1 A crane is used to raise a block of mass 600 kg vertically upwards at a constant speed through a height of 15 m. There is a resistance to the motion of the block, which the crane does 10 000 J of work to overcome.

(a) Find the total work done by the crane. [2]

$$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{out}$$

$$W + \tfrac{1}{2}(600)v^2 + 0 = \tfrac{1}{2}(600)v^2 + 600 \times 10 \times 15 + 10\,000$$

$$W + \cancel{300v^2} = \cancel{300v^2} + 90\,000 + 10\,000$$

$$W = \underline{100\,000\text{ J}}$$

(b) Given that the average power exerted by the crane is 12.5 kW, find the total time for which the block is in motion. [2]

$$\text{Power} = \frac{\text{Work done}}{\text{time}}$$

$$12\,500 = \frac{100\,000}{t}$$

$$12\,500t = 100\,000$$

$$t = \underline{8\text{ s}}$$

2 A particle P is projected vertically upwards from horizontal ground with speed $u\,\mathrm{m\,s^{-1}}$. P reaches a maximum height of 20 m above the ground.

(a) Find the value of u. [2]

$\uparrow +$ $s = 20$ | $v^2 = u^2 + 2as$

$u =$ | $0^2 = u^2 + 2(-10)(20)$

$v = 0$ | $0 = u^2 - 400$

$a = -10$ | $u^2 = 400$

$t =$ | $u = \underline{\underline{20\ \mathrm{ms^{-1}}}}$

(b) Find the total time for which P is at least 15 m above the ground. [3]

$\uparrow +$ $s = 15$ | $s = ut + \frac{1}{2}at^2$

$u = 20$ | $15 = 20t + \frac{1}{2}(-10) \times t^2$

$v =$ | $15 = 20t - 5t^2$

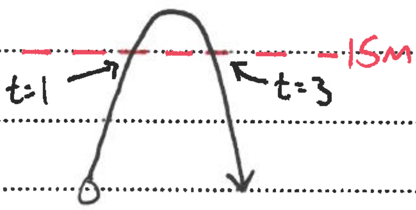
$a = -10$ | $5t^2 - 20t + 15 = 0$

$t =$ | $t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 3$ or $t = 1$

so P is 15m above ground at $t=1$ and $t=3$



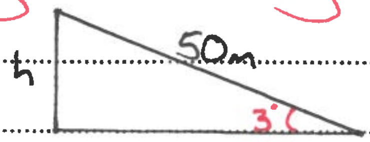
so it is above 15m for:

$3 - 1 = \underline{\underline{2\ \text{seconds}}}$

3 A car of mass m kg is towing a trailer of mass 300 kg down a straight hill inclined at 3° to the horizontal at a constant speed. There are resistance forces on the car and on the trailer, and the total work done against the resistance forces in a distance of 50 m is 40 000 J. The engine of the car is doing no work and the tow-bar is light and rigid.

(a) Find the value of m. [3]

Change in height when travelling 50m:



$\sin 3 = \frac{h}{50}$

$h = 50\sin 3$

$Work_{in} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{out}$

$0 + \frac{1}{2}(m+300)v^2 + 0 = \frac{1}{2}(m+300)v^2 + (m+300)\times 10 \times -50\sin 3 + 40\,000$

$\cancel{\frac{1}{2}(m+300)v^2} = \cancel{\frac{1}{2}(m+300)v^2} + (m+300)\times -500\sin 3 + 40\,000$

$(m+300)\times 500\sin 3 = 40\,000$

$m + 300 = 1528.59$

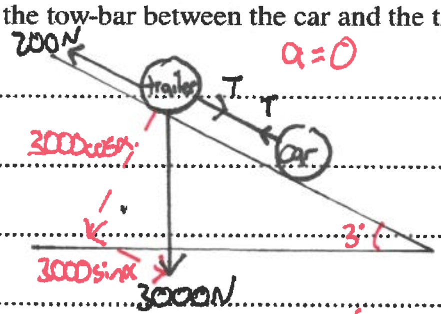
$m = 1228.59$

$= 1230\text{ kg}$

The resistance force on the trailer is 200 N.

(b) Find the tension in the tow-bar between the car and the trailer. [2]

Trailer:



$a = 0$

$R(\searrow)$: $F = ma$ ← $a = 0$ (constant speed)

$T + 3000\sin 3 - 200 = 0$

$T + 157.0 - 200 = 0$

$T - 43.0 = 0$

$T = 43.0\text{ N}$

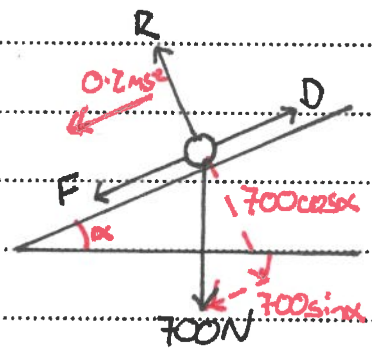
4 The total mass of a cyclist and her bicycle is 70 kg. The cyclist is riding with constant power of 180 W up a straight hill inclined at an angle α to the horizontal, where $\sin\alpha = 0.05$. At an instant when the cyclist's speed is $6\,\text{m s}^{-1}$, her acceleration is $-0.2\,\text{m s}^{-2}$. There is a constant resistance to motion of magnitude F N.

(a) Find the value of F. [4]

Driving Force of cyclist: Power = Dv

$180 = D \times 6$

$D = 30\text{N}$



$R(\nearrow)$: $30 - F - 700\sin\alpha = ma$

$30 - F - 700 \times 0{\cdot}05 = 70 \times -0{\cdot}2$

$30 - F - 35 = -14$

$-F - 5 = -14$

$-F = -9$

$\underline{\underline{F = 9\text{N}}}$

(b) Find the steady speed that the cyclist could maintain up the hill when working at this power. [2]

At steady speed, acceleration $= 0$:

$R(\nearrow)$: $D - F - 700\sin\alpha = ma$

$D - 9 - 35 = 0$

$D - 44 = 0$

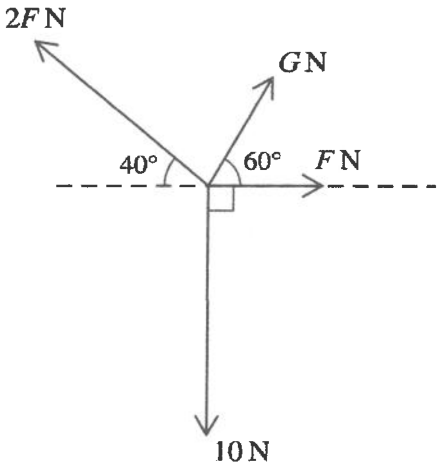
$D = 44\text{ N}$

Power $= Dv$

$180 = 44v$

$v = 4.09\text{ ms}^{-1}$

5



Four coplanar forces act at a point. The magnitudes of the forces are 10 N, F N, G N and $2F$ N. The directions of the forces are as shown in the diagram.

(a) Given that the forces are in equilibrium, find the values of F and G. [5]

$R(\uparrow)$: $G\sin 60 + 2F\sin 40 - 10 = 0$

$G\sin 60 + 2F\sin 40 = 10$

$G\sin 60 = 10 - 2F\sin 40$

$G = \dfrac{10 - 2F\sin 40}{\sin 60}$ ①

$R(\rightarrow)$: $F + G\cos 60 - 2F\cos 40 = 0$

$G\cos 60 = 2F\cos 40 - F$ ②

sub. ① into ②:

$\left(\dfrac{10 - 2F\sin 40}{\sin 60}\right)\cos 60 = 2F\cos 40 - F$

$\dfrac{10\cos 60}{\sin 60} - \dfrac{2F\sin 40\cos 60}{\sin 60} = 2F\cos 40 - F$

$\dfrac{10\sqrt{3}}{3} = \dfrac{2F\sin 40\cos 60}{\sin 60} + 2F\cos 40 - F$

$\dfrac{10\sqrt{3}}{3} = F\left(\dfrac{2\sin 40\cos 60}{\sin 60} + 2\cos 40 - 1\right)$

CONT →

$$F = \frac{10\sqrt{3}}{3} \div \left(\frac{2\sin 40 \cos 60}{\sin 60} + 2\cos 40 - 1\right)$$

$F = \underline{4.53\text{N}}$ STO

Sub into ①: $G = \dfrac{10 - 2(4.53)\sin 40}{\sin 60}$

$G = \underline{4.82\text{N}}$

(b) Given instead that $F = 3$, find the value of G for which the resultant of the forces is perpendicular to the 10 N force. [2]

If resultant is perpendicular to 10N force, then resultant in vertical direction = 0 :

$R(\uparrow)$: $G\sin 60 + 2F\sin 40 - 10 = 0$

$F = 3$

$G\sin 60 + 6\sin 40 - 10 = 0$

$G\sin 60 = 10 - 6\sin 40$

$G = \dfrac{10 - 6\sin 40}{\sin 60}$

$G = \underline{7.09\text{N}}$

6 A cyclist starts from rest at a fixed point O and moves in a straight line, before coming to rest k seconds later. The acceleration of the cyclist at time t s after leaving O is $a\,\mathrm{m\,s^{-2}}$, where $a = 2t^{-\frac{1}{2}} - \frac{3}{5}t^{\frac{1}{2}}$ for $0 < t \leqslant k$.

(a) Find the value of k. [4]

$V=0$ when $t=k$

$V = \int\left(2t^{-\frac{1}{2}} - \frac{3}{5}t^{\frac{1}{2}}\right)dt$

$V = 2\times 2t^{\frac{1}{2}} - \frac{2}{3}\times\frac{3}{5}t^{\frac{3}{2}} + C$

$V = 4t^{\frac{1}{2}} - \frac{2}{5}t^{\frac{3}{2}} + C$

$V=0$ at $t=0$:

$0 = 0 - 0 + C$

$C = 0$

$\rightarrow V = 4t^{\frac{1}{2}} - \frac{2}{5}t^{\frac{3}{2}}$

$V=0$:

$0 = 4t^{\frac{1}{2}} - \frac{2}{5}t^{\frac{3}{2}}$

$0 = t^{\frac{1}{2}}\left(4 - \frac{2}{5}t\right)$

$t^{\frac{1}{2}} = 0$ or $4 - \frac{2}{5}t = 0$

$t = 0$ $\quad\frac{2}{5}t = 4$

$2t = 20$

$t = 10$

so $k = 10$

(b) Find the maximum speed of the cyclist. [3]

max. speed is when $a = \frac{dv}{dt} = 0$:

$2t^{-\frac{1}{2}} - \frac{3}{5}t^{\frac{1}{2}} = 0$

$\frac{2}{\sqrt{t}} - \frac{3}{5}\sqrt{t} = 0$

$\frac{2}{\sqrt{t}} = \frac{3}{5}\sqrt{t}$

$2 = \frac{3}{5}t$

$3t = 10$

$t = \frac{10}{3}$

Sub into V:

$V = 4\left(\frac{10}{3}\right)^{\frac{1}{2}} - \frac{2}{5}\left(\frac{10}{3}\right)^{\frac{3}{2}}$

$V = 4.87\,\mathrm{ms^{-1}}$

(c) Find an expression for the displacement from O in terms of t. Hence find the total distance travelled by the cyclist from the time at which she reaches her maximum speed until she comes to rest. [4]

$S = \int \left(4t^{\frac{1}{2}} - \frac{2}{5}t^{\frac{3}{2}}\right) dt$

$S = \frac{2}{3} \times 4t^{\frac{3}{2}} - \frac{2}{5} \times \frac{2}{5}t^{\frac{5}{2}} + C$

$S = \frac{8}{3}t^{\frac{3}{2}} - \frac{4}{25}t^{\frac{5}{2}} + C$

$S = 0$ at $t = 0$, so:

$0 = 0 - 0 + C$

$C = 0$

$\rightarrow S = \frac{8}{3}t^{\frac{3}{2}} - \frac{4}{25}t^{\frac{5}{2}}$

displacement at maximum speed: $t = \frac{10}{3}$

$S = \frac{8}{3}\left(\frac{10}{3}\right)^{\frac{3}{2}} - \frac{4}{25}\left(\frac{10}{3}\right)^{\frac{5}{2}}$

$S = 12.983\,m$

displacement when she comes to rest: $t = 10$

$S = \frac{8}{3}(10)^{\frac{3}{2}} - \frac{4}{25}(10)^{\frac{5}{2}}$

$S = 33.731\,m$

distance travelled $= 33.731 - 12.983$

$= 20.7\,m$

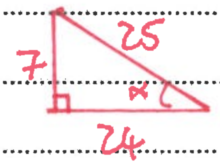
7 A bead, A, of mass 0.1 kg is threaded on a long straight rigid wire which is inclined at $\sin^{-1}(\frac{7}{25})$ to the horizontal. A is released from rest and moves down the wire. The coefficient of friction between A and the wire is μ. When A has travelled 0.45 m down the wire, its speed is $0.6\,\mathrm{m\,s^{-1}}$.

(a) Show that $\mu = 0.25$. [6]

Find acceleration:

+	
$s = 0.45$	$v^2 = u^2 + 2as$
$u = 0$	$0.6^2 = 0^2 + 2 \times a \times 0.45$
$v = 0.6$	$0.36 = 0.9a$
$a =$	$\underline{a = 0.4\,\mathrm{ms^{-2}}}$
$t =$	

$\alpha = \sin^{-1}\left(\frac{7}{25}\right) \rightarrow$



$\cos\alpha = \frac{24}{25}$

$\tan\alpha = \frac{7}{24}$

Force diagram:

$R(\nearrow)$: $R - 1\cos\alpha = 0$

$R - 1 \times \frac{24}{25} = 0$

$\underline{R = 0.96\,\mathrm{N}}$

$R(\searrow)$: $1\sin\alpha - F = ma$

$1 \times \frac{7}{25} - F = 0.1 \times 0.4$

$0.28 - F = 0.04$

$-F = -0.24$

$F = 0.24$

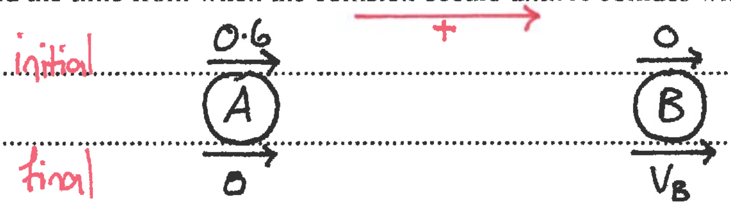
$\mu R = 0.24$

$\mu \times 0.96 = 0.24$

$\underline{\mu = 0.25}$ QED

Another bead, B, of mass 0.5 kg is also threaded on the wire. At the point where A has travelled 0.45 m down the wire, it hits B which is instantaneously at rest on the wire. A is brought to instantaneous rest in the collision. The coefficient of friction between B and the wire is 0.275.

(b) Find the time from when the collision occurs until A collides with B again. [6]



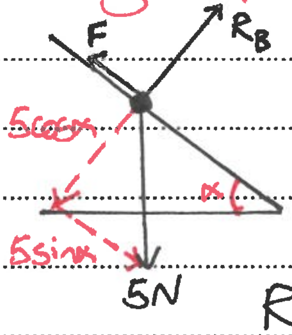
$m_A u_A + m_B u_B = m_A v_A + m_B v_B$

$0.1 \times 0.6 + 0 = 0 + 0.5 V_B$

$0.06 = 0.5 V_B$

$\underline{V_B = 0.12\,ms^{-1}}$

Force diagram for B:



$R(\nearrow): R_B - 5\cos\alpha = 0$

$R_B - 5 \times \frac{24}{25} = 0$

$\underline{R_B = 4.8N}$

$R(\searrow): 5\sin\alpha - F = ma$

$5 \times \frac{7}{25} - \mu R = 0.5a$

$1.4 - 0.275 \times 4.8 = 0.5a$

$1.4 - 1.32 = 0.5a$

$0.08 = 0.5a$

$\underline{a = 0.16\,ms^{-2}}$

Suvat for B:

$\searrow +$ $s = s_B$	$s_B = ut + \frac{1}{2}at^2$
$u = 0.12$	$= 0.12t + \frac{1}{2} \times 0.16t^2$
$v =$	$s_B = 0.12t + 0.08t^2$
$a = 0.16$	
$t =$	

continued...

Suvat for A:

$S = S_A$ | $S_A = ut + \frac{1}{2}at^2$

$u = 0$ | $= 0 + \frac{1}{2} \times 0.4 \times t^2$

$v =$ | $S_A = 0.2t^2$

$a = 0.4$

$t =$

When they collide, $S_A = S_B$:

$$0.2t^2 = 0.12t + 0.08t^2$$

$$0.12t^2 = 0.12t$$

$$0.12t^2 - 0.12t = 0 \quad \div 0.12$$

$$t^2 - t = 0$$

$$t(t-1) = 0$$

$$t = 0 \times \quad \text{or} \quad \underline{t = 1s} \ \checkmark$$