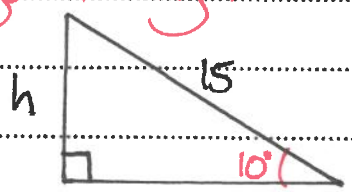
1 A particle of mass 0.6 kg is projected with a speed of $4\,\text{m}\,\text{s}^{-1}$ down a line of greatest slope of a smooth plane inclined at 10° to the horizontal.

Use an energy method to find the speed of the particle after it has moved 15 m down the plane. [3]

Change in height:



$\sin 10 = \frac{h}{15}$

$\underline{h = 15 \sin 10}$

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$0 + \frac{1}{2} \times 0.6 \times 4^2 + 0 = \frac{1}{2} \times 0.6 \times v^2 + 0.6 \times 10 \times -15 \sin 10 + 0$

$4.8 = 0.3v^2 - 90 \sin 10$

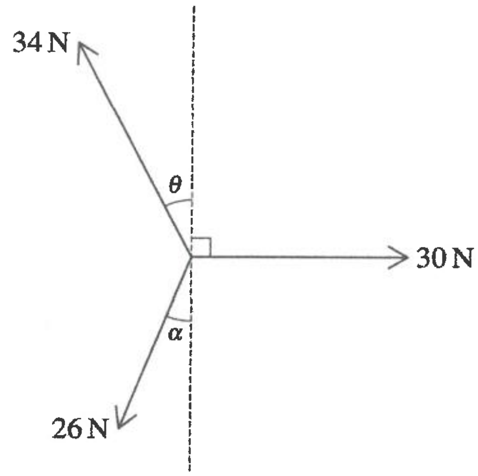
$4.8 + 90 \sin 10 = 0.3v^2$

$20.428.. = 0.3v^2$

$v^2 = 68.09..$

$v = \underline{8.25\,\text{ms}^{-1}}$

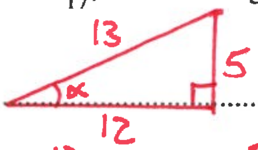
2



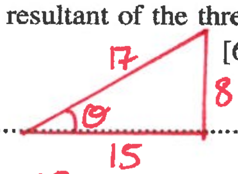
Coplanar forces of magnitudes 34 N, 30 N and 26 N act at a point in the directions shown in the diagram.

Given that $\sin\alpha = \frac{5}{13}$ and $\sin\theta = \frac{8}{17}$, find the magnitude and direction of the resultant of the three forces. [6]

$\sin\alpha = \frac{5}{13} \rightarrow$



$\sin\theta = \frac{8}{17} \rightarrow$



$\cos\alpha = \frac{12}{13}$ $\tan\alpha = \frac{5}{12}$ $\cos\theta = \frac{15}{17}$ $\tan\theta = \frac{8}{15}$

$R(\uparrow):\ 34\cos\theta - 26\cos\alpha$

$= 34\left(\frac{15}{17}\right) - 26\left(\frac{12}{13}\right)$

$= 30 - 24$

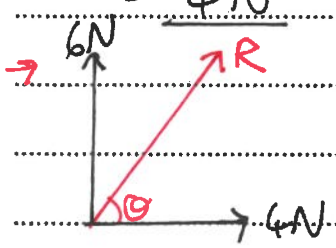
$= \underline{6\text{ N}}$

$R(\rightarrow):\ 30 - 34\sin\theta - 26\sin\alpha$

$= 30 - 34\left(\frac{8}{17}\right) - 26\left(\frac{5}{13}\right)$

$= 30 - 16 - 10$

$= \underline{4\text{ N}}$



$R = \sqrt{6^2 + 4^2}$

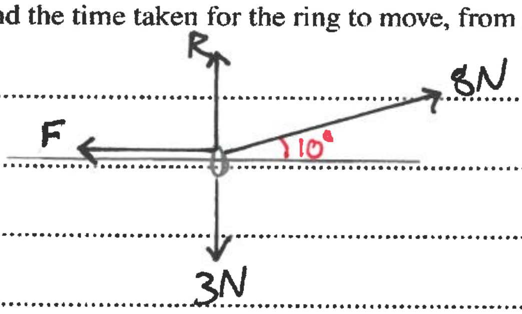
$= \underline{7.21\text{ N}}$

$\tan\theta = \frac{6}{4}$

$\theta = \underline{56.3^\circ \text{ above positive } x \text{ direction}}$

3 A ring of mass 0.3 kg is threaded on a horizontal rough rod. The coefficient of friction between the ring and the rod is 0.8. A force of magnitude 8 N acts on the ring. This force acts at an angle of 10° above the horizontal in the vertical plane containing the rod.

Find the time taken for the ring to move, from rest, 0.6 m along the rod. [6]



$R(\uparrow)$: $R + 8\sin 10 - 3 = 0$

$R = 3 - 8\sin 10$

$R = \underline{1.61\ N}$ STO

$R(\rightarrow)$: $8\cos 10 - F = ma$ (particle is in motion so $F = \mu R$)

$8\cos 10 - \mu R = 0.3a$

$8\cos 10 - 0.8 \times 1.61 = 0.3a$

$8\cos 10 - 1.289 = 0.3a$

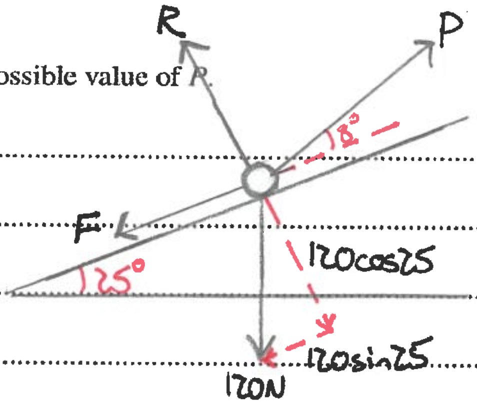
$a = (8\cos 10 - 1.289) \div 0.3$

$= \underline{21.966}$ STO

$s = 0.6$	$s = ut + \frac{1}{2}at^2$
$u = 0$	$0.6 = 0 + \frac{1}{2} \times 21.966t^2$
$v =$	$0.6 = 10.983t^2$
$a = 21.966$	$t^2 = 0.0546$
$t =$	$\underline{t = 0.234\text{s}}$

4 A particle of mass 12 kg is stationary on a rough plane inclined at an angle of 25° to the horizontal. A pulling force of magnitude P N acts at an angle of 8° above a line of greatest slope of the plane. This force is used to keep the particle in equilibrium. The coefficient of friction between the particle and the plane is 0.3.

Find the greatest possible value of P. [6]



Greatest value of P is when particle is in limiting equilibrium, about to move up the plane, so friction is acting down the plane.

R(↖): $R + P\sin 8 - 120\cos 25$

$R = 120\cos 25 - P\sin 8$

R(↗): $P\cos 8 - 120\sin 25 - F = 0$

$P\cos 8 - 120\sin 25 - \mu R = 0$

$P\cos 8 - 120\sin 25 - 0.3(120\cos 25 - P\sin 8) = 0$

$P\cos 8 - 120\sin 25 - 36\cos 25 + 0.3P\sin 8 = 0$

$P\cos 8 + 0.3P\sin 8 = 120\sin 25 + 36\cos 25$

$P(\cos 8 + 0.3\sin 8) = 120\sin 25 + 36\cos 25$

$$P = \frac{120\sin 25 + 36\cos 25}{\cos 8 + 0.3\sin 8}$$

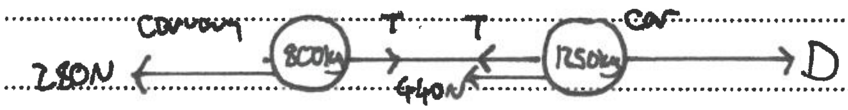
$P = \underline{80.8\text{N}}$

5 A car of mass 1250 kg is pulling a caravan of mass 800 kg along a straight road. The resistances to the motion of the car and caravan are 440 N and 280 N respectively. The car and caravan are connected by a light rigid tow-bar.

(a) The car and caravan move along a horizontal part of the road at a constant speed of 30 m s^{-1}.

(i) Calculate, in kW, the power developed by the engine of the car. [2]

$a = 0\,ms^{-2}$



Consider whole system:

$R(\rightarrow)$: $F = ma$ $\quad a = 0$

$D - T + T - 440 - 280 = ma$

$D - 720 = 0$

$D = 720N$

Power $= D \times V$

$= 720 \times 30$

$= 21600W$

(ii) Given that this power is suddenly decreased by 8 kW, find the instantaneous deceleration of the car and caravan and the tension in the tow-bar. [4]

New Power $= 21600 - 8000$

$= 13600\,W$

Power $= D \times V$

$13600 = D \times 30$

$D = 453.\dot{3}N$ STO

Whole system:

$R(\rightarrow)$: $F = ma$

$453.\dot{3} - 440 - 280 = 2050a$

$-266.\dot{6} = 2050a$

$a = -0.130\,ms^{-2}$

Caravan:

$R(\rightarrow)$: $T - 280 = ma$

$T - 280 = 800 \times (-0.130)$

$T - 280 = -104.1$

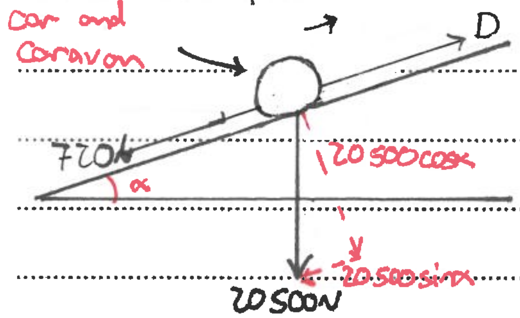
$T = 175.9$

$T = 176N$

$\sin\alpha = 0.06$

(b) The car and caravan now travel along a part of the road inclined at $\sin^{-1} 0.06$ to the horizontal. The car and caravan travel up the incline at constant speed with the engine of the car working at 28 kW.

(i) Find this constant speed. $a = 0\,ms^{-2}$ **[3]**



Whole system:

$R(\nearrow)$: $F = ma$ $\quad = 0$

$D - 720 - 20500\sin\alpha = 0$

$D - 720 - 20500 \times 0.06 = 0$

$D - 720 - 1230 = 0$

$D - 1950 = 0$

$D = 1950N$

Power $= D \times V$

$28000 = 1950 \times V$

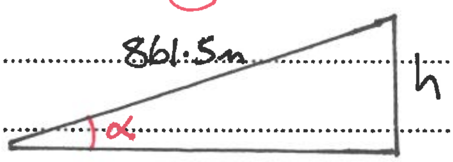
$V = 14.4\,ms^{-1}$ STO

(ii) Find the increase in the potential energy of the caravan in one minute. **[2]**

Distance travelled in 1 minute: distance $= 14.4 \times 60$

$= 861.5m$ STO

Increase in height in 1 minute:



$\sin\alpha = \dfrac{h}{861.5}$

$h = 861.5 \times 0.06$

$= 51.7m$ STO

Change in PE $= mg\Delta h$

$= 800 \times 10 \times 51.7$

$= 413538J$

$= 414000J$

6 A particle A is projected vertically upwards from level ground with an initial speed of $30\,\mathrm{m\,s^{-1}}$. At the same instant a particle B is released from rest 15 m vertically above A. The mass of one of the particles is twice the mass of the other particle. During the subsequent motion A and B collide and coalesce to form particle C.

Find the difference between the two possible times at which C hits the ground. [8]

Need to find the position and velocities of A and B when they collide:

$+\uparrow A$:		$+\downarrow B$:	
$s = s_A$	$s_A = ut + \frac{1}{2}at^2$	$s = s_B$	$s_B = ut + \frac{1}{2}at^2$
$u = 30$	$s_A = 30t - 5t^2$	$u = 0$	$s_B = 0 + 5t^2$
$v =$		$v =$	$s_B = 5t^2$
$a = -10$		$a = 10$	
$t =$		$t =$	

$$s_A + s_B = 15$$
$$30t - 5t^2 + 5t^2 = 15$$
$$30t = 15$$
$$t = 0.5\,\mathrm{s}$$

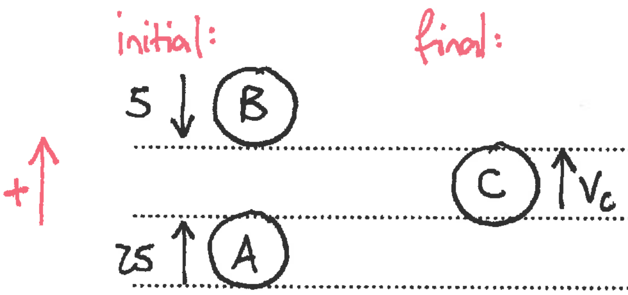
Find height above ground by substituting $t = 0.5$ into s_A:

$$s_A = 30(0.5) - 5(0.5)^2$$
$$= 15 - 1.25$$
$$= \underline{13.75\,\mathrm{m}} \text{ above ground}$$

Find velocities of A and B:

$+\uparrow A$:		$+\downarrow B$:	
$s =$	$v = u + at$	$s =$	$v = u + at$
$u = 30$	$= 30 + (-10) \times 0.5$	$u = 0$	$= 0 + 10(0.5)$
$v =$	$= 30 - 5$	$v =$	$= \underline{5\,\mathrm{ms^{-1}}}$
$a = -10$	$= \underline{25\,\mathrm{ms^{-1}}}$	$a = 10$	
$t = 0.5$		$t = 0.5$	

continued...



Scenario 1: $A = 2m$, $B = m$:

$$m_A u_A + m_B u_B = m_C V_C$$
$$2m \times 25 + m \times -5 = 3m V_C$$
$$50m - 5m = 3m V_C$$
$$45\cancel{m} = 3\cancel{m} V_C$$
$$45 = 3V_C$$
$$\underline{V_C = 15 ms^{-1}} \quad 1$$

Scenario 2: $A = m$, $B = 2m$:

$$m_A u_A + m_B u_B = m_C V_C$$
$$m \times 25 + 2m \times -5 = 3m V_C$$
$$25m - 10m = 3m V_C$$
$$15\cancel{m} = 3\cancel{m} V_C$$
$$15 = 3V_C$$
$$\underline{V_C = 5 ms^{-1}} \quad 2$$

Scenario 1:

$+\uparrow$ $s = -13.75$, $u = 15$, $v =$, $a = -10$, $t =$

$$s = ut + \tfrac{1}{2}at^2$$
$$-13.75 = 15t - 5t^2$$
$$5t^2 - 15t - 13.75 = 0$$
$$t = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(5)(-13.75)}}{2(5)}$$
$$\underline{t = 3.74} \checkmark \text{ or } t = -0.736 \times$$

STO

Scenario 2:

$+\uparrow$ $s = -13.75$, $u = 5$, $v =$, $a = -10$, $t =$

$$s = ut + \tfrac{1}{2}at^2$$
$$-13.75 = 5t - 5t^2$$
$$5t^2 - 5t - 13.75 = 0$$
$$t = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(5)(-13.75)}}{2(5)}$$
$$\underline{t = 2.23} \checkmark \text{ or } t = -1.23 \times$$

STO

$$\text{difference} = 3.74 - 2.23$$
$$= \underline{1.50s}$$

7 A particle P moving in a straight line starts from rest at a point O and comes to rest 16 s later. At time t s after leaving O, the acceleration a m s^{-2} of P is given by

$$a = 6 + 4t \qquad 0 \leqslant t < 2,$$
$$a = 14 \qquad 2 \leqslant t < 4,$$
$$a = 16 - 2t \qquad 4 \leqslant t \leqslant 16.$$

There is no sudden change in velocity at any instant.

(a) Find the values of t when the velocity of P is 55 m s^{-1}. [5]

① v for $0 \leqslant t < 2$:

$v = \int (6 + 4t)\,dt$

$v = 6t + 2t^2 + C$

$v = 0$ at $t = 0$:

$0 = 0 + 0 + C$

$C = 0$

→ $v = 6t + 2t^2$ ①

Sub. $t = 2$ to find v:

$v = 6(2) + 2(2)^2$

$= 20\text{ms}^{-1}$

② v for $2 \leqslant t < 4$:

$v = \int 14\,dt$

$v = 14t + C$

$v = 20$ when $t = 2$:

$20 = 14(2) + C$

$20 = 28 + C$

$C = -8$

→ $v = 14t - 8$ ②

Sub. $t = 4$ to find v:

$v = 14(4) - 8$

$= 48\text{ms}^{-1}$

③ v for $4 \leqslant t \leqslant 16$

$v = \int (16 - 2t)\,dt$

$v = 16t - t^2 + C$

$v = 48$ when $t = 4$:

$48 = 16(4) - 4^2 + C$

$48 = 48 + C$

$C = 0$

→ $v = 16t - t^2$ ③

Solve ① $= 55\text{ms}^{-1}$:

$2t^2 + 6t = 55$

$2t^2 + 6t - 55 = 0$

$t = \dfrac{-6 \pm \sqrt{6^2 - 4(2)(-55)}}{2(2)}$

$t = 3.95\text{s}$ ✗, $t = -6.95\text{s}$ ✗

outside $0 \leqslant t < 2$

Solve ② $= 55\text{ms}^{-1}$:

$14t - 8 = 55$

$14t = 63$

$t = 4.5\text{s}$ ✗ outside $2 \leqslant t < 4$

Continued →

Solve ③ = $55 ms^{-1}$:

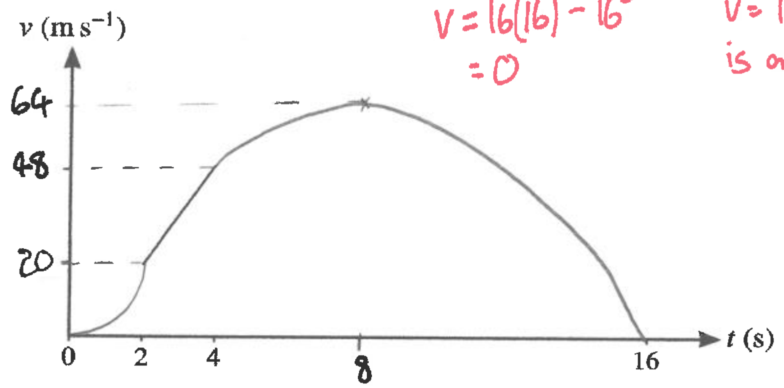
$16t - t^2 = 55$

$t^2 - 16t + 55 = 0$

$(t-5)(t-11) = 0$

$\underline{t = 5s}$ or $\underline{t = 11s}$

(b) Complete the sketch of the velocity-time diagram. [2]



$t=16$:
$v = 16(16) - 16^2$
$= 0$

turning point of $v = 16t - t^2$ is at $(8, 64)$

(c) Find the distance travelled by P when it is decelerating. [3]

P is decelerating between $t = 8$ and $t = 16$.

$s = \int_8^{16} (16t - t^2)\,dt$

$= \left[8t^2 - \frac{1}{3}t^3\right]_8^{16}$

$= \left[8(16)^2 - \frac{1}{3}(16)^3\right] - \left[8(8)^2 - \frac{1}{3}(8)^3\right]$

$= \frac{2048}{3} - \frac{1024}{3}$

$= \frac{1024}{3}$ m