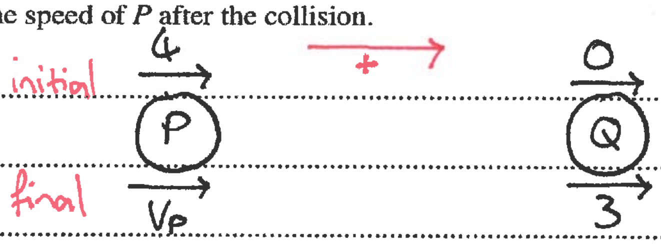
1 Two particles P and Q, of masses 0.3 kg and 0.2 kg respectively, are at rest on a smooth horizontal plane. P is projected at a speed of $4\,\text{m s}^{-1}$ directly towards Q. After P and Q collide, Q begins to move with a speed of $3\,\text{m s}^{-1}$.

(a) Find the speed of P after the collision. [2]



$M_P U_P + M_Q U_Q = M_P V_P + M_Q V_Q$

$0.3 \times 4 + 0 = 0.3V_P + 0.2 \times 3$

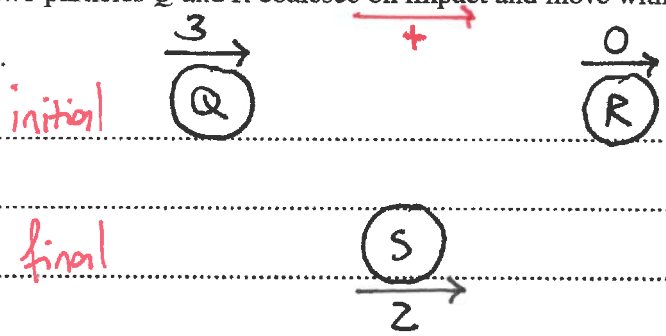
$1.2 = 0.3V_P + 0.6$

$0.6 = 0.3V_P$

$V_P = \underline{2\,\text{ms}^{-1}}$

After the collision, Q moves directly towards a third particle R, of mass m kg, which is at rest on the plane. The two particles Q and R coalesce on impact and move with a speed of $2\,\text{m s}^{-1}$.

(b) Find m. [2]



$M_Q U_Q + M_R U_R = M_S V_S$

$0.2 \times 3 + 0 = (m + 0.2) \times 2$

$0.6 = 2m + 0.4$

$0.2 = 2m$

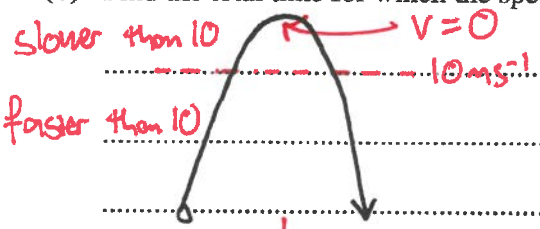
$m = \underline{0.1\text{ kg}}$

2 A particle P is projected vertically upwards from horizontal ground. P reaches a maximum height of 45 m. After reaching the ground, P comes to rest without rebounding.

(a) Find the speed at which P was projected. [2]

$\uparrow +$ $s = 45$	$v^2 = u^2 + 2as$
$u =$	$0^2 = u^2 + 2(-10)\times 45$
$v = 0$	$0 = u^2 - 900$
$a = -10$	$u^2 = 900$
$t =$	$u = 30\text{ms}^{-1}$

(b) Find the total time for which the speed of P is at least $10\,\text{m s}^{-1}$. [3]

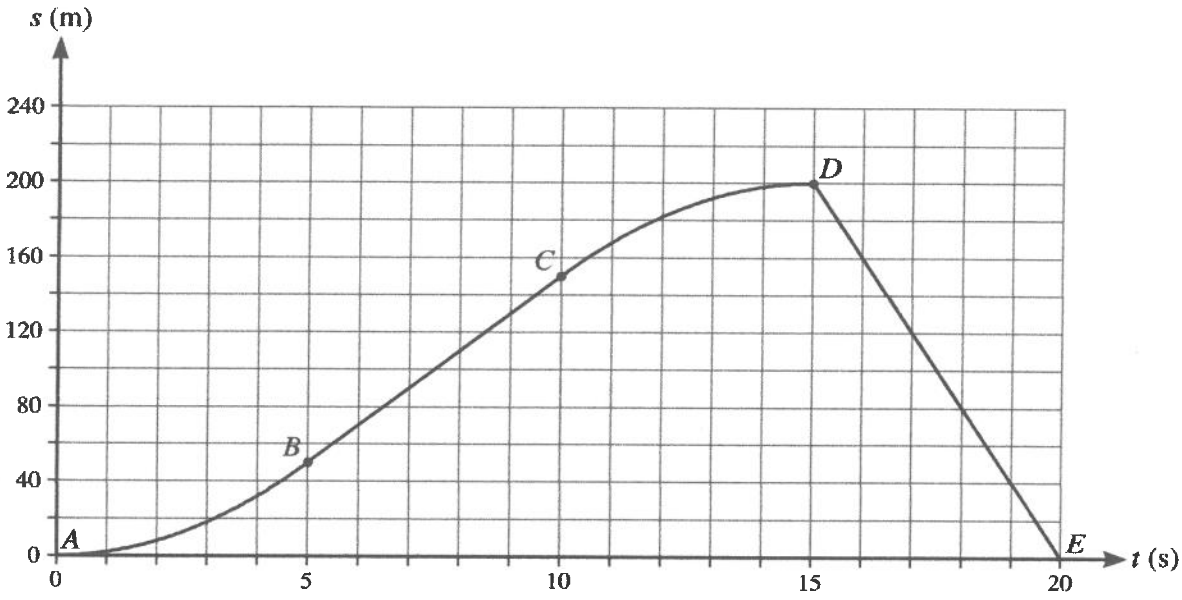


$s =$	$v = u + at$	$s =$	$v = u + at$
$u = 30$	$10 = 30 + (-10)\times t$	$u = 30$	$-10 = 30 + (-10)t$
$v = 10$	$10 = 30 - 10t$	$v = -10$	$-10 = 30 - 10t$
$a = -10$	$10t = 20$	$a = -10$	$10t = 40$
$t =$	$t = 2$	$t =$	$t = 4$

SO $v = 10\text{ms}^{-1}$ at $t = 2\text{s}$ and $t = 4\text{s}$

So it must be faster than 10ms^{-1} between 0–2s on the way up and 4–6s on the way down, so 4s in total.

3



The displacement of a particle moving in a straight line is s metres at time t seconds after leaving a fixed point O. The particle starts from rest and passes through points P, Q and R, at times $t = 5$, $t = 10$ and $t = 15$ respectively, and returns to O at time $t = 20$. The distances OP, OQ and OR are 50 m, 150 m and 200 m respectively.

The diagram shows a displacement-time graph which models the motion of the particle from $t = 0$ to $t = 20$. The graph consists of two curved segments AB and CD and two straight line segments BC and DE.

(a) Find the speed of the particle between $t = 5$ and $t = 10$. [1]

Speed is constant between B and C, so no acceleration, so we can use:

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{100}{5}$$

$$= \underline{\underline{20 \text{ ms}^{-1}}}$$

(b) Find the acceleration of the particle between $t = 0$ and $t = 5$, given that it is constant. [2]

$s = 50$ | $s = ut + \frac{1}{2}at^2$

$u = 0$ | $50 = 0 + \frac{1}{2} \times a \times 25$

$v =$ | $50 = 12.5a$

$a =$ | $a = 4ms^{-2}$

$t = 5$

(c) Find the average speed of the particle during its motion. [2]

average speed $= \frac{\text{total distance}}{\text{total time}}$

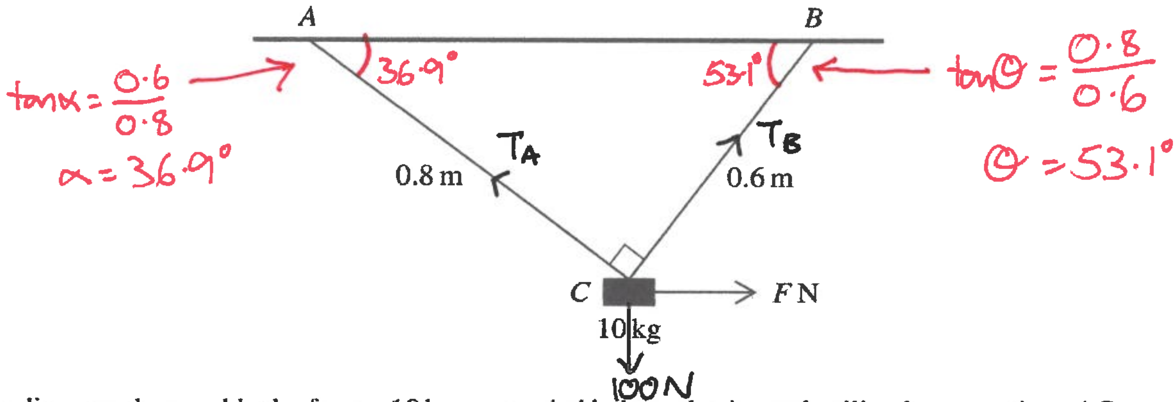
distance $= 200 + 200$

A→D D→E

av. speed $= \frac{400}{20}$

$= 20ms^{-1}$

4



The diagram shows a block of mass 10 kg suspended below a horizontal ceiling by two strings AC and BC, of lengths 0.8 m and 0.6 m respectively, attached to fixed points on the ceiling. Angle $ACB = 90°$. There is a horizontal force of magnitude F N acting on the block. The block is in equilibrium.

(a) In the case where $F = 20$, find the tensions in each of the strings. **[5]**

$R(\uparrow)$: $T_B \sin 53.1 + T_A \sin 36.9 - 100 = 0$

$0.8T_B + 0.6T_A - 100 = 0$

$0.8T_B + 0.6T_A = 100$ ①

$R(\rightarrow)$: $T_B \cos 53.1 + 20 - T_A \cos 36.9 = 0$

$0.6T_B + 20 - 0.8T_A = 0$

$0.6T_B - 0.8T_A = -20$

$0.8T_A = 0.6T_B + 20$

$T_A = 0.75T_B + 25$ ②

Sub ② into ①:

$0.8T_B + 0.6(0.75T_B + 25) = 100$

$0.8T_B + 0.45T_B + 15 = 100$

$1.25\,T_B = 85$

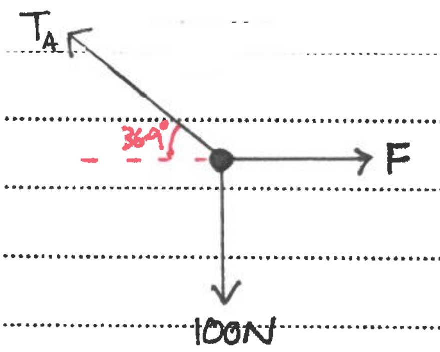
$T_B = 68N$

Sub into ②: $T_A = 0.75 \times 68 + 25$

$= 76N$

(b) Find the greatest value of F for which the block remains in equilibrium in the position shown. [3]

As F increases, T_B decreases until the point where $T_B = 0$:



$R(\uparrow)$: $T_A \sin 36.9 - 100 = 0$

$0.6 T_A = 100$

$T_A = \frac{500}{3}$ N

$R(\rightarrow)$: $F - T_A \cos 36.9 = 0$

$F - 0.8 T_A = 0$

$F - 0.8 \times \frac{500}{3} = 0$

$F = 0.8 \times \frac{500}{3}$

$= \frac{400}{3}$ N

5 A cyclist is riding along a straight horizontal road. The total mass of the cyclist and her bicycle is 70 kg. At an instant when the cyclist's speed is $4\,\mathrm{m\,s^{-1}}$, her acceleration is $0.3\,\mathrm{m\,s^{-2}}$. There is a constant resistance to motion of magnitude 30 N.

(a) Find the power developed by the cyclist. [3]

$R(\rightarrow):\quad F = ma$

$D - 30 = 70 \times 0.3$

$D - 30 = 21$

$\underline{D = 51\ \text{N}}$

$\text{Power} = D \times V$

$= 51 \times 4$

$= \underline{204\ \text{W}}$

The cyclist comes to the top of a hill inclined at 5° to the horizontal. The cyclist stops pedalling and freewheels down the hill (so that the cyclist is no longer supplying any power). The magnitude of the resistance force remains at 30 N. Over a distance of d m, the speed of the cyclist increases from $6\,\text{m s}^{-1}$ to $12\,\text{m s}^{-1}$.

(b) Find the change in kinetic energy. [2]

$KE_{initial} = \frac{1}{2} \times 70 \times 6^2$

$= 1260\text{ J}$

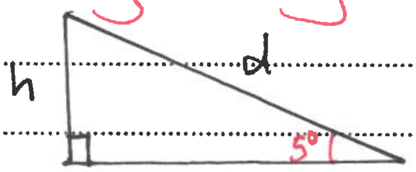
$KE_{final} = \frac{1}{2} \times 70 \times 12^2$

$= 5040$

Change in KE $= 5040 - 1260 = 3780\text{ J}$ increase

(c) Use an energy method to find d. [3]

Change in height over distance d:



$\sin 5 = \frac{h}{d}$

$h = d\sin 5$

$Work_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{OUT}$

$0 + 1260 + 0 = 5040 + 70 \times 10 \times -d\sin 5 + 30d$

$F \times d$

$1260 = 5040 - 700d\sin 5 + 30d$

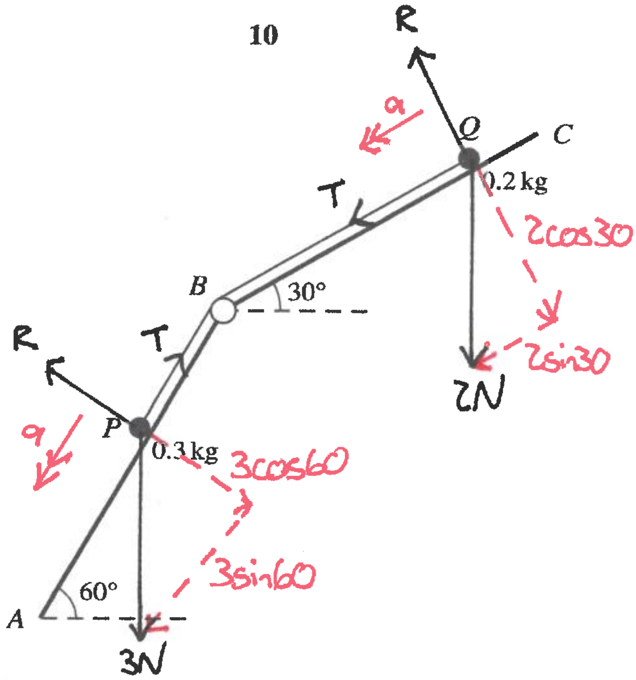
$-3780 = 30d - 700d\sin 5$

$-3780 = d(30 - 700\sin 5)$

$d = \frac{-3780}{30 - 700\sin 5}$

$= 122\text{ m}$

6



Two particles P and Q, of masses 0.3 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley at B which is attached to two inclined planes. P lies on a smooth plane AB which is inclined at 60° to the horizontal. Q lies on a plane BC which is inclined at 30° to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes (see diagram).

(a) It is given that the plane BC is smooth and that the particles are released from rest.

Find the tension in the string and the magnitude of the acceleration of the particles. [5]

P:

$R(\swarrow): 3\sin 60 - T = 0.3a$ ①

Q:

$R(\swarrow): 2\sin 30 + T = 0.2a$ ②

①+②: $3\sin 60 + 2\sin 30 = 0.5a$

$a = \dfrac{3\sin 60 + 2\sin 30}{0.5}$

$a = 7.20\ \text{ms}^{-2}$ STO

Sub into ②: $2\sin 30 + T = 0.2 \times 7.20$

$1 + T = 1.439$

$T = 0.439\ \text{N}$

(b) It is given instead that the plane BC is rough. A force of magnitude 3 N is applied to Q directly up the plane along a line of greatest slope of the plane.

Find the least value of the coefficient of friction between Q and the plane BC for which the particles remain at rest. [5]

P:

$R(\swarrow)$: $3\sin 60 - T = 0$

$T = 3\sin 60$

Q:

Resultant force up the plane = 3N

Resultant force down the plane = $3\sin 60 + 2\sin 30 = 3.60$N

So particle Q wants to move down the plane and friction acts upwards.

$R(\swarrow)$: $T + 2\sin 30 - F - 3 = 0$

$3\sin 60 + 2\sin 30 - \mu R - 3 = 0$

$\frac{3\sqrt{3}}{2} + 1 - \mu \times 2\cos 30 - 3 = 0$

$\mu \times 2\cos 30 = -2 + \frac{3\sqrt{3}}{2}$

$\mu = \dfrac{-2 + \frac{3\sqrt{3}}{2}}{2\cos 30}$

$\mu = 0.345$

7 A particle P moves in a straight line through a point O. The velocity $v\,\mathrm{m\,s^{-1}}$ of P, at time t s after passing O, is given by

$$v = \frac{9}{4} + \frac{b}{(t+1)^2} - ct^2,$$

where b and c are positive constants. At $t = 5$, the velocity of P is zero and its acceleration is $-\frac{13}{12}\,\mathrm{m\,s^{-2}}$.

(a) Show that $b = 9$ and find the value of c. [5]

$t=5, v=0$: $0 = \frac{9}{4} + \frac{b}{6^2} - c \times 5^2$

$0 = \frac{9}{4} + \frac{b}{36} - 25c$ $\times 36$

$0 = 81 + b - 900c$

$900c - b = 81$ (1)

$t=5, a=\frac{-13}{12}$:

$a = \frac{dv}{dt}$ $\left(v = \frac{9}{4} + b(t+1)^{-2} - ct^2\right)$

$a = -2b(t+1)^{-3} - 2ct$

$\frac{-13}{12} = \frac{-2b}{(6)^3} - 2c \times 5$

$\frac{-13}{12} = \frac{-2b}{216} - 10c$

$\frac{-13}{12} = \frac{-b}{108} - 10c$ $\times 108$

$-117 = -b - 1080c$ $\times -1$

$1080c + b = 117$ (2)

(1)+(2):

$1980c = 198$

$\underline{c = 0.1}$

Sub into (1): $900(0.1) - b = 81$

$90 - b = 81$

$-b = -9$

$\underline{b = 9}$ QED

(b) Given that the velocity of P is zero only at $t = 5$, find the distance travelled in the first 10 seconds of motion. [5]

If $v=0$ at $t=5$, there is a turning point here, so need to find s from $0 \to 5s$ and $5 \to 10s$.

$0 \to 5s$:

$$s = \int_0^5 \left(\frac{9}{4} + 9(t+1)^{-2} - \frac{1}{10}t^2\right) dt$$

$$= \left[\frac{9}{4}t - 9(t+1)^{-1} - \frac{1}{30}t^3\right]_0^5$$

$$= \left[\frac{9}{4}(5) - 9(5+1)^{-1} - \frac{1}{30}(5)^3\right] - \left[0 - 9(0+1)^{-1} - 0\right]$$

$$= \left[\frac{67}{12}\right] - \left[-9\right] = \underline{\frac{175}{12}\text{ m}}$$

$5 \to 10s$:

$$s = \int_5^{10} \left(\frac{9}{4} + 9(t+1)^{-2} - \frac{1}{10}t^2\right) dt$$

$$= \left[\frac{9}{4}t - 9(t+1)^{-1} - \frac{1}{30}t^3\right]_5^{10}$$

$$= \left[\frac{9}{4}(10) - 9(10+1)^{-1} - \frac{1}{30}(10)^3\right] - \left[\frac{67}{12}\right]$$

$$= \left[-\frac{769}{66}\right] - \left[\frac{67}{12}\right] = \underline{-\frac{2275}{132}\text{ m}}$$

$$\text{distance} = \frac{175}{12} + \frac{2275}{132}$$

$$= \underline{31.8\text{ m}}$$