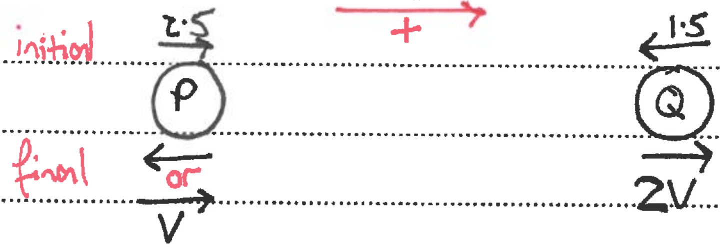
1 Particles P of mass 0.4 kg and Q of mass 0.5 kg are free to move on a smooth horizontal plane. P and Q are moving directly towards each other with speeds $2.5\,\mathrm{m\,s^{-1}}$ and $1.5\,\mathrm{m\,s^{-1}}$ respectively. After P and Q collide, the speed of Q is twice the speed of P.

Find the two possible values of the speed of P after the collision. [4]



(Q must be moving to the right, otherwise P and Q would coalesce)

- Scenario 1: P goes left:

$$M_P U_P + M_Q U_Q = M_P V_P + M_Q V_Q$$
$$0.4 \times 2.5 + 0.5 \times -1.5 = 0.4 \times -V + 0.5 \times 2V$$
$$1 - 0.75 = -0.4V + 1V$$
$$0.25 = 0.6V$$
$$V = \underline{0.417\,\mathrm{ms^{-1}}}$$

- Scenario 2: P goes right:

$$M_P U_P + M_Q U_Q = M_P V_P + M_Q V_Q$$
$$0.4 \times 2.5 + 0.5 \times -1.5 = 0.4 \times V + 0.5 \times 2V$$
$$1 - 0.75 = 0.4V + 1V$$
$$0.25 = 1.4V$$
$$V = \underline{0.179\,\mathrm{ms^{-1}}}$$

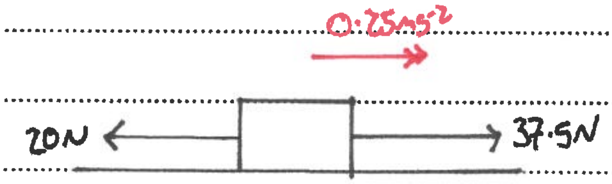
2 A cyclist is travelling along a straight horizontal road. She is working at a constant rate of 150 W. At an instant when her speed is $4\,\mathrm{m\,s^{-1}}$, her acceleration is $0.25\,\mathrm{m\,s^{-2}}$. The resistance to motion is 20 N.

(a) Find the total mass of the cyclist and her bicycle. [3]

Power = D × V

150 = D × 4

$\underline{D = 37.5\,N}$



$R(\rightarrow)$: F = ma

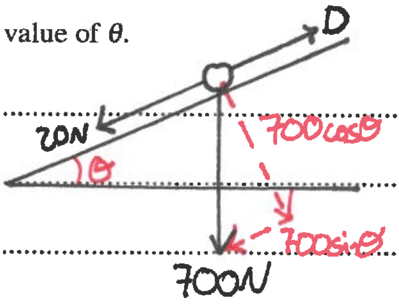
37.5 − 20 = m × 0.25

17.5 = m × 0.25

$\underline{\underline{m = 70\,kg}}$

The cyclist comes to a straight hill inclined at an angle θ above the horizontal. She ascends the hill at constant speed $3\,\mathrm{m\,s^{-1}}$. She continues to work at the same rate as before and the resistance force is unchanged.

(b) Find the value of θ. [2]



Power = D × V

150 = D × 3

$\underline{D = 50\,N}$

$R(\nearrow)$: 50 − 700sinθ − 20 = ma

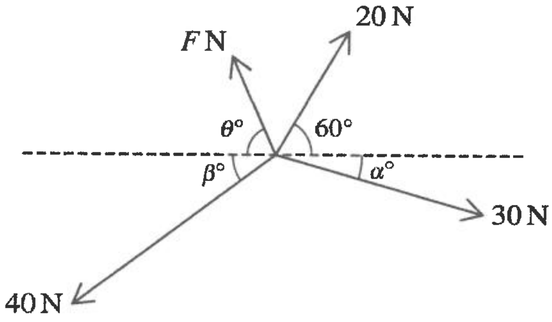
30 − 700sinθ = 0 ← a = 0 (constant speed)

700sinθ = 30

$\sin\theta = \frac{30}{700}$

$\underline{\underline{\theta = 2.5°}}$ (1dp)

3



Four coplanar forces act at a point. The magnitudes of the forces are 20 N, 30 N, 40 N and F N. The directions of the forces are as shown in the diagram, where $\sin\alpha° = 0.28$ and $\sin\beta° = 0.6$.

$\alpha = 16.26_{STO}$ $\quad \beta = 36.87_{STO}$

Given that the forces are in equilibrium, find F and θ. [6]

$R(\uparrow)$: $F\sin\theta + 20\sin60 - 40\sin\beta - 30\sin\alpha = 0$

$F\sin\theta + 20\sin60 - 40\times0.6 - 30\times0.28 = 0$

$F\sin\theta + 20\sin60 - 24 - 8.4 = 0$

$F\sin\theta = 32.4 - 20\sin60$ ①

$R(\rightarrow)$: $20\cos60 + 30\cos\alpha - F\cos\theta - 40\cos\beta = 0$

$20\cos60 + 30\cos(16.26) - F\cos\theta - 40\cos(36.87) = 0$

$F\cos\theta = 20\cos60 + 30\cos(16.26) - 40\cos(36.87)$

$F\cos\theta = 6.8$ ②

① ÷ ②: $\dfrac{F\sin\theta}{F\cos\theta} = \dfrac{32.4 - 20\sin60}{6.8}$

$\tan\theta = 2.218$

$\theta = \underline{\underline{65.7°}}_{STO}$

Sub into ②: $F\cos(65.7) = 6.8$

$F = \dfrac{6.8}{\cos(65.7)}$

$F = \underline{\underline{16.5\text{N}}}$

4 A particle is projected vertically upwards with speed u m s^{-1} from a point on horizontal ground. After 2 seconds, the height of the particle above the ground is 24 m.

(a) Show that $u = 22$. [2]

↑+ $s = 24$

$u =$

$v =$

$a = -10$

$t = 2$

$s = ut + \frac{1}{2}at^2$

$24 = 2u + \frac{1}{2}(-10) \times 2^2$

$24 = 2u - 20$

$44 = 2u$

$\underline{u = 22 \text{ ms}^{-1}}$ QED

(b) The height of the particle above the ground is more than h m for a period of 3.6 s.

Find h. [4]

Find maximum height:

↑+ $s =$

$u = 22$

$v = 0$

$a = -10$

$t =$

$v^2 = u^2 + 2as$

$0^2 = 22^2 + 2(-10) \times s$

$0 = 484 - 20s$

$20s = 484$

$s = 24.2$ m

Now find height 1.8s later, on the way down:

↖3.6 ÷ 2

↓+ $s =$

$u = 0$

$v =$

$a = 10$

$t = 1.8$

$s = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times 10 \times 1.8^2$

$= 16.2$m

→ Height above ground $= 24.2 - 16.2$

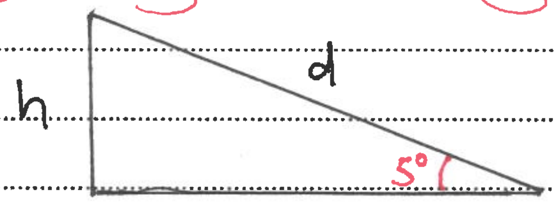
$= \underline{8\text{m}}$

5 A car of mass 1400 kg is towing a trailer of mass 500 kg down a straight hill inclined at an angle of 5° to the horizontal. The car and trailer are connected by a light rigid tow-bar. At the top of the hill the speed of the car and trailer is 20 m s^{-1} and at the bottom of the hill their speed is 30 m s^{-1}.

(a) It is given that as the car and trailer descend the hill, the engine of the car does 150 000 J of work, and there are no resistance forces.

Find the length of the hill. [5]

Change in height when travelling a distance d:



$\sin 5 = \frac{h}{d}$

$h = d\sin 5$

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$150\,000 + \frac{1}{2}\times 1900\times 20^2 + 0 = \frac{1}{2}\times 1900\times 30^2 + 1900\times 10\times -d\sin 5 + 0$

$150\,000 + 380\,000 = 855\,000 - 19000 d\sin 5$

$530\,000 = 855\,000 - 19000\sin 5\times d$

$-325\,000 = -19000\sin 5\times d$

$d = \frac{-325000}{-19000\sin 5}$

$= \underline{196\text{ m}}$

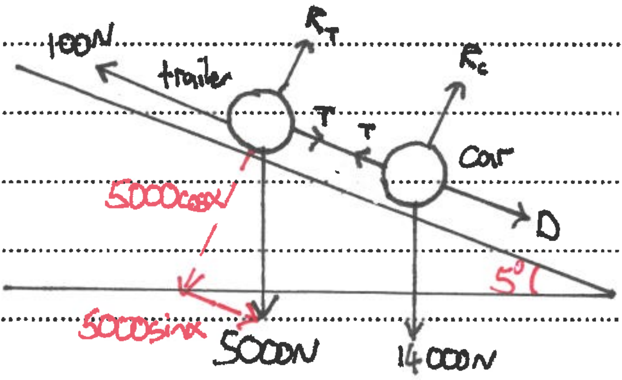
(b) It is given instead that there is a resistance force of 100 N on the trailer, the length of the hill is 200 m, and the acceleration of the car and trailer is constant.

Find the tension in the tow-bar between the car and trailer. [4]

Find acceleration using Suvat:

↘+

$s = 200$	$v^2 = u^2 + 2as$
$u = 20$	$30^2 = 20^2 + 2 \times a \times 200$
$v = 30$	$900 = 400 + 400a$
$a =$	$500 = 400a$
$t =$	$a = 1.25 ms^{-2}$



Consider Trailer:

$R(\searrow)$: $T + 5000\sin5 - 100 = ma$

$T + 5000\sin5 - 100 = 500 \times 1.25$

$T + 5000\sin5 - 100 = 625$

$T = 725 - 5000\sin5$

$= 289 N$

6 A particle moves in a straight line and passes through the point A at time $t = 0$. The velocity of the particle at time t s after leaving A is v m s^{-1}, where

$$v = 2t^2 - 5t + 3.$$

(a) Find the times at which the particle is instantaneously at rest. Hence or otherwise find the minimum velocity of the particle. [4]

$V = 0$:

$2t^2 - 5t + 3 = 0$

$(2t-3)(t-1) = 0$

$2t - 3 = 0$ or $t - 1 = 0$

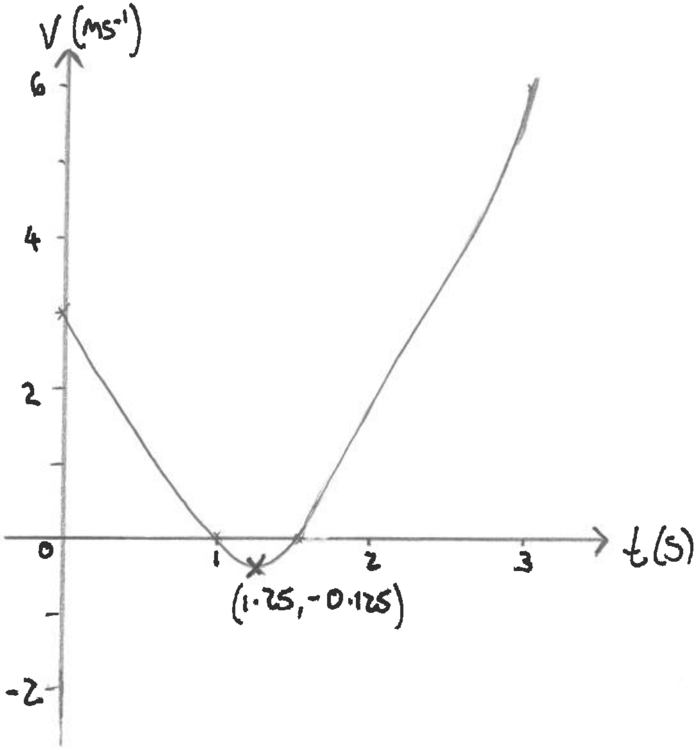
$t = 1.5$s or $t = 1$s

If ~~roots~~ roots are at $t=1$ and $t=1.5$, then turning point is at $t=1.25$:

$V = 2\times(1.25)^2 - 5(1.25) + 3$

$= -0.125$ ms^{-1}

(b) Sketch the velocity-time graph for the first 3 seconds of motion. [3]



(c) Find the distance travelled between the two times when the particle is instantaneously at rest. [3]

instantaneously at rest at $t = 1$ and $t = 1.5$:

$$S = \int_{1}^{1.5} (2t^2 - 5t + 3)\,dt$$

$$= \left[\frac{2}{3}t^3 - \frac{5}{2}t^2 + 3t\right]_1^{1.5}$$

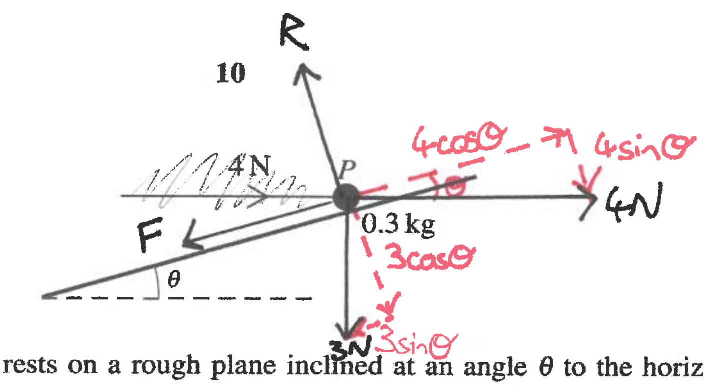
$$= \left[\frac{2}{3}(1.5)^3 - \frac{5}{2}(1.5)^2 + 3(1.5)\right] - \left[\frac{2}{3}(1)^3 - \frac{5}{2}(1)^2 + 3(1)\right]$$

$$= 1.125 - \frac{7}{6}$$

$$= -0.0417\text{m}$$ negative because it's below t-axis.

distance $= \underline{0.0417\text{m}}$

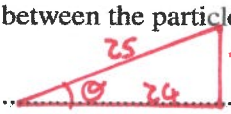
7



A particle P of mass 0.3 kg rests on a rough plane inclined at an angle θ to the horizontal, where $\sin\theta = \frac{7}{25}$. A horizontal force of magnitude 4 N, acting in the vertical plane containing a line of greatest slope of the plane, is applied to P (see diagram). The particle is on the point of sliding up the plane.

(a) Show that the coefficient of friction between the particle and the plane is $\frac{3}{4}$. [4]

$\sin\theta = \frac{7}{25}$ →



$\cos\theta = \frac{24}{25}$ $\tan\theta = \frac{7}{24}$

$R(\nwarrow): R - 3\cos\theta - 4\sin\theta = 0$

$R - 3\times\frac{24}{25} - 4\times\frac{7}{25} = 0$

$R - 4 = 0$

$\underline{R = 4\text{ N}}$

$R(\nearrow): 4\cos\theta - 3\sin\theta - F = 0$

$4\times\frac{24}{25} - 3\times\frac{7}{25} - \mu R = 0$

$3 - \mu R = 0$

$3 - 4\mu = 0$

$4\mu = 3$

$\underline{\mu = 3/4}$ QED

The force acting horizontally is replaced by a force of magnitude 4 N acting up the plane parallel to a line of greatest slope.

(b) Find the acceleration of P. [3]

$R(\nwarrow): R - 3\cos\theta = 0$

$R = 3\times\frac{24}{25}$

$R = \underline{2.88\text{N}}$

$R(\nearrow): 4 - 3\sin\theta - F = ma$

$4 - 3\times\frac{7}{25} - \mu R = 0.3a$

$4 - 0.84 - \frac{3}{4}\times 2.88 = 0.3a$

$4 - 0.84 - 2.16 = 0.3a$

$1 = 0.3a$

$a = 3.33 ms^{-2}$

STO

(c) Starting with P at rest, the force of 4 N parallel to the plane acts for 3 seconds and is then removed.

Find the total distance travelled until P comes to instantaneous rest. [3]

$S=$

$u=0$

$V=$

$a=\frac{10}{3}$ (3.33..)

$t=3$

$s=ut+\frac{1}{2}at^2$

$=0+\frac{1}{2}\times\left(\frac{10}{3}\right)\times 3^2$

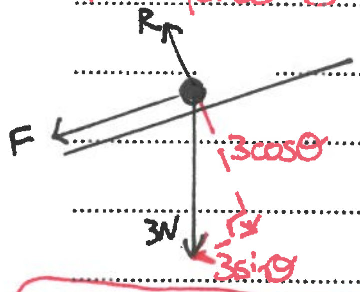
$s=15m$

$V=u+at$

$=0+\frac{10}{3}\times 3$

$V=10\ ms^{-1}$

After 3s, P has travelled 15m and is moving at 10ms⁻¹. 4N force is removed. Particle is still moving UP, so friction is acting down.



$R(\nearrow): -3\sin\theta - F = ma$

$-3\times\frac{7}{25} - \mu R = 0.3a$

$-0.84 - \frac{3}{4}\times 2.88 = 0.3a$

$-3 = 0.3a$

$a = -10$

$S=$

$u=10$

$v=0$

$a=-10$

$t=$

$v^2=u^2+2as$

$0^2=10^2+2(-10)s$

$0=100-20s$

$20s=100$

$s=5m$ → P moves $15+5=20m$