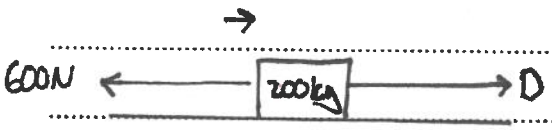
1 A crate of mass 200 kg is being pulled at constant speed along horizontal ground by a horizontal rope attached to a winch. The winch is working at a constant rate of 4.5 kW and there is a constant resistance to the motion of the crate of magnitude 600 N.

(a) Find the time that it takes for the crate to move a distance of 15 m. [2]



$R(\rightarrow)$: $F = ma$

$D - 600 = ma$ $=0$ (constant speed)

$D - 600 = 0$

$D = 600\,N$

Power $= D \times V$

$4500 = 600V$

$V = 7.5\,ms^{-1}$

No acceleration so time $= \frac{\text{distance}}{\text{speed}}$

time $= \frac{15}{7.5}$

$= 2s$

The rope breaks after the crate has moved 15 m.

(b) Find the time taken, after the rope breaks, for the crate to come to rest. [3]

$R(\rightarrow)$: $F = ma$

$-600 = 200a$

$a = -3ms^{-2}$

$\xrightarrow{+}$ $s =$	$V = u + at$
$u = 7.5$	$0 = 7.5 + -3 \times t$
$v = 0$	$0 = 7.5 - 3t$
$a = -3$	$3t = 7.5$
$t =$	$t = 2.5s$

2 A particle P is projected vertically upwards from horizontal ground with speed $15\,\mathrm{m\,s^{-1}}$.

(a) Find the speed of P when it is $10\,\mathrm{m}$ above the ground. [2]

↑+ $s = 10$ | $v^2 = u^2 + 2as$
$u = 15$ | $v^2 = 15^2 + 2(-10)(10)$
$v =$ | $v^2 = 225 - 200$
$a = -10$ | $v^2 = 25$
$t =$ | $v = 5\,\mathrm{ms^{-1}}$

At the same instant that P is projected, a second particle Q is dropped from a height of $18\,\mathrm{m}$ above the ground in the same vertical line as P.

(b) Find the height above the ground at which the two particles collide. [3]

P: ↑+ $s =$ | $S_P = ut + \frac{1}{2}at^2$
$u = 15$ | $= 15t + \frac{1}{2}(-10) \times t^2$
$v =$ | $S_P = 15t - 5t^2$
$a = -10$
$t =$

Q: ↓+ $s =$ | $S_Q = ut + \frac{1}{2}at^2$
$u = 0$ | $= 0 + \frac{1}{2}(10) \times t^2$
$v =$ | $S_Q = 5t^2$
$a = 10$
$t =$

$S_P + S_Q = 18$:

$15t - 5t^2 + 5t^2 = 18$
$15t = 18$
$t = 18/15$
$= 1.2\,\mathrm{s}$

Sub into S_P :
$S_P = 15(1.2) - 5(1.2)^2$
$= 10.8\,\mathrm{m}$

3 A particle moves in a straight line starting from rest from a point O. The acceleration of the particle at time t s after leaving O is a m s^{-2}, where $a = 4t^{\frac{1}{2}}$.

(a) Find the speed of the particle when $t = 9$. [2]

$V = \int 4t^{\frac{1}{2}}\,dt$

$V = \frac{2}{3} \times 4t^{3/2} + C$

$V = \frac{8}{3}t^{3/2} + C$

$V = 0$ at $t = 0$:

$0 = 0 + C$

$C = 0$

$\rightarrow V = \frac{8}{3}t^{3/2}$

sub. $t = 9$:

$V = \frac{8}{3} \times 9^{3/2}$

$V = \frac{8}{3} \times 27$

$V = 72\text{ms}^{-1}$

(b) Find the time after leaving O at which the speed (in metres per second) and the distance travelled (in metres) are numerically equal. [3]

$S = \int \frac{8}{3}t^{3/2}\,dt$

$S = \frac{2}{5} \times \frac{8}{3}t^{5/2} + C$

$S = \frac{16}{15}t^{5/2} + C$

$S = 0$ at $t = 0$:

$0 = 0 + C$

$C = 0$

$\rightarrow S = \frac{16}{15}t^{5/2}$

$S = V$:

$\frac{16}{15}t^{5/2} = \frac{8}{3}t^{3/2}$

$48t^{5/2} = 120t^{3/2}$

$2t^{5/2} = 5t^{3/2}$

$2t^{5/2} - 5t^{3/2} = 0$

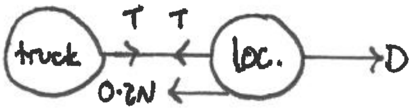
$t^{3/2}(2t - 5) = 0$

$t^{3/2} = 0$ or $2t - 5 = 0$

$t = 0$ ✗ $\quad 2t = 5$

$t = 2.5\text{s}$ ✓

4 A toy railway locomotive of mass 0.8 kg is towing a truck of mass 0.4 kg on a straight horizontal track at a constant speed of $2\,\mathrm{m\,s^{-1}}$. There is a constant resistance force of magnitude 0.2 N on the locomotive, but no resistance force on the truck. There is a light rigid horizontal coupling connecting the locomotive and the truck.



(a) State the tension in the coupling. [1]

Truck: $R(\rightarrow)$: $F = ma \leftarrow a = 0$ (constant speed)

$T = 0\,N$

(b) Find the power produced by the locomotive's engine. [1]

Locomotive: $R(\rightarrow)$: $F = ma$

$D - T - 0.2 = 0$

$D - 0 - 0.2 = 0$

$D = 0.2\,N$

Power $= D \times V$

$= 0.2 \times 2$

$= 0.4\,W$

The power produced by the locomotive's engine is now changed to 1.2 W.

(c) Find the magnitude of the tension in the coupling at the instant that the locomotive begins to accelerate. [5]

Power $= D \times V$

$1.2 = D \times 2$

$D = 0.6\,N$

Locomotive:

$R(\rightarrow)$: $F = ma$

$0.6 - T - 0.2 = 0.8a$

$0.4 - T = 0.8a$ ①

Truck:

$R(\rightarrow)$: $F = ma$

$T = 0.4a$ ②

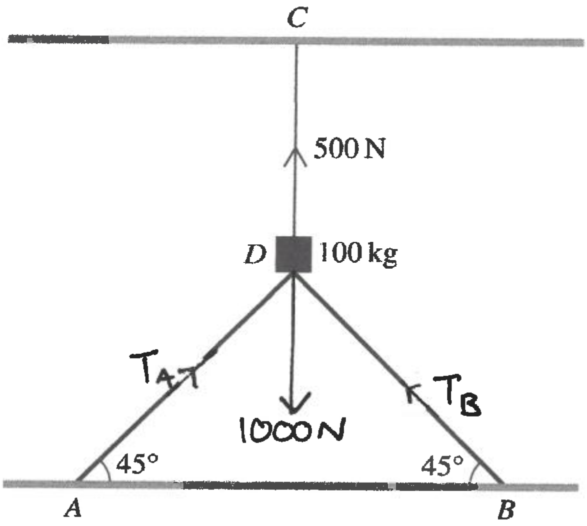
① + ②: $0.4 = 1.2a$

$a = \frac{1}{3}$

②: $T = 0.4 \times \frac{1}{3}$

$= 0.133\,N$

5



The diagram shows a block D of mass 100 kg supported by two sloping struts AD and BD, each attached at an angle of 45° to fixed points A and B respectively on a horizontal floor. The block is also held in place by a vertical rope CD attached to a fixed point C on a horizontal ceiling. The tension in the rope CD is 500 N and the block rests in equilibrium.

(a) Find the magnitude of the force in each of the struts AD and BD. [3]

$R(\rightarrow)$: $T_B \cos 45 = T_A \cos 45$

$T_B = T_A \; (= T)$

$R(\uparrow)$: $500 - 1000 + T\sin 45 + T \sin 45 = 0$

$-500 + \frac{\sqrt{2}}{2}T + \frac{\sqrt{2}}{2}T = 0$

$-500 + \sqrt{2}T = 0$

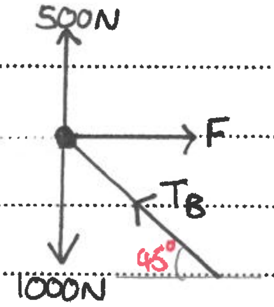
$\sqrt{2}T = 500$

$T = \frac{500}{\sqrt{2}}$

$T = 354\,N$

A horizontal force of magnitude F N is applied to the block in a direction parallel to AB.

(b) Find the value of F for which the magnitude of the force in the strut AD is zero. [3]



$R(\uparrow)$: $500 + T_B \sin 45 - 1000 = 0$

$T_B \sin 45 - 500 = 0$

$T_B \sin 45 = 500$

$T_B = \underline{707.1\text{N}}$ STO

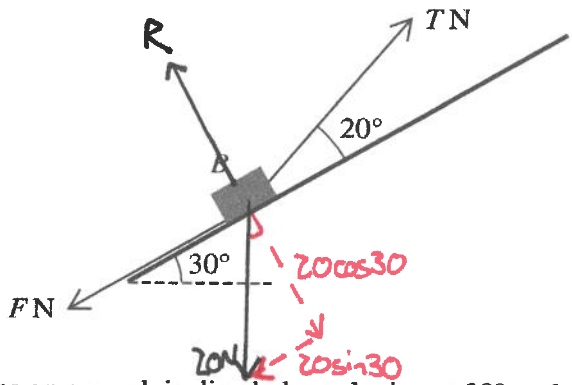
$R(\rightarrow)$: $F - T_B \cos 45 = 0$

$F = T_B \cos 45$

$= 707.1 \times \cos 45$

$= \underline{\underline{500\text{ N}}}$

6



A block B, of mass 2 kg, lies on a rough inclined plane sloping at 30° to the horizontal. A light rope, inclined at an angle of 20° above a line of greatest slope, is attached to B. The tension in the rope is T N. There is a friction force of F N acting on B (see diagram). The coefficient of friction between B and the plane is μ.

(a) It is given that $F = 5$ and that the acceleration of B up the plane is $1.2\,\mathrm{m\,s^{-2}}$.

(i) Find the value of T. [3]

$R(\nearrow)$: $T\cos 20 - 20\sin 30 - F = ma$

$T\cos 20 - 10 - 5 = 2 \times 1.2$

$T\cos 20 - 15 = 2.4$

$T\cos 20 = 17.4$

$T = \dfrac{17.4}{\cos 20}$

$T = 18.5\,N$ STO

(ii) Find the value of μ. [3]

$R(\nwarrow)$: $R + T\sin 20 - 20\cos 30 = 0$

$R + 18.5\sin 20 - 20\cos 30 = 0$

$R = 20\cos 30 - 18.5\sin 20$

$R = 10.987\,N$ STO

Block is in motion so $F = \mu R$:

$5 = \mu \times 10.987$

$\mu = 0.455$

(b) It is given instead that $\mu = 0.8$ and $T = 15$.

Determine whether B will move up the plane. [3]

$R(\nwarrow)$: $R + 15\sin 20 - 20\cos 30 = 0$

$R = 20\cos 30 - 15\sin 20$

$\underline{R = 12.19\ N}$ STO

Max possible value of friction, $F = \mu R$:

$F = 0.8 \times 12.19$

$= \underline{9.75\ N}$ STO

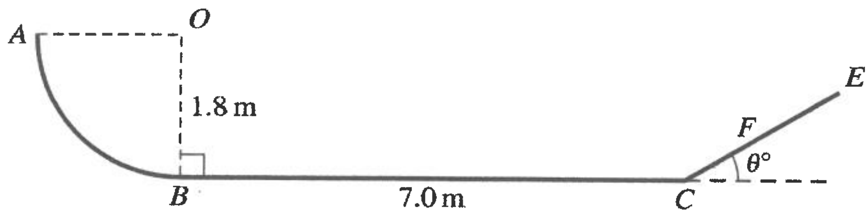
Resolve up the plane (without friction)

$R(\nearrow)$: $15\cos 20 - 20\sin 30$

$= 4.095\ N$

→ Resultant force up the plane (without friction) is 4.095 N, but the maximum possible value of friction is greater than this (9.75 N) so the block won't move.

7



The diagram shows a smooth track which lies in a vertical plane. The section AB is a quarter circle of radius 1.8 m with centre O. The section BC is a horizontal straight line of length 7.0 m and OB is perpendicular to BC. The section CFE is a straight line inclined at an angle of $\theta°$ above the horizontal.

A particle P of mass 0.5 kg is released from rest at A. Particle P collides with a particle Q of mass 0.1 kg which is at rest at B. Immediately after the collision, the speed of P is $4\,\text{m s}^{-1}$ in the direction BC. You should assume that P is moving horizontally when it collides with Q.

(a) Show that the speed of Q immediately after the collision is $10\,\text{m s}^{-1}$. [4]

P: $Work_W + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{out}$

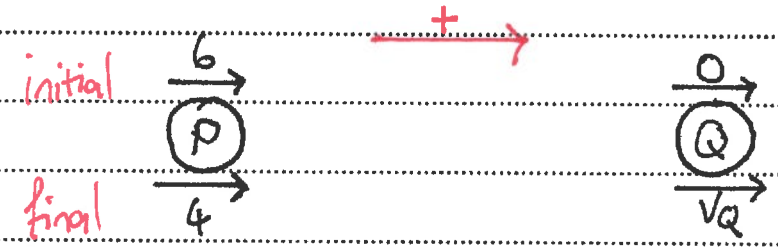
$0 + 0 + 0 = \frac{1}{2}\times 0.5\times V^2 + 0.5\times 10\times -1.8 + 0$

$0 = 0.25V^2 - 9$

$0.25V^2 = 9$

$V^2 = 36$

$V = 6\,\text{ms}^{-1}$



$m_P u_P + m_Q u_Q = m_P V_P + m_Q V_Q$

$0.5\times 6 + 0 = 0.5\times 4 + 0.1\times V_Q$

$3 = 2 + 0.1V_Q$

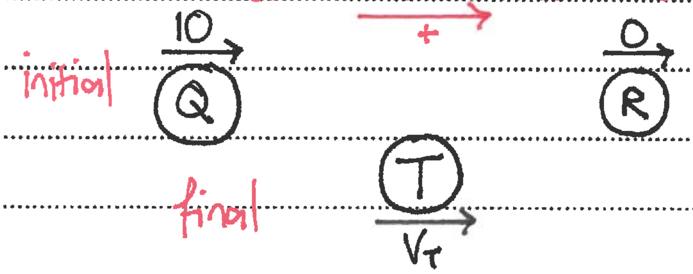
$1 = 0.1V_Q$

$V_Q = 10\,\text{ms}^{-1}$ QED

When Q reaches C, it collides with a particle R of mass 0.4 kg which is at rest at C. The two particles coalesce. The combined particle comes instantaneously to rest at F. You should assume that there is no instantaneous change in speed as the combined particle leaves C, nor when it passes through C again as it returns down the slope.

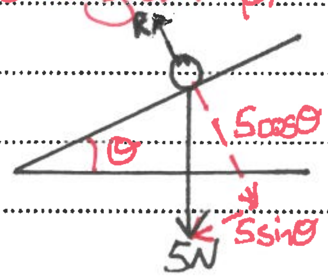
(b) Given that the distance CF is 0.4 m, find the value of θ. **[4]**

B→C: Track is smooth and no other horizontal forces, so acceleration = 0



$$M_Q U_Q + M_R U_R = M_T V_T$$
$$0.1 \times 10 + 0 = 0.5 V_T$$
$$1 = 0.5 V_T$$
$$V_T = 2\,\text{ms}^{-1}$$

Force diagram for T on incline:



$R(\nearrow)$: $F = ma$
$$-5\sin\theta = 0.5a$$
$$a = -10\sin\theta$$

Suvat on incline for T:

$s = 0.4$	$v^2 = u^2 + 2as$
$u = 2$	$0^2 = 2^2 + 2 \times -10\sin\theta \times 0.4$
$v = 0$	$0 = 4 - 8\sin\theta$
$a = -10\sin\theta$	$8\sin\theta = 4$
$t =$	$\sin\theta = \frac{1}{2}$

$$\theta = 30^\circ$$

[Question 7 continues on the next page.]

(c) Find the distance from B at which P collides with the combined particle. [5]

Time taken for Q to go from B to C:

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$
$$= \frac{7}{10}$$
$$= 0.7\,\text{s}$$

Time taken for coalesced particle (T) to go from $C \to F \to C$:

$(a = -10\sin 30 = -5\,\text{ms}^{-2})$

$+\nearrow$

$s = 0$	$s = ut + \frac{1}{2}at^2$
$u = 2$	$0 = 2t + \frac{1}{2} \times -5 \times t^2$
$v =$	$0 = 2t - 2.5t^2$
$a = -5$	$2.5t^2 - 2t = 0$
$t =$	$5t^2 - 4t = 0$

$$t(5t - 4) = 0$$
$$t = 0^{\times}, \quad 5t - 4 = 0$$
$$5t = 4$$
$$t = 0.8\,\text{s}$$

So after 1.5s, T is at C and travelling towards B with $v = 2\,\text{ms}^{-1}$.

How far along BC is P after 1.5s?

$$\text{distance} = \text{speed} \times \text{time}$$
$$= 4 \times 1.5$$
$$= 6\,\text{m}$$

→ They are 1m apart.

P: distance = speed × time

$$d_P = 4t$$

T: distance = speed × time

$$d_T = 2t$$

$$4t + 2t = 1$$
$$6t = 1$$
$$t = \frac{1}{6}$$

sub into d_P: $d_P = 4 \times \frac{1}{6}$

$$= \frac{2}{3}\,\text{m}$$

distance from B $= 6 + \frac{2}{3}$

$$= 6\frac{2}{3}\,\text{m}$$