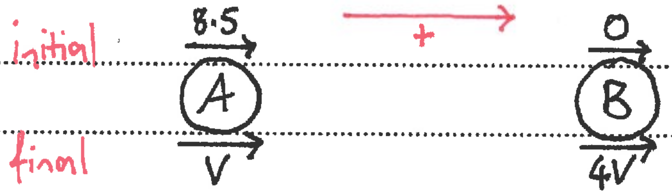
**1** Small smooth spheres $A$ and $B$, of equal radii and of masses 5 kg and 3 kg respectively, lie on a smooth horizontal plane. Initially $B$ is at rest and $A$ is moving towards $B$ with speed $8.5\,\mathrm{m\,s^{-1}}$. The spheres collide and after the collision $A$ continues to move in the same direction but with a quarter of the speed of $B$.

**(a)** Find the speed of $B$ after the collision. [3]




$$m_A u_A + m_B u_B = m_A V_A + m_B V_B$$

$$5 \times 8.5 + 0 = 5V + 3 \times 4V$$

$$42.5 = 17V$$

$$V = 2.5\,\mathrm{ms^{-1}}$$

$$V_B = 4V = \underline{10\,\mathrm{ms^{-1}}}$$

**(b)** Find the loss of kinetic energy of the system due to the collision. [2]

$$KE_{initial} = \frac{1}{2} \times 5 \times 8.5^2$$

$$= \underline{180.625\,\mathrm{J}}$$

$$KE_{final} = \frac{1}{2} \times 5 \times 2.5^2 + \frac{1}{2} \times 3 \times 10^2$$

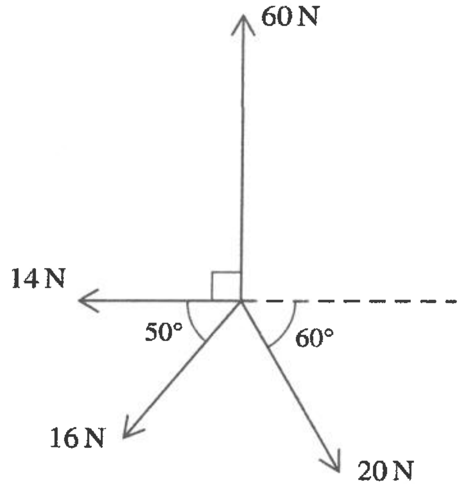
$$= 15.625 + 150$$

$$= 165.625\,\mathrm{J}$$

$$\text{Loss in KE} = 180.625 - 165.625$$

$$= \underline{15\,\mathrm{J}}$$

**2**




Coplanar forces of magnitudes 60 N, 20 N, 16 N and 14 N act at a point in the directions shown in the diagram.

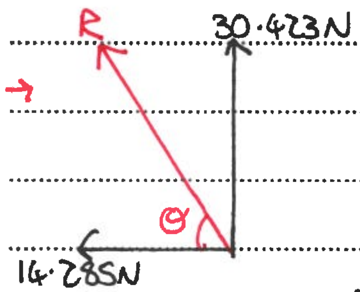
Find the magnitude and direction of the resultant force. [6]

$R(\uparrow)$: $60 - 20\sin 60 - 16\sin 50$

$= \underline{30.423\ N}$ STO

$R(\rightarrow)$: $20\cos 60 - 16\cos 50 - 14$

$= \underline{-14.285\ N}$ STO




$R = \sqrt{30.423^2 + 14.285^2}$

$= \underline{33.6\ N}$

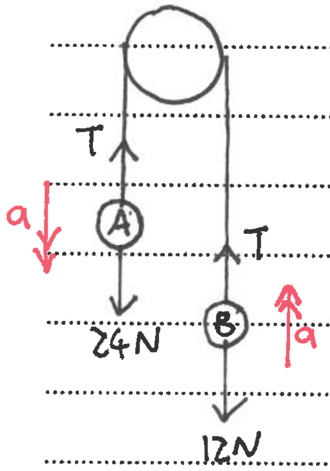
$\tan\theta = \dfrac{30.423}{14.285}$

$\theta = 64.8°$ above negative $x$ axis

(note mistake in mark scheme)

**3** Two particles $A$ and $B$, of masses 2.4 kg and 1.2 kg respectively, are connected by a light inextensible string which passes over a fixed smooth pulley. $A$ is held at a distance of 2.1 m above a horizontal plane and $B$ is 1.5 m above the plane. The particles hang vertically and are released from rest. In the subsequent motion $A$ reaches the plane and does not rebound and $B$ does not reach the pulley.

**(a)** Show that the tension in the string before $A$ reaches the plane is 16 N and find the magnitude of the acceleration of the particles before $A$ reaches the plane. [4]




A:

$R(\downarrow): 24 - T = 2.4a$ ①

B:

$R(\uparrow): T - 12 = 1.2a$ ②

①+②: $12 = 3.6a$

$a = \frac{10}{3} \text{ ms}^{-2}$

sub. into ②: $T - 12 = 1.2 \times \frac{10}{3}$

$T - 12 = 4$

$T = 16 \text{ N}$ QED

**(b)** Find the greatest height of $B$ above the plane. [3]

Before A hits ground:

| | |
|---|---|
| $\uparrow +$ $s = 2.1$ | $v^2 = u^2 + 2as$ |
| $u = 0$ | $v^2 = 0^2 + 2\left(\frac{10}{3}\right) \times 2.1$ |
| $v =$ | $v^2 = 14$ |
| $a = \frac{10}{3}$ | $v = \sqrt{14}$ |
| $t =$ | |

Once A has hit the ground, B moves under gravity.

| | |
|---|---|
| $\uparrow +$ $s =$ | $v^2 = u^2 + 2as$ |
| $u = \sqrt{14}$ | $0^2 = (\sqrt{14})^2 + 2(-10) \times s$ |
| $v = 0$ | $0 = 14 - 20s$ |
| $a = -10$ | $20s = 14$ |
| $t =$ | $s = 0.7\text{m}$ |

Height above plane $= 1.5 + 2.1 + 0.7 = \underline{4.3\text{m}}$

initial height

moved before A hits ground

after A hit ground

**4** A particle $A$, moving along a straight horizontal track with constant speed $8\,\text{m}\,\text{s}^{-1}$, passes a fixed point $O$. Four seconds later, another particle $B$ passes $O$, moving along a parallel track in the same direction as $A$. Particle $B$ has speed $20\,\text{m}\,\text{s}^{-1}$ when it passes $O$ and has a constant deceleration of $2\,\text{m}\,\text{s}^{-2}$. $B$ comes to rest when it returns to $O$.

**(a)** Find expressions, in terms of $t$, for the displacement from $O$ of each particle $t$ seconds after $B$ passes $O$. [3]

A: In the 4 seconds before B passes O, A travels:

$8 \times 4 = 32\,\text{m}$

After that A travels: $8 \times t = 8t$

So for A: $S_A = \underline{\underline{32 + 8t}}$

B: $s =$

$u = 20$

$v =$

$a = -2$

$t =$

$S_B = ut + \frac{1}{2}at^2$

$= 20t + \frac{1}{2}(-2)t^2$

$= 20t - t^2$

$\rightarrow S_B = \underline{\underline{20t - t^2}}$

**(b)** Find the values of $t$ when the particles are the same distance from $O$. [3]

$S_A = S_B$:

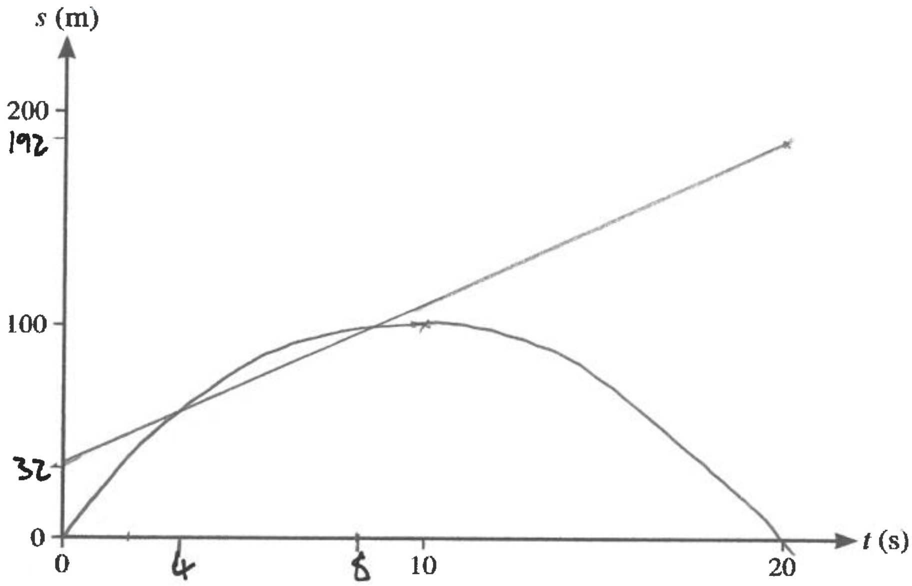
$32 + 8t = 20t - t^2$

$t^2 - 12t + 32 = 0$

$(t-4)(t-8) = 0$

$\underline{t = 4\text{s}}$ and $\underline{t = 8\text{s}}$

**(c)** On the given axes, sketch the displacement-time graphs for both particles, for values of $t$ from 0 to 20. [3]




A: at $t = 20$:

$S_A = 32 + 8 \times 20$

$= \underline{192}$

B: roots: $20t - t^2 = 0$

$t(20 - t) = 0$

$\underline{t = 0}$ or $\underline{t = 20}$

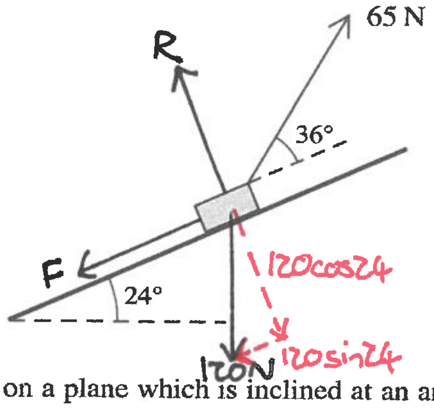
turning point: $t = 10$:

$S = 20 \times 10 - 10^2$

$= \underline{100}$

$\rightarrow (10, 100)$

5




A block of mass 12 kg is placed on a plane which is inclined at an angle of 24° to the horizontal. A light string, making an angle of 36° above a line of greatest slope, is attached to the block. The tension in the string is 65 N (see diagram). The coefficient of friction between the block and plane is $\mu$. The block is in limiting equilibrium and is on the point of sliding up the plane.

Find $\mu$. [6]

$R(\nwarrow): R + 65\sin 36 - 120\cos 24 = 0$

$R = 120\cos 24 - 65\sin 36$

$= \underline{71.419\ N}$ STO

$R(\nearrow): 65\cos 36 - 120\sin 24 - F = 0$

$65\cos 36 - 120\sin 24 - \mu R = 0$

$65\cos 36 - 120\sin 24 - 71.419\mu = 0$

$71.419\mu = 65\cos 36 - 120\sin 24$

$\mu = \dfrac{65\cos 36 - 120\sin 24}{71.419}$

$\mu = \underline{0.0529}$

$\sin\alpha = 0.12$

6 A car of mass 900 kg is moving up a hill inclined at $\sin^{-1} 0.12$ to the horizontal. The initial speed of the car is $11\,\text{m}\,\text{s}^{-1}$. After 12 s, the car has travelled 150 m up the hill and has speed $16\,\text{m}\,\text{s}^{-1}$. The engine of the car is working at a constant rate of 24 kW.

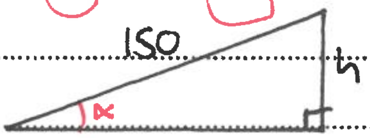
**(a)** Find the work done against the resistive forces during the 12 s. [5]

$$\text{Power} = \frac{\text{Work done}}{\text{time}}$$

$$24000 = \frac{\text{work done}}{12}$$

$$\text{Work done} = \underline{288000\,\text{J}}$$

Change in height when car travels 150m:




$$\sin\alpha = \frac{h}{150} \rightarrow h = 150 \times 0.12$$
$$= \underline{18\,\text{m}}$$

$$\text{Work}_{in} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{out}$$

$$288000 + \tfrac{1}{2} \times 900 \times 11^2 + 0 = \tfrac{1}{2} \times 900 \times 16^2 + 900 \times 10 \times 18 + W$$

$$288000 + 54450 = 115200 + 162000 + W$$

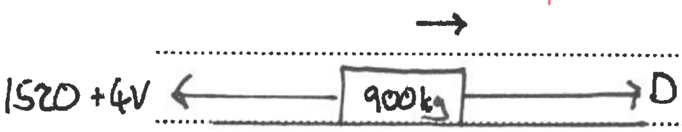
$$342450 = 277200 + W$$

$$W = \underline{65250\,\text{J}}$$

The car then travels along a straight horizontal road. There is a resistance to the motion of the car of $(1520 + 4v)$ N when the speed of the car is $v\,\mathrm{m\,s^{-1}}$. The car travels at a constant speed with the engine working at a constant rate of 32 kW.

**(b)** Find this speed. [3]

At constant speed, $a = 0$:




$R(\rightarrow)$: $D - (1520 + 4v) = m\underbrace{a}_{=0}$

$D - 1520 - 4V = 0$

$D = 1520 + 4v$

$\text{Power} = D \times V$

$32000 = (1520 + 4v)v$

$32000 = 1520v + 4v^2$

$4v^2 + 1520v - 32000 = 0$

$v^2 + 380v - 8000 = 0$

$(v + 400)(v - 20) = 0$

$\cancel{v = -400\,\mathrm{ms^{-1}}}$ or $v = 20\,\mathrm{ms^{-1}}$

wrong direction

**7** A particle $P$ moves in a straight line. The velocity $v\,\mathrm{m\,s^{-1}}$ at time $t$ seconds is given by

$$v = 0.5t \qquad \text{for } 0 \leqslant t \leqslant 10,$$
$$v = 0.25t^2 - 8t + 60 \qquad \text{for } 10 \leqslant t \leqslant 20.$$

**(a)** Show that there is an instantaneous change in the acceleration of the particle at $t = 10$. [3]

a from first expression:

$a = \frac{dv}{dt}$

$a = 0.5\,ms^{-2}$ (constant acceleration)

a from second expression:

$a = \frac{dv}{dt}$

$a = 0.5t - 8$

when $t = 10$:

$a = 0.5(10) - 8$

$= 5 - 8$

$= -3\,ms^{-2}$

So at 10s, the acceleration istantaneously changes from $0.5\,ms^{-2}$ to $-3\,ms^{-2}$.

**(b)** Find the total distance covered by $P$ in the interval $0 \leqslant t \leqslant 20$. [6]

0-10s: $s = \int_0^{10} 0.5t\,dt$

$= [0.25t^2]_0^{10}$

$= [0.25 \times 10^2] - [0]$

$= \underline{25m}$

10-20s: Careful - need to find when $v = 0$: these are the stationary points on $s(t)$ and tell us when a particle stops and starts moving in the opposite direction:

$0.25t^2 - 8t + 60 = 0$

$t^2 - 32t + 240 = 0$

$(t-12)(t-20) = 0$

$t = 12 \quad , \quad t = 20$

→ turning point at $t=12$, so need to integrate from 10→12 and 12→20

10-12s: $\int_{10}^{12} (0.25t^2 - 8t + 60)dt = \left[\frac{1}{12}t^3 - 4t^2 + 60t\right]_{10}^{12}$

$= \left[\frac{1}{12}(12)^3 - 4(12)^2 + 60(12)\right] - \left[\frac{1}{12}(10)^3 - 4(10)^2 + 60(10)\right]$

$= [288] - \left[\frac{850}{3}\right] = \underline{\frac{14}{3}m}$

12-20s: $\int_{12}^{20} (0.25t^2 - 8t + 60)dt = \left[\frac{1}{12}t^3 - 4t^2 + 60t\right]_{12}^{20}$

$= \left[\frac{1}{12}(20)^3 - 4(20)^2 + 60(20)\right] - \left[\frac{1}{12}(12)^3 - 4(12)^2 + 60(12)\right]$

$= \left[\frac{800}{3}\right] - [288] = \underline{-\frac{64}{3}m}$

distance $= 25 + \frac{14}{3} + \frac{64}{3} = \underline{51m}$