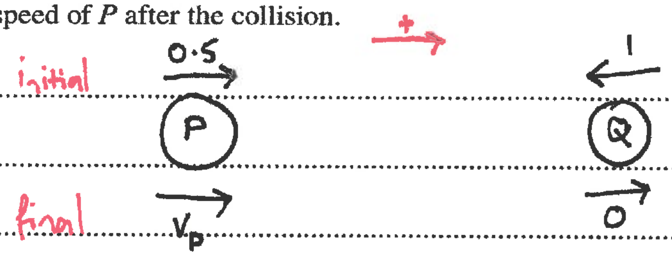
1 Two particles P and Q of masses 0.2 kg and 0.3 kg respectively are free to move in a horizontal straight line on a smooth horizontal plane. P is projected towards Q with speed $0.5\,\mathrm{m\,s^{-1}}$. At the same instant Q is projected towards P with speed $1\,\mathrm{m\,s^{-1}}$. Q comes to rest in the resulting collision.

Find the speed of P after the collision. [3]



$$M_P U_P + M_Q U_Q = M_P V_P + M_Q V_Q$$

$$0.2 \times 0.5 + 0.3 \times -1 = 0.2 V_P + 0$$

$$0.1 - 0.3 = 0.2 V_P$$

$$-0.2 = 0.2 V_P$$

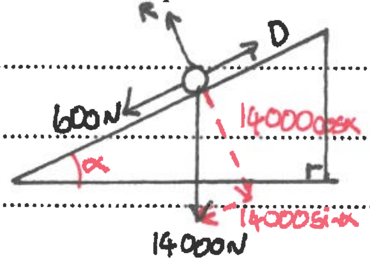
$$V_P = -1\,\mathrm{ms^{-1}}$$

$$\text{speed} = \underline{1\,\mathrm{ms^{-1}}}$$

2 A car of mass 1400 kg is travelling at constant speed up a straight hill inclined at α to the horizontal, where $\sin\alpha = 0.1$. There is a constant resistance force of magnitude 600 N. The power of the car's engine is 22 500 W.

Constant speed so $a = 0$

(a) Show that the speed of the car is $11.25\ \mathrm{m\,s^{-1}}$. [3]



$R(\nearrow): D - 600 - 14000\sin\alpha = ma$

$D - 600 - 14000 \times 0.1 = 0$

$D - 600 - 1400 = 0$

$D - 2000 = 0$

$D = 2000\text{N}$

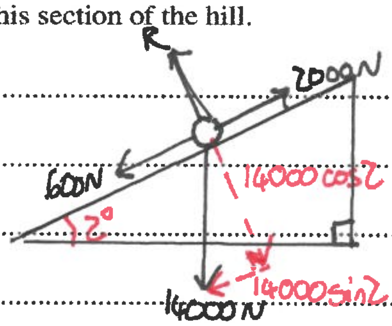
$\text{Power} = Dv$

$22500 = 2000v$

$v = 11.25\ \mathrm{ms^{-1}}$ QED

The car, moving with speed $11.25\ \mathrm{m\,s^{-1}}$, comes to a section of the hill which is inclined at 2° to the horizontal.

(b) Given that the power and resistance force do not change, find the initial acceleration of the car up this section of the hill. [3]

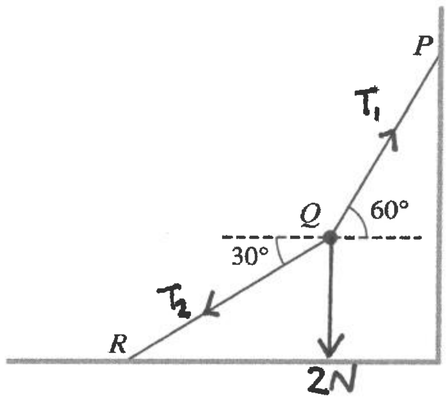


$R(\nearrow): 2000 - 600 - 14000\sin 2 = ma$

$911.407 = 1400a$

$a = 0.651\ \mathrm{ms^{-2}}$

3



A particle Q of mass 0.2 kg is held in equilibrium by two light inextensible strings PQ and QR. P is a fixed point on a vertical wall and R is a fixed point on a horizontal floor. The angles which strings PQ and QR make with the horizontal are 60° and 30° respectively (see diagram).

Find the tensions in the two strings. [5]

$R(\rightarrow)$: $T_1 \cos 60 - T_2 \cos 30 = 0$

$T_1 \cos 60 = T_2 \cos 30$

$T_1 = \dfrac{T_2 \cos 30}{\cos 60}$

$T_1 = \sqrt{3} T_2$ ①

$R(\uparrow)$: $T_1 \sin 60 - T_2 \sin 30 - 2 = 0$ ②

Sub. ① into ②:

$\sqrt{3} T_2 \sin 60 - T_2 \sin 30 - 2 = 0$

$\sqrt{3} T_2 \times \frac{\sqrt{3}}{2} - T_2 \times \frac{1}{2} - 2 = 0$

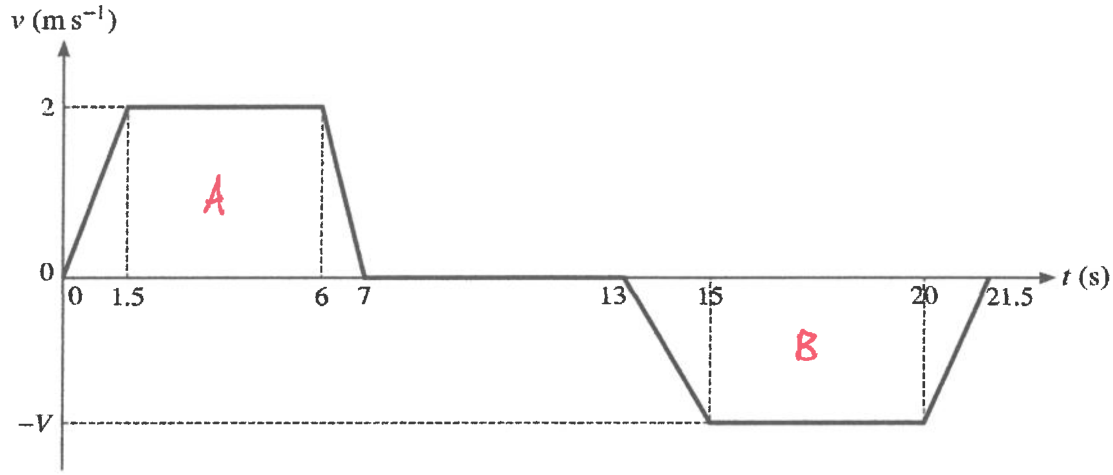
$\frac{3}{2} T_2 - \frac{1}{2} T_2 = 2$

$T_2 = 2\text{ N}$

Sub. into ①: $T_1 = \sqrt{3} \times 2$

$= 2\sqrt{3}\text{ N}$

4



An elevator moves vertically, supported by a cable. The diagram shows a velocity-time graph which models the motion of the elevator. The graph consists of 7 straight line segments.

The elevator accelerates upwards from rest to a speed of 2 m s^{-1} over a period of 1.5 s and then travels at this speed for 4.5 s, before decelerating to rest over a period of 1 s.

The elevator then remains at rest for 6 s, before accelerating to a speed of V m s^{-1} downwards over a period of 2 s. The elevator travels at this speed for a period of 5 s, before decelerating to rest over a period of 1.5 s.

(a) Find the acceleration of the elevator during the first 1.5 s. [1]

$s=$	$V = u + at$
$u = 0$	$2 = 0 + 1.5a$
$v = 2$	$a = \frac{2}{1.5}$
$a =$	$= \frac{4}{3}\ \text{ms}^{-2}$
$t = 1.5$	

(b) Given that the elevator starts and finishes its journey on the ground floor, find V. [2]

If elevator starts and finishes in same place, displacements A and B are equal: Area A = Area B

$\frac{1}{2}(4.5 + 7) \times 2 = \frac{1}{2}(5 + 8.5) \times V$

$\frac{1}{2}(11.5) \times 2 = \frac{1}{2}(13.5) \times V$

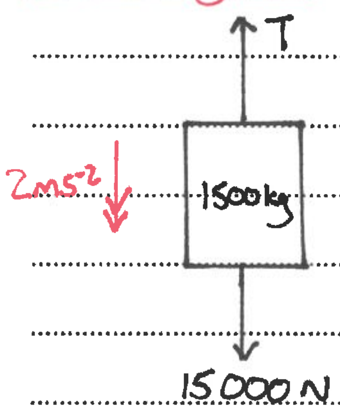
$11.5 = 6.75V$

$V = 1.70\ \text{ms}^{-1}$

(c) The combined weight of the elevator and passengers on its upward journey is 1500 kg. Assuming that there is no resistance to motion, find the tension in the elevator cable on its upward journey when the elevator is decelerating. [3]

$s =$	$v = u + at$
$u = 2$	$0 = 2 + a \times 1$
$v = 0$	$0 = 2 + a$
$a =$	$a = -2\ ms^{-2}$
$t = 1$	

Force diagram:



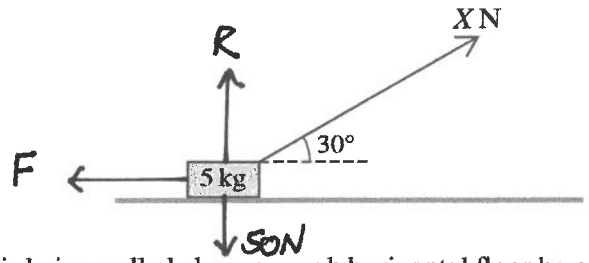
$R(\uparrow)$: $T - 15000 = ma$

$T - 15000 = 1500 \times -2$

$T - 15000 = -3000$

$T = 12000\ N$

5



A block of mass 5 kg is being pulled along a rough horizontal floor by a force of magnitude X N acting at 30° above the horizontal (see diagram). The block starts from rest and travels 2 m in the first 5 s of its motion.

(a) Find the acceleration of the block. [2]

$s = 2$, $u = 0$, $v =$, $a =$, $t = 5$

$s = ut + \frac{1}{2}at^2$

$2 = 0 + \frac{1}{2} \times a \times 5^2$

$2 = 12.5a$

$a = \underline{0.16 \text{ ms}^{-2}}$

(b) Given that the coefficient of friction between the block and the floor is 0.4, find X. [4]

$R(\uparrow)$: $X\sin 30 + R - 50 = 0$

$R = 50 - X\sin 30$

$R = 50 - \frac{1}{2}X$ ①

$R(\rightarrow)$: $X\cos 30 - F = ma$ (particle is in motion so $F = \mu R$)

$X\cos 30 - \mu R = 5 \times 0.16$

$\frac{\sqrt{3}}{2}X - 0.4R = 0.8$ ②

Sub. ①→②: $\frac{\sqrt{3}}{2}X - 0.4(50 - \frac{1}{2}X) = 0.8$

$\frac{\sqrt{3}}{2}X - 20 + 0.2X = 0.8$

$\frac{\sqrt{3}}{2}X + 0.2X = 20.8$

$X(\frac{\sqrt{3}}{2} + 0.2) = 20.8$

$X = 20.8 \div (\frac{\sqrt{3}}{2} + 0.2)$

$= \underline{19.5 \text{ N}}$

The block is now placed on a part of the floor where the coefficient of friction between the block and the floor has a different value. The value of X is changed to 25, and the block is now in limiting equilibrium.

(c) Find the value of the coefficient of friction between the block and this part of the floor. [3]

$R(\uparrow)$: $25\sin 30 + R - 50 = 0$

$R = 50 - 25\sin 30$

$= 37.5\,N$

$R(\rightarrow)$: $25\cos 30 - F = 0$

$25\cos 30 - \mu R = 0$

$25\cos 30 - 37.5\mu = 0$

$37.5\mu = 25\cos 30$

$\mu = \dfrac{25\cos 30}{37.5}$

$= 0.577$

(limiting equilibrium, so $F = \mu R$)

6 A particle moves in a straight line. It starts from rest from a fixed point O on the line. Its velocity at time t s after leaving O is v m s^{-1}, where $v = t^2 - 8t^{\frac{3}{2}} + 10t$.

(a) Find the displacement of the particle from O when $t = 1$. [4]

$s = \int (t^2 - 8t^{3/2} + 10t)\,dt$

$s = \frac{1}{3}t^3 - \frac{2}{5} \times 8t^{5/2} + 5t^2 + C$

$s = \frac{1}{3}t^3 - 3.2t^{5/2} + 5t^2 + C$

$s = 0$ when $t = 0$

$0 = 0 - 0 + 0 + C$

$C = 0$

$\rightarrow s = \frac{1}{3}t^3 - 3.2t^{5/2} + 5t^2$

sub. $t = 1$:

$s = \frac{1}{3} \times 1^3 - 3.2 \times 1^{5/2} + 5 \times 1^2$

$= \frac{1}{3} - 3.2 + 5$

$= 2.13\text{m}$

(b) Show that the minimum velocity of the particle is $-125\,\mathrm{m\,s^{-1}}$. [7]

at stationary point, $\frac{dv}{dt} = 0$:

$\frac{dv}{dt} = 2t - \frac{3}{2} \times 8t^{\frac{1}{2}} + 10$

$= 2t - 12t^{\frac{1}{2}} + 10$

SP: $\frac{dv}{dt} = 0$:

$2t - 12t^{\frac{1}{2}} + 10 = 0$

$t - 6t^{\frac{1}{2}} + 5 = 0$

Let $Y = t^{\frac{1}{2}}$:

$Y^2 - 6Y + 5 = 0$

$(Y-5)(Y-1) = 0$

$Y = 5$ or $Y = 1$

$t^{\frac{1}{2}} = 5 \qquad t^{\frac{1}{2}} = 1$

$t = 25 \qquad t = 1$

Which one is minimum point?

$\frac{d^2v}{dt^2} = 2 - 6t^{-\frac{1}{2}}$

Sub. $t = 25$:

$2 - 6(25)^{-\frac{1}{2}}$

$= 2 - 6 \times \frac{1}{5}$

$= 0.8$ >0 so minimum ✓

Sub. $t = 1$:

$2 - 6(1)^{-\frac{1}{2}}$

$= 2 - 6 \times 1$

$= -4$ <0 so maximum ✗

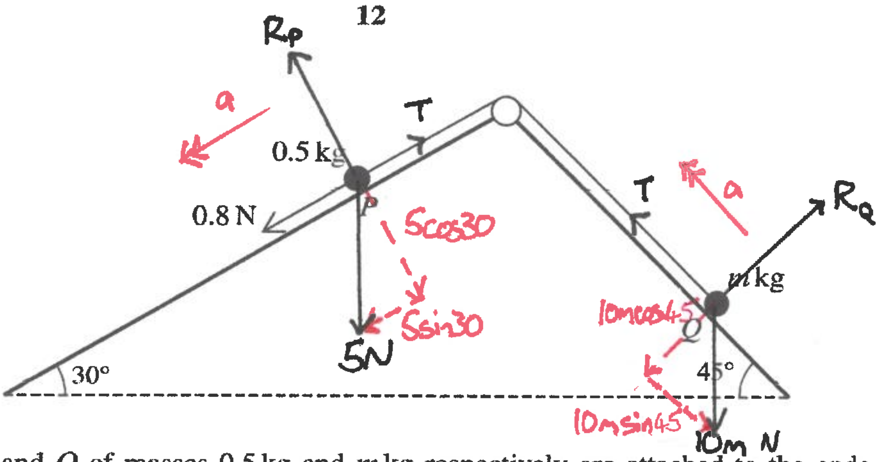
Sub. $t = 25$ into V:

$V = 25^2 - 8(25)^{\frac{3}{2}} + 10(25)$

$= 625 - 1000 + 250$

$= -125\ \mathrm{ms^{-1}}$ QED

7



Two particles P and Q of masses 0.5 kg and m kg respectively are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to the top of two inclined planes. The particles are initially at rest with P on a smooth plane inclined at 30° to the horizontal and Q on a plane inclined at 45° to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes. A force of magnitude 0.8 N is applied to P acting down the plane, causing P to move down the plane (see diagram).

(a) It is given that $m = 0.3$, and that the plane on which Q rests is smooth.

Find the tension in the string. [5]

P: R(↙): $0.8 + 5\sin 30 - T = ma$

$0.8 + 2.5 - T = 0.5a$

$3.3 - T = 0.5a$ ①

Q: R(↖): $T - 3\sin 45 = ma$

$T - \frac{3\sqrt{2}}{2} = 0.3a$ ②

① + ②: $3.3 - \frac{3\sqrt{2}}{2} = 0.8a$

$1.179 = 0.8a$

$\underline{a = 1.473\ ms^{-2}}$ STO

Sub. into ②: $T - \frac{3\sqrt{2}}{2} = 0.3 \times 1.473$

$T - \frac{3\sqrt{2}}{2} = 0.442$

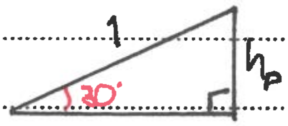
$T = \underline{2.56\ N}$

(b) It is given instead that the plane on which Q rests is rough, and that after each particle has moved a distance of 1 m, their speed is $0.6\,\mathrm{m\,s^{-1}}$. The work done against friction in this part of the motion is 0.5 J.

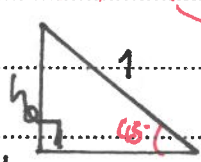
Use an energy method to find the value of m. **[5]**

If both particles have moved 1m along their planes:

P:



Q:



$h_P = 1\sin 30$ height moved down by P

$h_Q = 1\sin 45$ height moved up by Q

P: $\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$$0.8\times 1 + 0 + 0 = \tfrac{1}{2}\times 0.5\times 0.6^2 + 0.5\times 10\times -1\sin 30 + T\times 1$$

$$0.8 = 0.09 - 2.5 + T$$

$$0.8 = -2.41 + T$$

$$\underline{T = 3.21\text{ N}}$$

Q: $\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$$T\times 1 + 0 + 0 = \tfrac{1}{2}\times m\times 0.6^2 + m\times 10\times 1\sin 45 + 0.5$$

$$T = 0.18m + 5\sqrt{2}\,m + 0.5$$

T=3.21: $3.21 = (0.18 + 5\sqrt{2})m + 0.5$

$$2.71 = (0.18 + 5\sqrt{2})m$$

$$m = \frac{2.71}{0.18 + 5\sqrt{2}}$$

$$m = \underline{0.374\text{ kg}}$$