2 A particle P of mass 0.4 kg is projected vertically upwards from horizontal ground with speed $10\,\text{m}\,\text{s}^{-1}$.

(a) Find the greatest height above the ground reached by P. [2]

$+\uparrow$ $s =$ | $v^2 = u^2 + 2as$

$u = 10$ | $0^2 = 10^2 + 2 \times -10 \times s$

$v = 0$ | $0 = 100 - 20s$

$a = -10$ | $20s = 100$

$t =$ | $\underline{s = 5\text{m}}$

When P reaches the ground again, it bounces vertically upwards. At the first instant that it hits the ground, P loses 7.2 J of energy.

(b) Find the time between the first and second instants at which P hits the ground. [4]

When P reaches the ground it will have the same speed as when it was projected: $V = 10\text{ms}^{-1}$

KE before it hits ground $= \frac{1}{2} \times 0.4 \times 10^2$

$= \underline{20\text{J}}$

KE after $= 20 - 7.2$

$= \underline{12.8\text{J}}$

$\frac{1}{2} \times 0.4 \times v^2 = 12.8$

$0.2v^2 = 12.8$

$v^2 = 64$

$\underline{v = 8\text{ms}^{-1}}$

$+\uparrow$ $s = 0$ | $s = ut + \frac{1}{2}at^2$

$u = 8$ | $0 = 8t - 5t^2$

$v =$ | $0 = t(8 - 5t)$

$a = -10$ | $t = 0$ or $8 - 5t = 0$

$t =$ | $5t = 8$

$\underline{t = 1.6\text{s}}$

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1 A particle of mass 1.6 kg is dropped from a height of 9 m above horizontal ground. The speed of the particle at the instant before hitting the ground is $12\,\mathrm{m\,s^{-1}}$.

Find the work done against air resistance. [3]

$\text{Work}_{\text{IN}} + KE_{\text{init}} + PE_{\text{init}} = KE_{\text{fin}} + PE_{\text{fin}} + \text{Work}_{\text{OUT}}$

$0 + 0 + 0 = \frac{1}{2} \times 1{\cdot}6 \times 12^2 + 1{\cdot}6 \times 10 \times -9 + W$

$0 = 115{\cdot}2 - 144 + W$

$0 = -28{\cdot}8 + W$

$W = 28{\cdot}8\text{J}$

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1 A cyclist is riding a bicycle along a straight horizontal road AB of length 50 m. The cyclist starts from rest at A and reaches a speed of $6\,\mathrm{m\,s^{-1}}$ at B. The cyclist produces a constant driving force of magnitude 100 N. There is a resistance force, and the work done against the resistance force from A to B is 3560 J.

Find the total mass of the cyclist and bicycle. [3]

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$100 \times 50 + 0 + 0 = \frac{1}{2} \times m \times 6^2 + 0 + 3560$

Work = F×d

$5000 = 18m + 3560$

$18m = 1440$

$m = \underline{80\,\text{kg}}$

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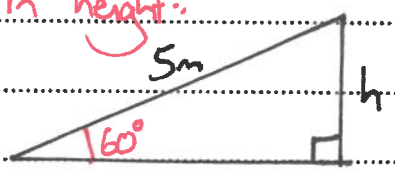
1 A winch operates by means of a force applied by a rope. The winch is used to pull a load of mass 50 kg up a line of greatest slope of a plane inclined at 60° to the horizontal. The winch pulls the load a distance of 5 m up the plane at constant speed. There is a constant resistance to motion of 100 N.

Find the work done by the winch. [3]

Change in height:



$\sin 60 = \frac{h}{5}$

$\underline{h = 5\sin 60}$

$Work_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{OUT}$

$W + \frac{1}{2}\times 50\times v^2 + 0 = \frac{1}{2}\times 50\times v^2 + 50\times 10\times 5\sin 60 + 100\times 5$

$\uparrow W = F\times d$

$W + \cancel{25}v^2 = \cancel{25}v^2 + 2500\sin 60 + 500$

$W = 2500\sin 60 + 500$

$= \underline{\underline{2670\,J}}$

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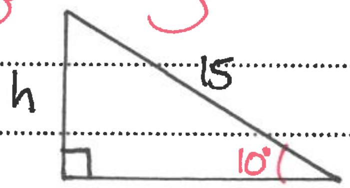
1 A particle of mass 0.6 kg is projected with a speed of $4\,\text{m}\,\text{s}^{-1}$ down a line of greatest slope of a smooth plane inclined at 10° to the horizontal.

Use an energy method to find the speed of the particle after it has moved 15 m down the plane. [3]

Change in height:



$\sin 10 = \frac{h}{15}$

$\underline{h = 15\sin 10}$

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$0 + \frac{1}{2}\times 0.6\times 4^2 + 0 = \frac{1}{2}\times 0.6\times v^2 + 0.6\times 10\times -15\sin 10 + 0$

$4.8 = 0.3v^2 - 90\sin 10$

$4.8 + 90\sin 10 = 0.3v^2$

$20.428.. = 0.3v^2$

$v^2 = 68.09..$

$v = \underline{8.25\,\text{ms}^{-1}}$

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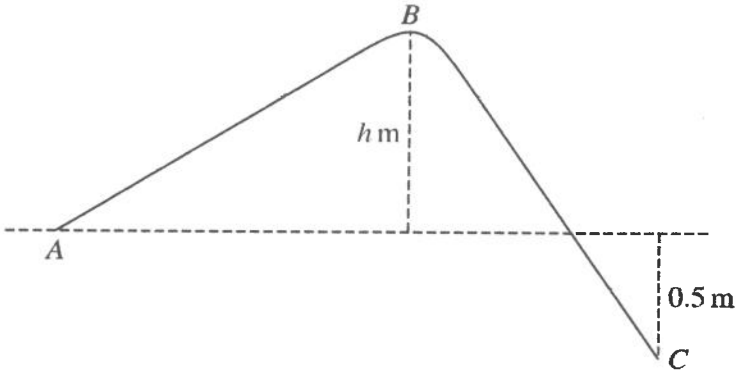
3



The diagram shows the vertical cross-section of a surface. A, B and C are three points on the cross-section. The level of B is h m above the level of A. The level of C is 0.5 m below the level of A. A particle of mass 0.2 kg is projected up the slope from A with initial speed 5 m s^{-1}. The particle remains in contact with the surface as it travels from A to C.

(a) Given that the particle reaches B with a speed of 3 m s^{-1} and that there is no resistance force, find h. [3]

$$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$$

$$0 + \tfrac{1}{2}\times 0.2\times 5^2 + 0 = \tfrac{1}{2}\times 0.2\times 3^2 + 0.2\times 10\times h + 0$$

$$2.5 = 0.9 + 2h$$

$$2h = 1.6$$

$$h = \underline{\underline{0.8\text{m}}}$$

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(b) It is given instead that there is a resistance force and that the particle does 3.1 J of work against the resistance force as it travels from A to C.

Find the speed of the particle when it reaches C. [3]

$\text{Work}_{in} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{out}$

$0 + \frac{1}{2}\times 0.2\times 5^2 + 0 = \frac{1}{2}\times 0.2\times V^2 + 0.2\times 10\times -0.5 + 3.1$

$2.5 = 0.1V^2 - 1 + 3.1$

$2.5 = 0.1V^2 + 2.1$

$0.4 = 0.1V^2$

$V^2 = 4$

$V = \underline{\underline{2\,ms^{-1}}}$

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2 A box of mass 5 kg is pulled at a constant speed a distance of 15 m up a rough plane inclined at an angle of 20° to the horizontal. The box moves along a line of greatest slope against a frictional force of 40 N. The force pulling the box is parallel to the line of greatest slope.

(a) Find the work done against friction. [1]

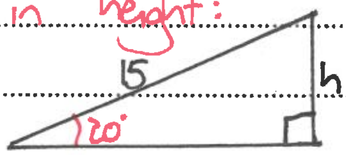
Work done $= F \times d$

$= 40 \times 15$

$= \underline{600\,J}$

(b) Find the change in gravitational potential energy of the box. [2]

Change in height:



$\sin 20 = \frac{h}{15}$

$h = 15 \sin 20$

Change in P.E. $= mg\Delta h$

$= 5 \times 10 \times 15 \sin 20$

$= \underline{257\,J}$

STO

(c) Find the work done by the pulling force. [1]

$Work_{W} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{out}$

$W + \frac{1}{2} \times 5 \times v^2 + 0 = \frac{1}{2} \times 5 \times v^2 + 257 + 600$

$W + \cancel{2.5v^2} = \cancel{2.5v^2} + 857$

$W = \underline{857\,J}$

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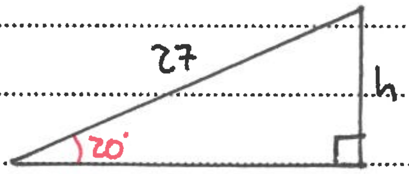
2 A box of mass 5 kg is pulled at a constant speed of $1.8\,\mathrm{m\,s^{-1}}$ for 15 s up a rough plane inclined at an angle of 20° to the horizontal. The box moves along a line of greatest slope against a frictional force of 40 N. The force pulling the box is parallel to the line of greatest slope.

(a) Find the change in gravitational potential energy of the box. [2]

Change in height: Constant speed, so distance = speed × time

$= 1.8 \times 15$

$= 27\text{m}$



$\sin 20 = \frac{h}{27}$

$h = 27\sin 20$

Change in PE $= mg\Delta h$

$= 5 \times 10 \times 27\sin 20$

$= 462\text{J}$

STO

(b) Find the work done by the pulling force. [2]

$\text{Work}_{in} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{out}$

$W + \frac{1}{2}\times 5 \times 1.8^2 + 0 = \frac{1}{2}\times 5\times 1.8^2 + 462 + 40\times 27$

work = F×d

$W + \cancel{8.1} = \cancel{8.1} + 462 + 1080$

$W = 462 + 1080$

$= 1540\text{J}$ (3sf)

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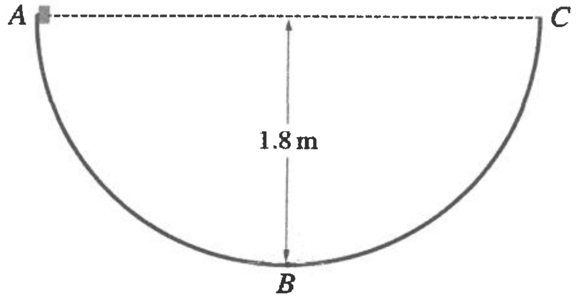
3



The diagram shows a semi-circular track ABC of radius 1.8 m which is fixed in a vertical plane. The points A and C are at the same horizontal level and the point B is at the bottom of the track. The section AB is smooth and the section BC is rough. A small block is released from rest at A.

(a) Show that the speed of the block at B is $6\,\mathrm{m\,s^{-1}}$. [2]

$A \to B$ $\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$0 + 0 + 0 = \frac{1}{2} \times m \times v^2 + m \times 10 \times -1.8 + 0$

$0 = \frac{1}{2}mv^2 - 18m$

$\frac{1}{2}mv^2 = 18m$

$\cancel{m}v^2 = 36\cancel{m}$

$v^2 = 36$ $\rightarrow$ $\underline{\underline{v = 6\,\mathrm{ms^{-1}}}}$

The block comes to instantaneous rest for the first time at a height of 1.2 m above the level of B. The work done against the resistance force during the motion of the block from B to this point is 4.5 J.

(b) Find the mass of the block. [3]

$B \to C$: $\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$0 + \frac{1}{2}m \times 6^2 + 0 = 0 + m \times 10 \times 1.2 + 4.5$

$18m = 12m + 4.5$

$6m = 4.5$

$m = \underline{\underline{0.75\ \mathrm{kg}}}$

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3 A ball of mass 1.6 kg is released from rest at a point 5 m above horizontal ground. When the ball hits the ground it instantaneously loses 8 J of kinetic energy and starts to move upwards.

(a) Use an energy method to find the greatest height that the ball reaches after hitting the ground. [3]

On the way down: $Work_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{OUT}$

$0 + 0 + 0 = KE_{fin} + 1{\cdot}6 \times 10 \times -5 + 0$

$0 = KE_{fin} - 80$

$KE_{fin} = 80\text{J}$

KE after bounce $= 72\text{J}$

after bounce: $Work_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{OUT}$

$0 + 72 + 0 = 0 + 1{\cdot}6 \times 10 \times h + 0$

$v = 0$

$72 = 16h$

$h = \underline{4{\cdot}5\text{m}}$

(b) Find the total time taken, from the initial release of the ball until it reaches this greatest height. [3]

On the way down:

$\downarrow +$	
$s = 5$	$s = ut + \frac{1}{2}at^2$
$u = 0$	$5 = 0 + \frac{1}{2} \times 10 \times t^2$
$v =$	$5 = 5t^2$
$a = 10$	$t^2 = 1$
$t =$	$\underline{t = 1\text{s}}$

On the way up:

$\uparrow +$	
$s = 4{\cdot}5$	$s = vt - \frac{1}{2}at^2$
$u =$	$4{\cdot}5 = 0 - \frac{1}{2} \times -10 \times t^2$
$v = 0$	$4{\cdot}5 = 5t^2$
$a = -10$	$t^2 = 0{\cdot}9$
$t =$	$\underline{t = 0{\cdot}949\text{s}}$

Total time $= 1 + 0{\cdot}949$

$= \underline{1{\cdot}95\text{s}}$

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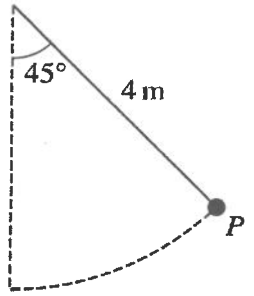
5

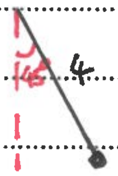


A child of mass 35 kg is swinging on a rope. The child is modelled as a particle P and the rope is modelled as a light inextensible string of length 4 m. Initially P is held at an angle of 45° to the vertical (see diagram).

(a) Given that there is no resistance force, find the speed of P when it has travelled half way along the circular arc from its initial position to its lowest point. [4]

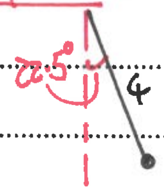
Change in height when P has travelled half way, ie. angle = 22.5°:

initial:



$h = 4\cos 45$

final:



$h = 4\cos 22.5$

$$\text{change in height} = 4\cos 45 - 4\cos 22.5$$
$$= \underline{-0.867\text{m}}$$

STO

$$\text{Work}_{\text{in}} + KE_{\text{init}} + PE_{\text{init}} = KE_{\text{fin}} + PE_{\text{fin}} + \text{Work}_{\text{out}}$$
$$0 + 0 + 0 = \tfrac{1}{2} \times 35 \times v^2 + 35 \times 10 \times -0.867 + 0$$

$$0 = 17.5v^2 - 303.48..$$
$$17.5v^2 = 303.48$$
$$v^2 = 17.34$$
$$v = \underline{4.16\text{ms}^{-1}}$$

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(b) It is given instead that there is a resistance force. The work done against the resistance force as P travels from its initial position to its lowest point is X J. The speed of P at its lowest point is $4\,\mathrm{m\,s^{-1}}$.

Find X. [3]

Change in height from start to lowest point:

change in height $= 4\cos 45 - 4$

$= \underline{-1.1716}$ STO

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$0 + 0 + 0 = \frac{1}{2} \times 35 \times 4^2 + 35 \times 10 \times -1.1716 + X$

$0 = 280 - 410.05 + X$

$0 = -130.05 + X$

$X = \underline{130\text{ J}}$

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5 A block B of mass 4 kg is pushed up a line of greatest slope of a smooth plane inclined at 30° to the horizontal by a force applied to B, acting in the direction of motion of B. The block passes through points P and Q with speeds $12\,\text{m}\,\text{s}^{-1}$ and $8\,\text{m}\,\text{s}^{-1}$ respectively. P and Q are 10 m apart with P below the level of Q.

(a) Find the decrease in kinetic energy of the block as it moves from P to Q. [2]

$KE_{init} = \frac{1}{2} \times 4 \times 12^2$

$= 288\,J$

$KE_{fin} = \frac{1}{2} \times 4 \times 8^2$

$= 128\,J$

Change in KE $= 288 - 128$

$= 160\,J$ decrease

(b) Hence find the work done by the force pushing the block up the slope as the block moves from P to Q. [3]

$Work_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{out}$

$W + 288 + 0 = 128 + 4 \times 10 \times 10\sin 30 + 0$

$W + 288 = 128 + 400\sin 30$

$W + 288 = 128 + 200$

$W + 288 = 328$

$W = 40\,J$

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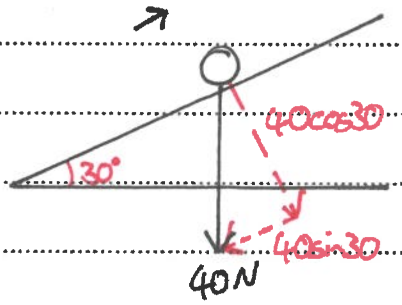
(c) At the instant the block reaches Q, the force pushing the block up the slope is removed.

Find the time taken, after this instant, for the block to return to P. [4]



$R(\nearrow): F = ma$

$-40\sin 30 = 4a$

$-20 = 4a$

$a = -5 \text{ ms}^{-2}$

$+\nearrow$

$s = -10$ | $s = ut + \frac{1}{2}at^2$

$u = 8$ | $-10 = 8t + \frac{1}{2}(-5) \times t^2$

$v =$ | $-10 = 8t - 2.5t^2$

$a = -5$ | $2.5t^2 - 8t - 10 = 0$

$t =$ | $5t^2 - 16t - 20 = 0$

$$t = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(5)(-20)}}{2(5)}$$

$t = 4.16\text{s}$ ✓ or $t = -0.961\text{s}$ ✗

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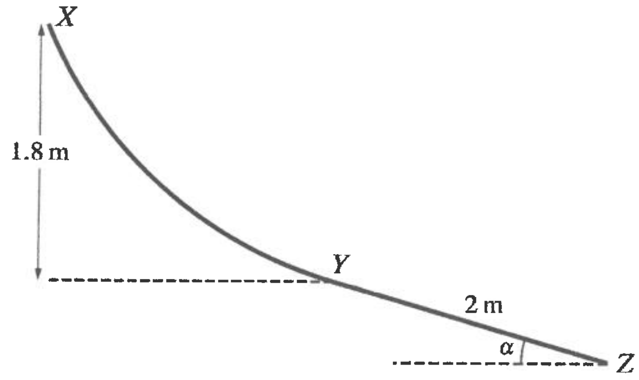
7



The diagram shows the vertical cross-section XYZ of a rough slide. The section YZ is a straight line of length 2 m inclined at an angle of α to the horizontal, where $\sin\alpha = 0.28$. The section YZ is tangential to the curved section XY at Y, and X is 1.8 m above the level of Y. A child of mass 25 kg slides down the slide, starting from rest at X. The work done by the child against the resistance force in moving from X to Y is 50 J.

(a) Find the speed of the child at Y. [4]

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$0 + 0 + 0 = \frac{1}{2} \times 25 \times v^2 + 25 \times 10 \times -1.8 + 50$

$0 = 12.5v^2 - 450 + 50$

$0 = 12.5v^2 - 400$

$12.5v^2 = 400$

$v^2 = 32$

$v = 5.66\ ms^{-1}$

STO

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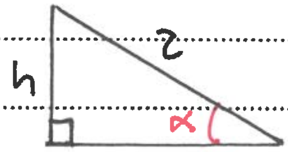
It is given that the child comes to rest at Z.

(b) Use an energy method to find the coefficient of friction between the child and YZ, giving your answer as a fraction in its simplest form. [6]

Change in height:



$\sin\alpha = \frac{h}{2}$

$h = 2\sin\alpha$

$h = 2 \times 0.28 = \underline{0.56\text{m}}$

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$0 + \frac{1}{2} \times 25 \times 5.66^2 + 0 = 0 + 25 \times 10 \times -0.56 + F \times 2$

↳ $v=0$ ↳ Work = F×d

$400 = -140 + 2F$

$540 = 2F$

$\underline{F = 270\text{N}}$

Force diagram:

$\sin\alpha = 0.28$

$\underline{\alpha = 16.26°}$ STO

$R(\uparrow): R - 250\cos(16.26) = 0$

$R = 250\cos(16.26)$

$\underline{R = 240\text{ N}}$

$F = \mu R$

$270 = \mu \times 240$

$\underline{\mu = 1.125}$

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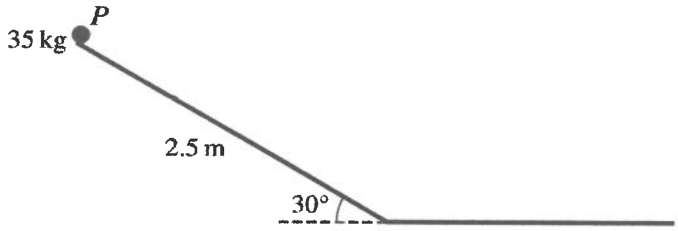
7



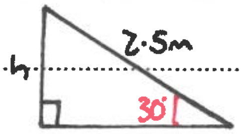
A slide in a playground descends at a constant angle of 30° for 2.5 m. It then has a horizontal section in the same vertical plane as the sloping section. A child of mass 35 kg, modelled as a particle P, starts from rest at the top of the slide and slides straight down the sloping section. She then continues along the horizontal section until she comes to rest (see diagram). There is no instantaneous change in speed when the child goes from the sloping section to the horizontal section.

The child experiences a resistance force on the horizontal section of the slide, and the work done against the resistance force on the horizontal section of the slide is 250 J per metre.

(a) It is given that the sloping section of the slide is smooth.

(i) Find the speed of the child when she reaches the bottom of the sloping section. [3]

Change in height:



$\sin 30 = \frac{h}{2.5}$

$h = 2.5 \sin 30 = 1.25\text{m}$

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$0 + 0 + 0 = \frac{1}{2} \times 35 \times v^2 + 35 \times 10 \times -1.25 + 0$

$0 = 17.5v^2 - 437.5$

$17.5v^2 = 437.5$

$v^2 = 25$

$v = 5\text{ms}^{-1}$

(ii) Find the distance that the child travels along the horizontal section of the slide before she comes to rest. [2]

$\text{Work}_{IN} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + \text{Work}_{OUT}$

$0 + \frac{1}{2} \times 35 \times 5^2 + 0 = 0 + 0 + 250d$

because $V = 0$

$437.5 = 250d$

$d = 1.75\text{m}$

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(b) It is given instead that the sloping section of the slide is rough and that the child comes to rest on the slide 1.05 m after she reaches the horizontal section.

Find the coefficient of friction between the child and the sloping section of the slide. [6]

Horizontal section: Find initial velocity:

$Work_{in} + KE_{init} + PE_{init} = KE_{fin} + PE_{fin} + Work_{out}$

$0 + \frac{1}{2} \times 35 \times v^2 + 0 = 0 + 0 + 250 \times 1.05$

$17.5v^2 = 262.5$

$v^2 = 15$

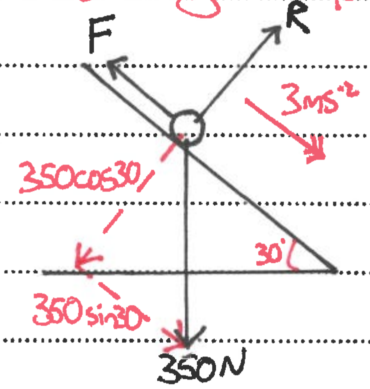
$v = \sqrt{15}\ ms^{-1}$

Use suvat on inclined section to find acceleration:

+

$s = 2.5$	$v^2 = u^2 + 2as$
$u = 0$	$(\sqrt{15})^2 = 0^2 + 2 \times a \times 2.5$
$v = \sqrt{15}$	$15 = 0 + 5a$
$a =$	$15 = 5a$
$t =$	$a = 3ms^{-2}$

Force diagram for inclined section:



$R(\nearrow): R - 350\cos 30 = 0$

$R = 350\cos 30$

$R(\searrow): 350\sin 30 - F = ma$

$175 - \mu R = 35 \times 3$

$175 - \mu R = 105$

$-\mu R = -70$

$\mu R = 70$

$\mu \times 350\cos 30 = 70$

$\mu = 0.231$

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