1 A car starts from rest and moves in a straight line with constant acceleration for a distance of 200 m, reaching a speed of $25\,\mathrm{m\,s^{-1}}$. The car then travels at this speed for 400 m, before decelerating uniformly to rest over a period of 5 s.

(a) Find the time for which the car is accelerating. [2]

$s = 200$ | $s = \frac{1}{2}(u+v)t$

$u = 0$ | $200 = \frac{1}{2}(25) \times t$

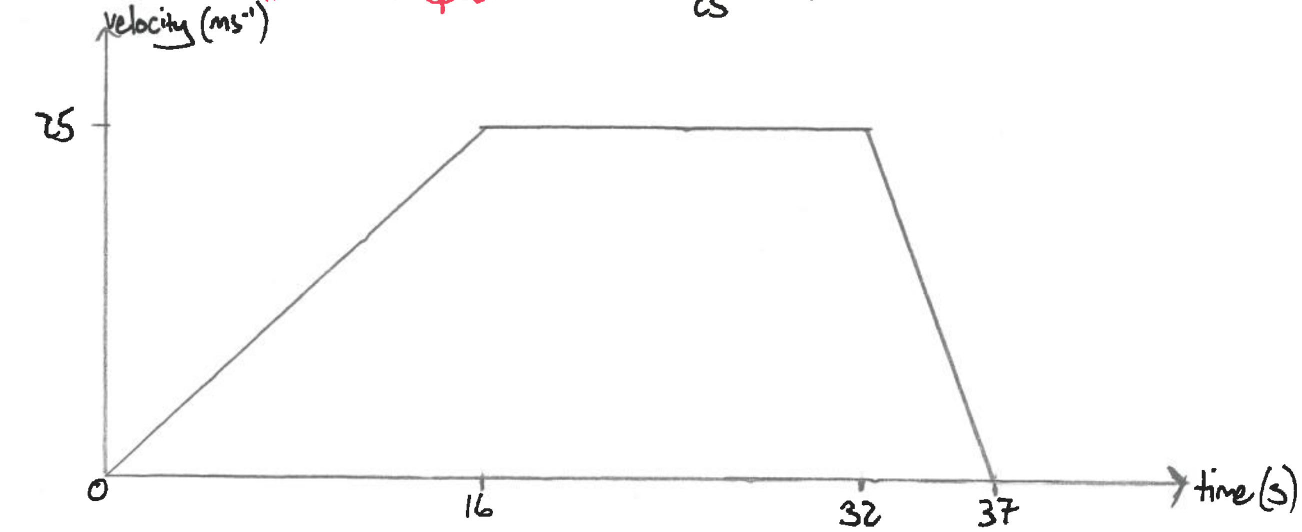
$v = 25$ | $200 = 12.5t$

$a =$ | $t = 16\,\mathrm{s}$

$t =$

(b) Sketch the velocity–time graph for the motion of the car, showing the key points. [2]

Time travelled at constant speed: time $= \frac{400}{25} = 16\mathrm{s}$



(c) Find the average speed of the car during its motion. [2]

Total distance $= \frac{1}{2}(16+37) \times 25$

$= \frac{1}{2} \times 53 \times 25$

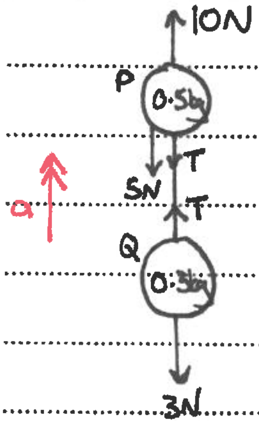
$= 662.5\mathrm{m}$

Average speed $= \frac{\text{total distance}}{\text{total time}}$

$= \frac{662.5}{37} = 17.9\,\mathrm{ms^{-1}}$

2 Two particles P and Q, of masses 0.5 kg and 0.3 kg respectively, are connected by a light inextensible string. The string is taut and P is vertically above Q. A force of magnitude 10 N is applied to P vertically upwards.

Find the acceleration of the particles and the tension in the string connecting them. [5]



Q:

$R(\uparrow): T - 3 = ma$

$T - 3 = 0.3a$ (1)

P:

$R(\uparrow): 10 - 5 - T = ma$

$5 - T = 0.5a$ (2)

(1) + (2):

$5 - 3 = 0.3a + 0.5a$

$2 = 0.8a$

$a = 2.5 \text{ ms}^{-2}$

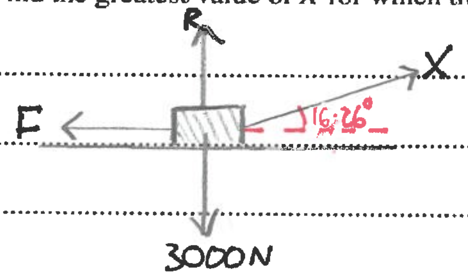
Sub into (1): $T - 3 = 0.3 \times 2.5$

$T - 3 = 0.75$

$T = 3.75 \text{ N}$

3 A crate of mass 300 kg is at rest on rough horizontal ground. The coefficient of friction between the crate and the ground is 0.5. A force of magnitude X N, acting at an angle α above the horizontal, is applied to the crate, where $\sin\alpha = 0.28$.

Find the greatest value of X for which the crate remains at rest. [5]



$\sin\alpha = 0.28$

$\alpha = 16.26$ STO

Greatest value of X for which crate remains at rest is at limiting equilibrium

$R(\uparrow)$: $R + X\sin\alpha - 3000 = 0$

$R + 0.28X - 3000 = 0$

$R = 3000 - 0.28X$

$R(\rightarrow)$: $X\cos\alpha - F = 0$ (limiting eqm so $F = \mu R$)

$X\cos(16.26) - \mu R = 0$

$0.96X - 0.5(3000 - 0.28X) = 0$

$0.96X - 1500 + 0.14X = 0$

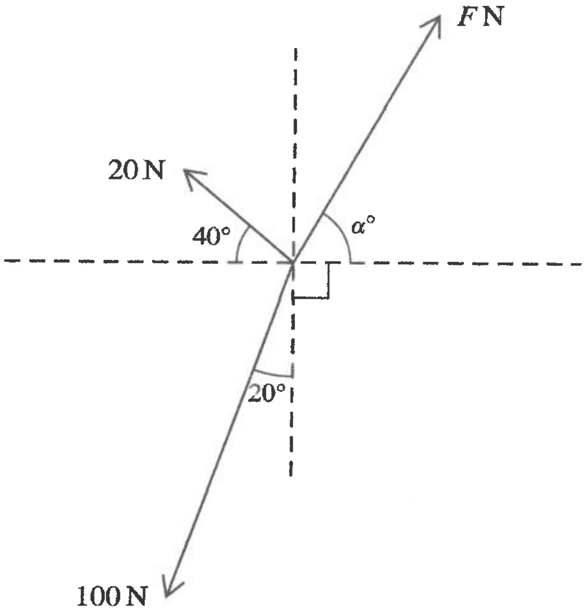
$1.1X - 1500 = 0$

$1.1X = 1500$

$X = 1363.6$

$= \underline{\underline{1360\ N}}$ (3sf)

4



Three coplanar forces of magnitudes 20 N, 100 N and F N act at a point. The directions of these forces are shown in the diagram.

Given that the three forces are in equilibrium, find F and α. [6]

$R(\uparrow)$: $F\sin\alpha + 20\sin 40 - 100\cos 20 = 0$

$F\sin\alpha = 100\cos 20 - 20\sin 40$

$F\sin\alpha = 81.114$ STO (1)

$R(\rightarrow)$: $F\cos\alpha - 20\cos 40 - 100\sin 20 = 0$

$F\cos\alpha = 100\sin 20 + 20\cos 40$

$F\cos\alpha = 49.523$ STO (2)

(1) ÷ (2): $\dfrac{F\sin\alpha}{F\cos\alpha} = \dfrac{81.114}{49.523}$

$\tan\alpha = 1.638$

$\alpha = \underline{58.6°}$ STO

Sub into (1): $F\sin(58.6) = 81.114$

$F = \dfrac{81.114}{\sin(58.6)}$

$= \underline{95.0\text{ N}}$

5 Two racing cars A and B are at rest alongside each other at a point O on a straight horizontal test track. The mass of A is 1200 kg. The engine of A produces a constant driving force of 4500 N. When A arrives at a point P its speed is $25\,\mathrm{m\,s^{-1}}$. The distance OP is d m. The work done against the resistance force experienced by A between O and P is 75 000 J.

(a) Show that $d = 100$. [3]

$$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{out}$$

$$4500d + 0 + 0 = \tfrac{1}{2} \times 1200 \times 25^2 + 0 + 75000$$

$$4500d = 375000 + 75000$$

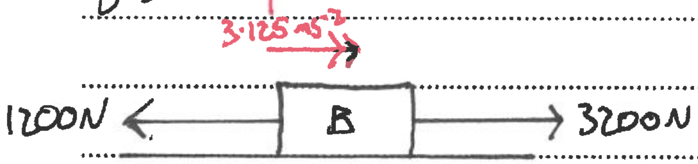
$$4500d = 450000$$

$$d = 100\text{m}$$

Car B starts off at the same instant as car A. The two cars arrive at P simultaneously and with the same speed. The engine of B produces a driving force of 3200 N and the car experiences a constant resistance to motion of 1200 N.

(b) Find the mass of B. [3]

$s = 100$ | $v^2 = u^2 + 2as$
$u = 0$ | $25^2 = 0^2 + 2 \times a \times 100$
$v = 25$ | $625 = 200a$
$a =$ | $a = 3.125 \text{ ms}^{-2}$
$t =$



$R(\rightarrow)$: $3200 - 1200 = ma$
$2000 = m \times 3.125$
$m = 640 \text{ kg}$

(c) Find the steady speed which B can maintain when its engine is working at the same rate as it is at P. [3]

Power $= D \times V$
$= 3200 \times 25$
$= 80\,000 \text{ W}$

At steady speed, acceleration $= 0$, so driving force equals resistance to motion $= 1200$ N:

Power $= D \times V$
$80\,000 = 1200 \times V$
$V = 66.7 \text{ ms}^{-1}$

6 A particle starts from a point O and moves in a straight line. The velocity $v\,\mathrm{m\,s^{-1}}$ of the particle at time t s after leaving O is given by

$$v = k(3t^2 - 2t^3),$$

where k is a constant.

(a) Verify that the particle returns to O when $t = 2$. [4]

$V = 3kt^2 - 2kt^3$

$S = \int (3kt^2 - 2kt^3)\,dt$

$S = kt^3 - \frac{1}{2}kt^4 + C$

$S=0$ when $t=0$:

$0 = 0 - 0 + C$

$C = 0$

$\rightarrow S = kt^3 - \frac{1}{2}kt^4$

when particle returns to O, $S=0$:

$0 = kt^3 - \frac{1}{2}kt^4$

$0 = kt^3(1 - \frac{1}{2}t)$

$t^3 = 0$ or $1 - \frac{1}{2}t = 0$

$t = 0$ $\quad \frac{t}{2} = 1$

$t = 2$

so particle returns to O when $t=2$.

(b) It is given that the acceleration of the particle is $-13.5\,\mathrm{m\,s^{-2}}$ for the positive value of t at which $v = 0$.

Find k and hence find the total distance travelled in the first two seconds of motion. [6]

$V=0$:

$3kt^2 - 2kt^3 = 0$

$kt^2(3 - 2t) = 0$

$t^2 = 0$ or $3 - 2t = 0$

$t = 0$ ✗ $\quad 2t = 3$

$\underline{t = 1.5}$ ✓

$a = \frac{dv}{dt} = 6kt - 6kt^2$

$a = -13.5$ when $t = 1.5$:

$6k(1.5) - 6k(1.5)^2 = -13.5$

$9k - 13.5k = -13.5$

$-4.5k = -13.5$

$\underline{k = 3}$

→ $S = 3t^3 - \frac{3}{2}t^4$

$V=0$ when $t = 1.5$, so turning point is at $t = 1.5$, meaning particle travels away from point O for 1.5s, then returns to starting point in the next 0.5s.

→ Find distance travelled in first 1.5s and multiply by 2:

sub. $t = 1.5$:

$S = 3 \times 1.5^3 - \frac{3}{2} \times 1.5^4$

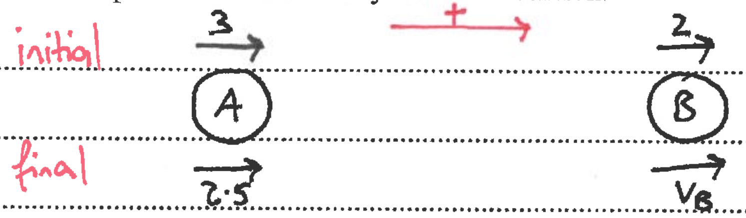
$= 2.53125$

Total distance $= 2 \times 2.53125$

$= \underline{5.0625\,\mathrm{m}}$

7 Two particles A and B, of masses 0.4 kg and 0.2 kg respectively, are moving down the same line of greatest slope of a smooth plane. The plane is inclined at 30° to the horizontal, and A is higher up the plane than B. When the particles collide, the speeds of A and B are $3\,\text{m s}^{-1}$ and $2\,\text{m s}^{-1}$ respectively. In the collision between the particles, the speed of A is reduced to $2.5\,\text{m s}^{-1}$.

(a) Find the speed of B immediately after the collision. [2]



$M_A u_A + M_B u_B = M_A V_A + M_B V_B$

$0.4 \times 3 + 0.2 \times 2 = 0.4 \times 2.5 + 0.2 V_B$

$1.2 + 0.4 = 1 + 0.2 V_B$

$1.6 = 1 + 0.2 V_B$

$0.6 = 0.2 V_B$ $\qquad V_B = 3\,\text{ms}^{-1}$

After the collision, when B has moved 1.6 m down the plane from the point of collision, it hits a barrier and returns back up the same line of greatest slope. B hits the barrier 0.4 s after the collision, and when it hits the barrier, its speed is reduced by 90%. The two particles collide again 0.44 s after their previous collision, and they then coalesce on impact.

(b) Show that the speed of B immediately after it hits the barrier is $0.5\,\text{m s}^{-1}$. Hence find the speed of the combined particle immediately after the second collision between A and B. [7]

B: ↓+

$s = 1.6$	$s = \frac{1}{2}(u+v) \times t$
$u = 3$	$1.6 = \frac{1}{2}(3+v) \times 0.4$
$v =$	$3.2 = (3+v) \times 0.4$
$a =$	$8 = 3 + v$
$t = 0.4$	$v = 5\,\text{ms}^{-1}$

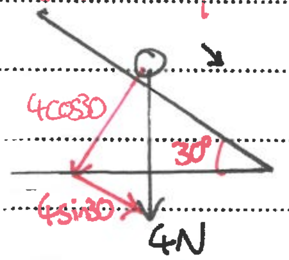
speed of B is reduced by 90%:

$0.9 \times 5 = 4.5 \qquad 5 - 4.5 = 0.5\,\text{ms}^{-1}$ QED

continued...

Need to find velocity of particle A at point of impact:

A:



$R(\searrow)$: $F = ma$

$4\sin 30 = 0.4a$

$2 = 0.4a$

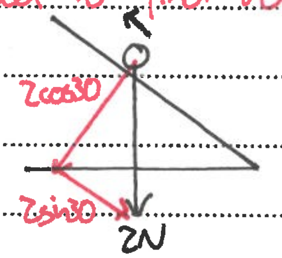
$\underline{a = 5\text{ms}^{-2}}$

A: $\searrow +$

$s =$	$v = u + at$
$u = 2.5$	$= 2.5 + 5 \times 0.44$
$v =$	$= 2.5 + 2.2$
$a = 5$	$= \underline{4.7\text{ms}^{-1}}$
$t = 0.44$	

Need to find velocity of particle B 0.04s after hitting barrier:

B:



$R(\nwarrow)$: $F = ma$

$-2\sin 30 = 0.2a$

$-1 = 0.2a$

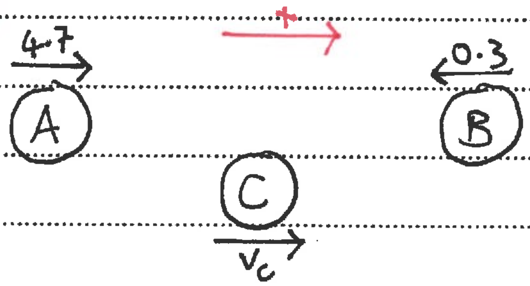
$\underline{a = -5\text{ms}^{-2}}$

B: $\nwarrow +$

$s =$	$v = u + at$
$u = 0.5$	$= 0.5 + (-5) \times 0.04$
$v =$	$= 0.5 - 0.2$
$a = -5$	$= \underline{0.3\text{ms}^{-1}}$
$t = 0.04$	

initial

final



$m_A u_A + m_B u_B = m_C V_C$

$0.4 \times 4.7 + 0.2 \times -0.3 = 0.6V_C$

$1.88 - 0.06 = 0.6V_C$

$1.82 = 0.6V_C$

$\underline{\underline{V_C = 3.03\text{ms}^{-1}}}$

long!!