 N, 15 N, 33 N and P N act at a point in the directions shown in the diagram, where $\tan\alpha = \frac{4}{3}$. The system is in equilibrium.

(a) Show that $\left(\frac{14.4}{30-P}\right)^2 + \left(\frac{28.8}{P+30}\right)^2 = 1.$ [4]

$\tan\alpha = \frac{4}{3}$ →



$\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{3}{5}$

$R(\uparrow)$: $30\sin\theta + 15\sin\alpha - P\sin\theta - 33\sin\alpha = 0$

$30\sin\theta + 15\times\frac{4}{5} - P\sin\theta - 33\times\frac{4}{5} = 0$

$30\sin\theta + 12 - P\sin\theta - 26.4 = 0$

$30\sin\theta - P\sin\theta = 14.4$

$\sin\theta(30 - P) = 14.4$

$\sin\theta = \frac{14.4}{30-P}$ ①

$R(\rightarrow)$: $15\cos\alpha + 33\cos\alpha - 30\cos\theta - P\cos\theta = 0$

$15\times\frac{3}{5} + 33\times\frac{3}{5} - 30\cos\theta - P\cos\theta = 0$

$9 + 19.8 - 30\cos\theta - P\cos\theta = 0$

$28.8 - 30\cos\theta - P\cos\theta = 0$

$P\cos\theta + 30\cos\theta = 28.8$

$\cos\theta(P + 30) = 28.8$

$\cos\theta = \frac{28.8}{P+30}$ ②

from Trigonometry: $\sin^2\theta + \cos^2\theta = 1$

Sub. ① and ②: $\left(\frac{14.4}{30-P}\right)^2 + \left(\frac{28.8}{P+30}\right)^2 = 1$ QED

(b) Verify that $P = 6$ satisfies this equation and find the value of θ. [2]

Sub. $P = 6$:

$$\left(\frac{14.4}{24}\right)^2 + \left(\frac{28.8}{36}\right)^2$$

$$= \frac{9}{25} + \frac{16}{25}$$

$$= 1$$ QED

Sub. $P = 6$ into ①:

$$\sin\theta = \frac{14.4}{24}$$

$$\theta = 36.9°$$

4 An athlete of mass 84 kg is running along a straight road.

(a) Initially the road is horizontal and he runs at a constant speed of $3\,\mathrm{m\,s^{-1}}$. The athlete produces a constant power of 60 W.

Find the resistive force which acts on the athlete. [1]

Power $= D \times V$

$60 = D \times 3$

$D = 20N$

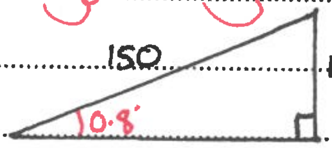
Constant speed, so $a = 0$, so D = resistive force:

resistive force $= 20N$

(b) The athlete then runs up a 150 m section of the road which is inclined at 0.8° to the horizontal. The speed of the athlete at the start of this section of road is $3\,\mathrm{m\,s^{-1}}$ and he now produces a constant driving force of 24 N. The total resistive force which acts on the athlete along this section of road has constant magnitude 13 N.

Use an energy method to find the speed of the athlete at the end of the 150 m section of road. [6]

Change in height over 150m:



$\sin 0.8 = \frac{h}{150}$

$h = 150 \sin 0.8$

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$24 \times 150 + \frac{1}{2} \times 84 \times 3^2 + 0 = \frac{1}{2} \times 84 \times V^2 + 84 \times 10 \times 150 \sin 0.8 + 13 \times 150$

Fxd

Fxd

$3600 + 378 = 42V^2 + 1759.2 + 1950$

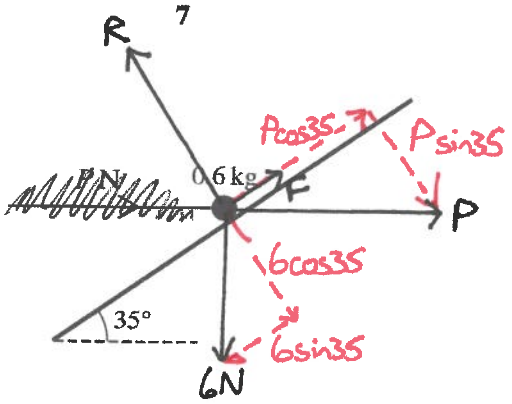
$3978 = 42V^2 + 3709.2$

$268.77 = 42V^2$

$V^2 = 6.399$

$V = 2.53\,\mathrm{ms^{-1}}$

5



A particle of mass 0.6 kg is placed on a rough plane which is inclined at an angle of 35° to the horizontal. The particle is kept in equilibrium by a horizontal force of magnitude P N acting in a vertical plane containing a line of greatest slope (see diagram). The coefficient of friction between the particle and plane is 0.4.

Find the least possible value of P. [6]

For the least possible value of P, the particle is on the point of moving down the plane and friction is acting up the plane.

$R(\nwarrow)$: $R - 6\cos35 - P\sin35 = 0$

$R = 6\cos35 + P\sin35$

$R(\swarrow)$: $6\sin35 - P\cos35 - F = 0$

$6\sin35 - P\cos35 - \mu R = 0$

$6\sin35 - P\cos35 - 0.4(6\cos35 + P\sin35) = 0$

$6\sin35 - P\cos35 - 2.4\cos35 - 0.4P\sin35 = 0$

$6\sin35 - 2.4\cos35 = P\cos35 + 0.4P\sin35$

$6\sin35 - 2.4\cos35 = P(\cos35 + 0.4\sin35)$

$$P = \frac{6\sin35 - 2.4\cos35}{\cos35 + 0.4\sin35}$$

$P = 1.41$ N

6 A particle P starts at rest and moves in a straight line from a point O. At time t s after leaving O, the velocity of P, $v\,\mathrm{m\,s^{-1}}$, is given by $v = bt + ct^{\frac{3}{2}}$, where b and c are constants. P has velocity $8\,\mathrm{m\,s^{-1}}$ when $t = 4$ and has velocity $13.5\,\mathrm{m\,s^{-1}}$ when $t = 9$.

(a) Show that $b = 3$ and $c = -0.5$. [1]

$V=8,\ t=4:$ $8 = 4b + c\times 4^{3/2}$

$8 = 4b + 8c$ ①

$V=13.5,\ t=9:$ $13.5 = 9b + c\times 9^{3/2}$

$13.5 = 9b + 27c$ ②

① × 9: $72 = 36b + 72c$

② × 4: $54 = 36b + 108c$

$18 = -36c$

$c = -0.5$

Sub into ①: $8 = 4b + 8(-0.5)$

$8 = 4b - 4$

$4b = 12 \rightarrow b = 3$

(b) Find the acceleration of P when $t = 1$. [2]

$a = \frac{dv}{dt}$ $(V = 3t - 0.5t^{3/2})$

$a = 3 - \frac{3}{4}t^{\frac{1}{2}}$

$t=1:$

$a = 3 - \frac{3}{4}(1)^{\frac{1}{2}}$

$= 3 - \frac{3}{4}$

$= 2.25\,\mathrm{ms^{-2}}$

(c) Find the positive value of t when P is at instantaneous rest and find the distance of P from O at this instant. [5]

$V=0:$ $3t - \frac{1}{2}t^{3/2} = 0$ $\times 2$

$6t - t^{3/2} = 0$

$t(6 - t^{\frac{1}{2}}) = 0$

$t=0$ or $6 - t^{\frac{1}{2}} = 0$

$t^{\frac{1}{2}} = 6$

$t = 36\,\mathrm{s}$

distance from O at 36s:

$$S = \int_0^{36} \left(3t - \frac{1}{2}t^{3/2}\right) dt$$

$$= \left[\frac{3}{2}t^2 - \frac{1}{5}t^{5/2}\right]_0^{36}$$

$$= \left[\frac{3}{2}(36)^2 - \frac{1}{5}(36)^{5/2}\right] - [0]$$

$$= [1944 - 1555.2]$$

$$= \underline{388.8\text{m}}$$

(d) Find the speed of P at the instant it returns to O. [3]

when it returns to O, s=0:

$$\frac{3}{2}t^2 - \frac{1}{5}t^{5/2} = 0 \quad \times 10$$

$$15t^2 - 2t^{5/2} = 0$$

$$t^2(15 - 2t^{1/2}) = 0$$

$$t^2 = 0 \quad \text{or} \quad 15 - 2t^{1/2} = 0$$

$$t = 0 \qquad 2t^{1/2} = 15$$

$$t^{1/2} = 7.5$$

$$t = \underline{56.25}$$

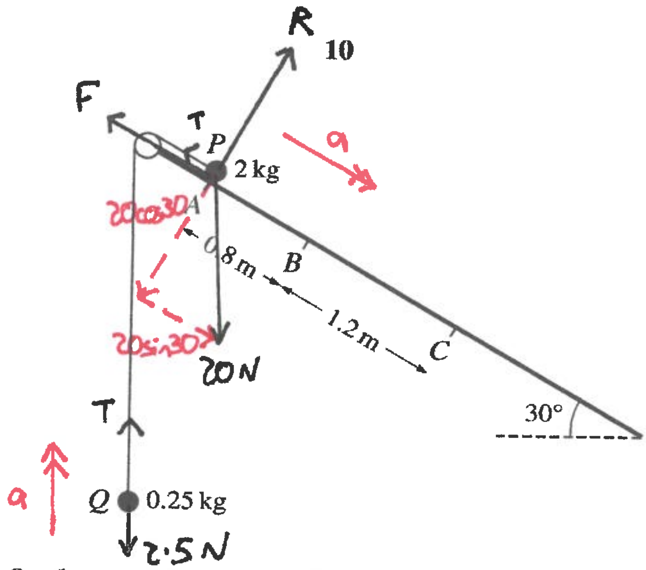
Sub. into V:

$$v = 3(56.25) - 0.5(56.25)^{3/2}$$

$$= -42.2\text{ms}^{-1}$$

$$\underline{\text{Speed} = 42.2\text{ms}^{-1}}$$

7



Two particles P and Q, of masses 2 kg and 0.25 kg respectively, are connected by a light inextensible string that passes over a fixed smooth pulley. Particle P is on an inclined plane at an angle of 30° to the horizontal. Particle Q hangs below the pulley. Three points A, B and C lie on a line of greatest slope of the plane with $AB = 0.8$ m and $BC = 1.2$ m (see diagram).

Particle P is released from rest at A with the string taut and slides down the plane. During the motion of P from A to C, Q does not reach the pulley. The part of the plane from A to B is rough, with coefficient of friction 0.3 between the plane and P. The part of the plane from B to C is smooth.

(a) **(i)** Find the acceleration of P between A and B. [4]

Q:

$R(\uparrow)$: $T - 2.5 = ma$

$T - 2.5 = 0.25a$ (1)

P:

$R(\nearrow)$: $R - 20\cos 30 = 0$

$R = 10\sqrt{3}$ N

$R(\searrow)$: $20\sin 30 - T - F = ma$

$10 - T - \mu R = 2a$

$10 - T - 0.3 \times 10\sqrt{3} = 2a$

$10 - T - 3\sqrt{3} = 2a$ (2)

(1) + (2): $10 - 2.5 - 3\sqrt{3} = 2.25a$

$7.5 - 3\sqrt{3} = 2.25a$

$a = \dfrac{7.5 - 3\sqrt{3}}{2.25}$

$a = 1.02\ \text{ms}^{-2}$

STO

(ii) Hence, find the speed of P at C. [5]

Find speed at B:

A→B

$s = 0.8$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2(1.02) \times 0.8$
$v =$	$v^2 = 1.638$
$a = 1.02$	$v = \underline{1.28\ ms^{-1}}$ STO
$t =$	

Find acceleration from B to C by resolving forces:

Q:

$R(\uparrow): T - 2.5 = 0.25a$ ①

P:

$R(\searrow): 20\sin 30 - T = 2a$ ②

①+②: $20\sin 30 - 2.5 = 2.25a$

$7.5 = 2.25a$

$a = \underline{\frac{10}{3}}\ ms^{-2}$

B→C

$s = 1.2$	$v^2 = u^2 + 2as$
$u = 1.28$	$v^2 = 1.28^2 + 2\left(\frac{10}{3}\right) \times 1.2$
$v =$	$v^2 = 9.638$
$a = \frac{10}{3}$	$v = \underline{3.10\ ms^{-1}}$ STO
$t =$	

(b) Find the time taken for P to travel from A to C. [4]

A→B

$s = 0.8$	$v = u + at$
$u = 0$	$1.28 = 0 + 1.02t$
$v = 1.28$	$t = \underline{1.25s}$ STO
$a = 1.02$	
$t =$	

B→C

$s = 1.2$	$v = u + at$
$u = 1.28$	$3.10 = 1.28 + \frac{10}{3}t$
$v = 3.10$	$\frac{10}{3}t = 1.8246$
$a = \frac{10}{3}$	$t = \underline{0.547}$ STO
$t =$	

Time taken $= 1.25 + 0.547$

$= \underline{1.80s}$