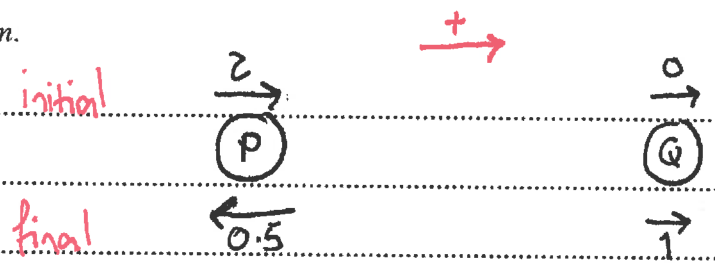
1 Particles P of mass m kg and Q of mass 0.2 kg are free to move on a smooth horizontal plane. P is projected at a speed of $2\,\text{m}\,\text{s}^{-1}$ towards Q which is stationary. After the collision P and Q move in opposite directions with speeds of $0.5\,\text{m}\,\text{s}^{-1}$ and $1\,\text{m}\,\text{s}^{-1}$ respectively.

Find m. [3]



$$M_P u_P + M_Q u_Q = M_P V_P + M_Q V_Q$$

$$m \times 2 + 0 = m \times -0.5 + 0.2 \times 1$$

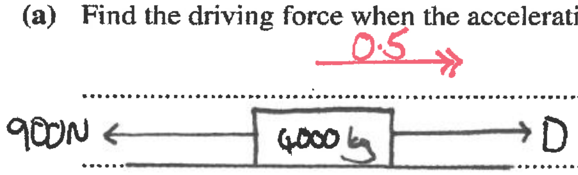
$$2m = -0.5m + 0.2$$

$$2.5m = 0.2$$

$$m = \underline{0.08\text{ kg}}$$

2 A minibus of mass 4000 kg is travelling along a straight horizontal road. The resistance to motion is 900 N.

(a) Find the driving force when the acceleration of the minibus is $0.5\,\mathrm{m\,s^{-2}}$. [2]



$R(\rightarrow)$: $F = ma$

$D - 900 = 4000 \times 0.5$

$D - 900 = 2000$

$D = \underline{2900\ N}$

(b) Find the power required for the minibus to maintain a constant speed of $25\,\mathrm{m\,s^{-1}}$. [2]

At constant speed, $a = 0$:

$R(\rightarrow)$: $D - 900 = 0$

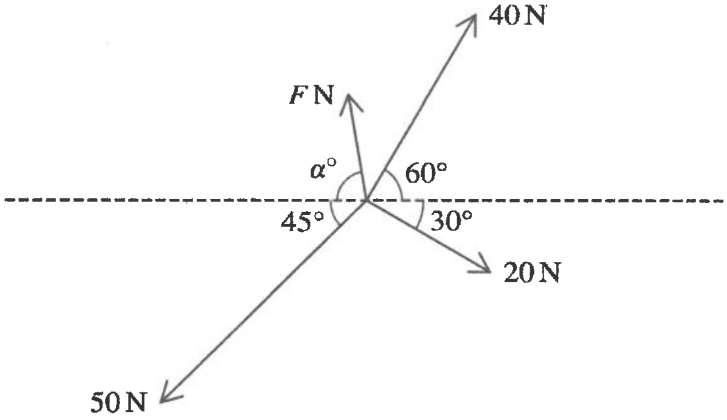
$D = 900N$

$\text{Power} = D \times V$

$= 900 \times 25$

$= \underline{22\,500\ W}$

3



Four coplanar forces of magnitudes 40 N, 20 N, 50 N and F N act at a point in the directions shown in the diagram. The four forces are in equilibrium.

Find F and α. [6]

$R(\uparrow)$: $40\sin 60 + F\sin\alpha - 20\sin 30 - 50\sin 45 = 0$

$F\sin\alpha = 20\sin 30 + 50\sin 45 - 40\sin 60$

$F\sin\alpha = 10.714$ STO ①

$R(\rightarrow)$: $40\cos 60 + 20\cos 30 - F\cos\alpha - 50\cos 45 = 0$

$F\cos\alpha = 40\cos 60 + 20\cos 30 - 50\cos 45$

$F\cos\alpha = 1.965$ STO ②

① ÷ ②: $\dfrac{F\sin\alpha}{F\cos\alpha} = \dfrac{10.714}{1.965}$

$\tan\alpha = 5.452\ldots$

$\alpha = 79.6°$ STO

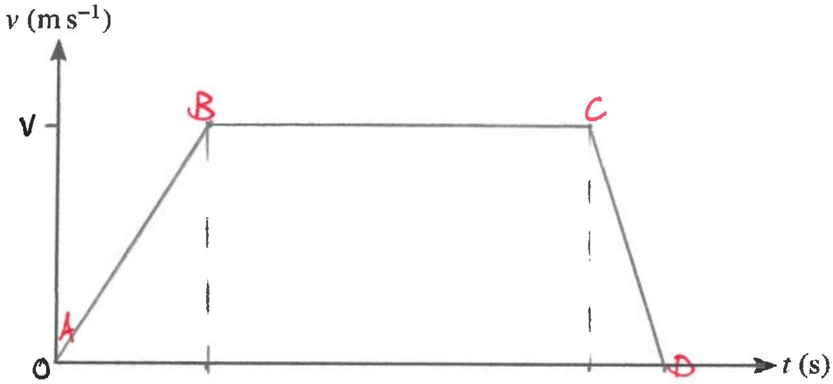
Sub into ①: $F\sin(79.6) = 10.714$

$F = \dfrac{10.714}{\sin(79.6)}$

$= 10.9\text{ N}$

4 A car starts from rest and moves in a straight line with constant acceleration $a\,\mathrm{m\,s^{-2}}$ for a distance of 50 m. The car then travels with constant velocity for 500 m for a period of 25 s, before decelerating to rest. The magnitude of this deceleration is $2a\,\mathrm{m\,s^{-2}}$.

(a) Sketch the velocity-time graph for the motion of the car. [1]



(b) Find the value of a. [3]

during constant velocity section (BC): speed $= \frac{500}{25}$

$= 20\,\mathrm{ms^{-1}}$

s: 50 | $V^2 = u^2 + 2as$
u: 0 | $20^2 = 0^2 + 2 \times a \times 50$
v = 20 | $400 = 100a$
a = | $a = 4\,\mathrm{ms^{-2}}$
t =

(c) Find the total time for which the car is in motion. [3]

AB: s = 50 | $V = u + at$
u: 0 | $20 = 0 + 4t$
V: 20 | $t = 5\,\mathrm{s}$
a = 4
t =

CD: s = | $V = u + at$
u = 20 | $0 = 20 - 8t$
V = 0 | $8t = 20$
a = −8 | $t = 2.5\,\mathrm{s}$
t =

total time $= 5 + 25 + 2.5$

$= 32.5\,\mathrm{s}$

5 A block B of mass 4 kg is pushed up a line of greatest slope of a smooth plane inclined at 30° to the horizontal by a force applied to B, acting in the direction of motion of B. The block passes through points P and Q with speeds $12\,\text{m}\,\text{s}^{-1}$ and $8\,\text{m}\,\text{s}^{-1}$ respectively. P and Q are 10 m apart with P below the level of Q.

(a) Find the decrease in kinetic energy of the block as it moves from P to Q. [2]

$KE_{init} = \frac{1}{2} \times 4 \times 12^2$

$= 288\text{J}$

$KE_{fin} = \frac{1}{2} \times 4 \times 8^2$

$= 128\text{J}$

Change in $KE = 288 - 128$

$= 160\text{J}$ decrease

(b) Hence find the work done by the force pushing the block up the slope as the block moves from P to Q. [3]

$Work_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{out}$

$W + 288 + 0 = 128 + 4 \times 10 \times 10\sin 30 + 0$

$W + 288 = 128 + 400\sin 30$

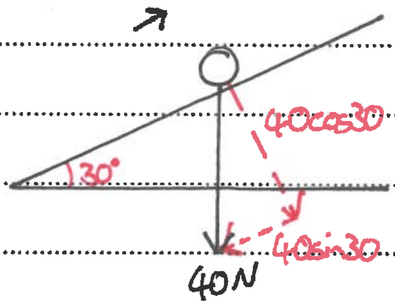
$W + 288 = 128 + 200$

$W + 288 = 328$

$W = 40\text{J}$

(c) At the instant the block reaches Q, the force pushing the block up the slope is removed.

Find the time taken, after this instant, for the block to return to P. [4]



$R(\nearrow)$: $F = ma$

$-40\sin 30 = 4a$

$-20 = 4a$

$a = -5 \text{ ms}^{-2}$

$+\nearrow$

$s = -10$ | $s = ut + \frac{1}{2}at^2$

$u = 8$ | $-10 = 8t + \frac{1}{2}(-5) \times t^2$

$v =$ | $-10 = 8t - 2.5t^2$

$a = -5$ | $2.5t^2 - 8t - 10 = 0$

$t =$ | $5t^2 - 16t - 20 = 0$

$$t = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(5)(-20)}}{2(5)}$$

$t = 4.16\text{s}$ ✓ or $t = -0.961\text{s}$ ✗

6 A particle travels in a straight line PQ. The velocity of the particle t s after leaving P is v m s^{-1}, where

$$v = 4.5 + 4t - 0.5t^2.$$

(a) Find the velocity of the particle at the instant when its acceleration is zero. [3]

$a = \frac{dv}{dt} = 4 - t$

$a = 0$:

$4 - t = 0$

$t = 4$s

Sub. $t = 4$ into V:

$V = 4.5 + 4(4) - 0.5(4)^2$

$= 4.5 + 16 - 8$

$= 12.5$ ms^{-1}

The particle comes to instantaneous rest at Q.

(b) Find the distance PQ. [6]

$V = 0$:

$4.5 + 4t - 0.5t^2 = 0$ $\times 2$

$9 + 8t - t^2 = 0$ $\times -1$

$t^2 - 8t - 9 = 0$

$(t-9)(t+1) = 0$

$t = 9$ ✓ or $t = -1$ ✗

$S = \int (4.5 + 4t - \frac{1}{2}t^2)\,dt$

$S = 4.5t + 2t^2 - \frac{1}{6}t^3 + C$

at P, $t = 0$:

$S = 0 + 0 - 0 + C$

$\underline{S = C}$

at Q, $t = 9$:

$S = 4.5(9) + 2(9)^2 - \frac{1}{6}(9)^3 + C$

$S = 40.5 + 162 - 121.5 + C$

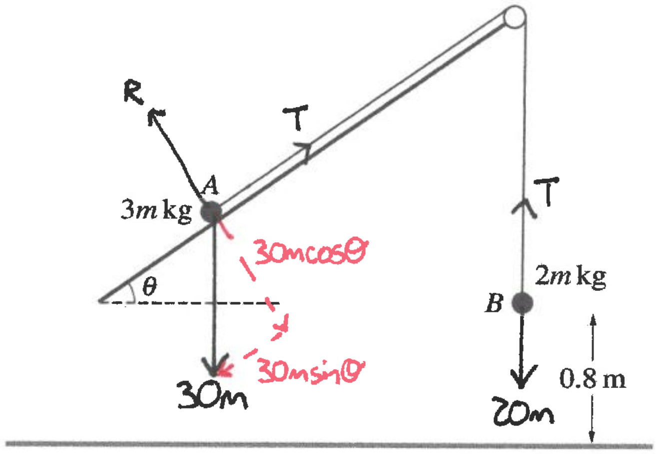
$\underline{S = 81 + C}$

distance between P and Q:

distance $= 81 + C - C$

$= \underline{81\text{m}}$

7



Two particles A and B, of masses $3m$ kg and $2m$ kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to the edge of a plane. The plane is inclined at an angle θ to the horizontal. A lies on the plane and B hangs vertically, 0.8 m above the floor, which is horizontal. The string between A and the pulley is parallel to a line of greatest slope of the plane (see diagram). Initially A and B are at rest.

(a) Given that the plane is smooth, find the value of θ for which A remains at rest. [3]

B: A:

$R(\downarrow)$: $20m - T = 0$ ① $R(\nearrow)$: $T - 30m\sin\theta = 0$ ②

①+②:

$20m - 30m\sin\theta = 0$ $\sin\theta = \frac{2}{3}$

$30m\sin\theta = 20m$ $\theta = 41.8°$

$\sin\theta = \frac{20m}{30m}$

It is given instead that the plane is rough, $\theta = 30°$ and the acceleration of A up the plane is $0.1\ \text{m s}^{-2}$.

(b) Show that the coefficient of friction between A and the plane is $\frac{1}{10}\sqrt{3}$. [5]

B: A:

$R(\downarrow)$: $20m - T = 2ma$ $R(\nwarrow)$: $R - 30m\cos 30 = 0$

$20m - T = 2m \times 0.1$ $R = 30m\cos 30$

$20m - T = 0.2m$ $R = 15\sqrt{3}m$

$T = 19.8m$ ①

CONT →

A:

R(↗): $T - 30m\sin 30 - F = 3ma$

$T - 15m - \mu R = 3m \times 0.1$

$T - 15m - \mu \times 15\sqrt{3}m = 0.3m$

$T - 15m - 15\sqrt{3}m\mu = 0.3m$

$T = 15.3m + 15\sqrt{3}m\mu$ ②

Sub. ① into ②: $19.8m = 15.3m + 15\sqrt{3}m\mu$

$19.8 = 15.3 + 15\sqrt{3}\mu$

$15\sqrt{3}\mu = 4.5$

$\mu = \frac{4.5}{15\sqrt{3}}$

$\mu = \frac{3}{10\sqrt{3}} \frac{\times\sqrt{3}}{\times\sqrt{3}}$

$\mu = \frac{3\sqrt{3}}{30}$

$\mu = \frac{\sqrt{3}}{10}$ QED

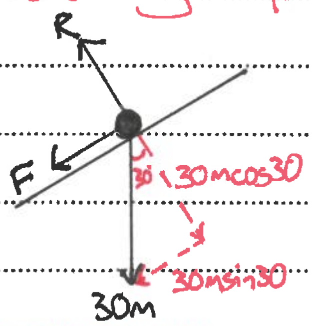
(c) When B reaches the floor it comes to rest.

Find the length of time after B reaches the floor for which A is moving up the plane. [You may assume that A does not reach the pulley.] [4]

Before B hits floor:

$s = 0.8$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2(0.1) \times 0.8$
$v =$	$v^2 = 0.16$
$a = 0.1$	$v = 0.4\,ms^{-1}$
$t =$	

Force diagram for A once string goes slack:



R(↗): $-30m\sin 30 - F = 3ma$

$-15m - \mu R = 3ma$

$-15m - \frac{\sqrt{3}}{10} \times 15\sqrt{3}m = 3ma$

$-15m - 4.5m = 3ma$

$-19.5m = 3ma$

$a = \frac{-19.5m}{3m}$

$= -6.5\,ms^{-2}$

$s =$	$v = u + at$
$u = 0.4$	$0 = 0.4 + -6.5 \times t$
$v = 0$	$6.5t = 0.4$
$a = -6.5$	$t = 0.0615\,s$
$t =$	