1 Two particles P and Q, of masses 0.2 kg and 0.5 kg respectively, are at rest on a smooth horizontal plane. P is projected towards Q with speed $2\,\text{m s}^{-1}$.

(a) Write down the momentum of P. [1]

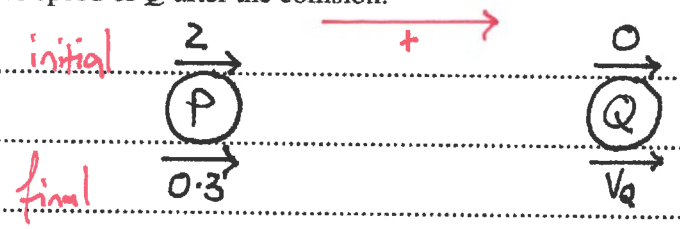
Momentum $= mV$

$= 0.2 \times 2$

$= 0.4\ \text{kgms}^{-1}$

(b) After the collision P continues to move in the same direction with speed $0.3\,\text{m s}^{-1}$.

Find the speed of Q after the collision. [2]



$m_P u_P + m_Q u_Q = m_P V_P + m_Q V_Q$

$0.2 \times 2 + 0 = 0.2 \times 0.3 + 0.5 \times V_Q$

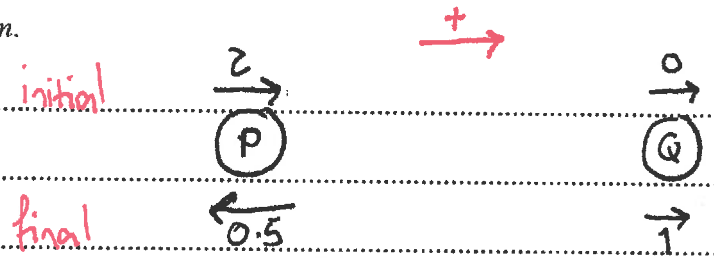
$0.4 = 0.06 + 0.5 V_Q$

$0.34 = 0.5 V_Q$

$V_Q = 0.68\ \text{ms}^{-1}$

1 Particles P of mass m kg and Q of mass 0.2 kg are free to move on a smooth horizontal plane. P is projected at a speed of $2\,\mathrm{m\,s^{-1}}$ towards Q which is stationary. After the collision P and Q move in opposite directions with speeds of $0.5\,\mathrm{m\,s^{-1}}$ and $1\,\mathrm{m\,s^{-1}}$ respectively.

Find m. [3]



$m_P u_P + m_Q u_Q = m_P v_P + m_Q v_Q$

$m \times 2 + 0 = m \times -0.5 + 0.2 \times 1$

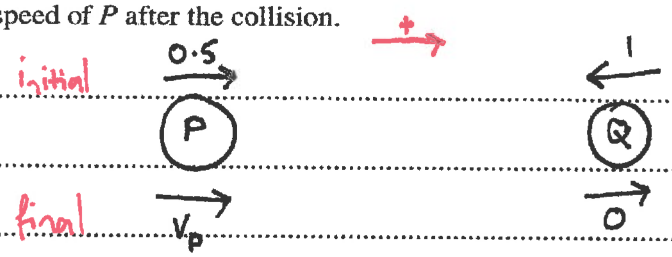
$2m = -0.5m + 0.2$

$2.5m = 0.2$

$m = \underline{0.08\text{ kg}}$

1 Two particles P and Q of masses 0.2 kg and 0.3 kg respectively are free to move in a horizontal straight line on a smooth horizontal plane. P is projected towards Q with speed $0.5\ \mathrm{m\,s^{-1}}$. At the same instant Q is projected towards P with speed $1\ \mathrm{m\,s^{-1}}$. Q comes to rest in the resulting collision.

Find the speed of P after the collision. [3]



$$M_P U_P + M_Q U_Q = M_P V_P + M_Q V_Q$$

$$0.2 \times 0.5 + 0.3 \times -1 = 0.2 V_P + 0$$

$$0.1 - 0.3 = 0.2 V_P$$

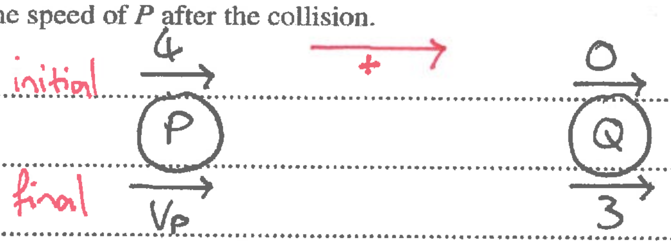
$$-0.2 = 0.2 V_P$$

$$V_P = -1\ \mathrm{ms^{-1}}$$

$$\text{speed} = \underline{1\ \mathrm{ms^{-1}}}$$

1 Two particles P and Q, of masses 0.3 kg and 0.2 kg respectively, are at rest on a smooth horizontal plane. P is projected at a speed of $4\,\mathrm{m\,s^{-1}}$ directly towards Q. After P and Q collide, Q begins to move with a speed of $3\,\mathrm{m\,s^{-1}}$.

(a) Find the speed of P after the collision. [2]



$M_P u_P + M_Q u_Q = M_P V_P + M_Q V_Q$

$0.3 \times 4 + 0 = 0.3 V_P + 0.2 \times 3$

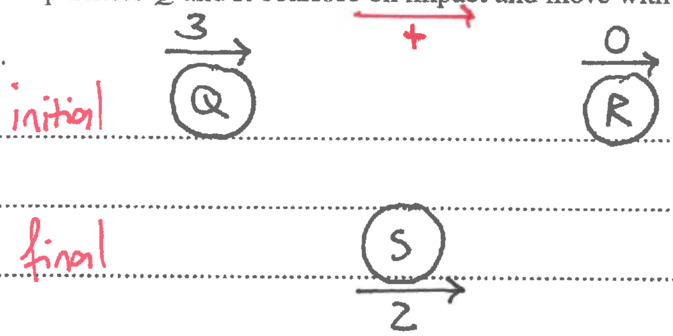
$1.2 = 0.3 V_P + 0.6$

$0.6 = 0.3 V_P$

$V_P = \underline{\underline{2\,\mathrm{ms^{-1}}}}$

After the collision, Q moves directly towards a third particle R, of mass m kg, which is at rest on the plane. The two particles Q and R coalesce on impact and move with a speed of $2\,\mathrm{m\,s^{-1}}$.

(b) Find m. [2]



$M_Q u_Q + M_R u_R = M_S V_S$

$0.2 \times 3 + 0 = (m + 0.2) \times 2$

$0.6 = 2m + 0.4$

$0.2 = 2m$

$m = \underline{\underline{0.1\,\mathrm{kg}}}$

1 Two particles P and Q, of masses m kg and 0.3 kg respectively, are at rest on a smooth horizontal plane. P is projected at a speed of $5\,\text{m}\,\text{s}^{-1}$ directly towards Q. After P and Q collide, P moves with a speed of $2\,\text{m}\,\text{s}^{-1}$ in the same direction as it was originally moving.

(a) Find, in terms of m, the speed of Q after the collision. [2]

$m_P u_P + m_Q u_Q = m_P v_P + m_Q v_Q$

$m \times 5 + 0 = m \times 2 + 0.3 v_Q$

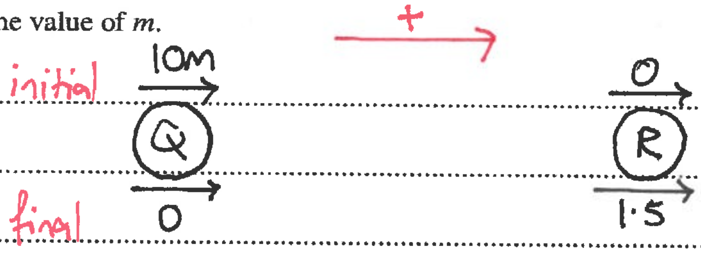
$5m = 2m + 0.3 v_Q$

$3m = 0.3 v_Q$

$v_Q = \underline{10m \text{ ms}^{-1}}$

After this collision, Q moves directly towards a third particle R, of mass 0.6 kg, which is at rest on the plane. Q is brought to rest in the collision with R, and R begins to move with a speed of $1.5\,\text{m}\,\text{s}^{-1}$.

(b) Find the value of m. [2]



$m_Q u_Q + m_R u_R = m_Q v_Q + m_R v_R$

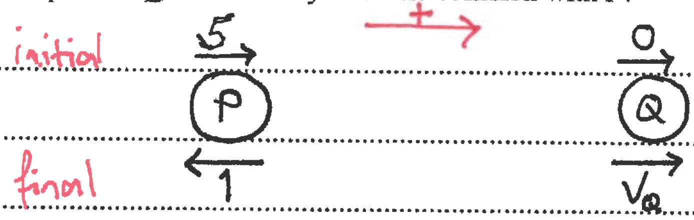
$0.3 \times 10m + 0 = 0 + 0.6 \times 1.5$

$3m = 0.9$

$m = \underline{0.3 \text{ kg}}$

3 Three particles P, Q and R, of masses 0.1 kg, 0.2 kg and 0.5 kg respectively, are at rest in a straight line on a smooth horizontal plane. Particle P is projected towards Q at a speed of $5\,\mathrm{m\,s^{-1}}$. After P and Q collide, P rebounds with speed $1\,\mathrm{m\,s^{-1}}$.

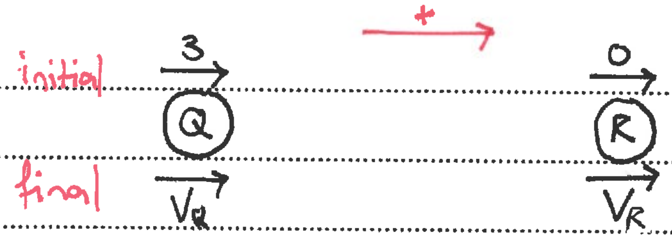
(a) Find the speed of Q immediately after the collision with P. [3]



$$m_P u_P + m_Q u_Q = m_P v_P + m_Q v_Q$$
$$0.1 \times 5 + 0 = 0.1 \times -1 + 0.2V_Q$$
$$0.5 = -0.1 + 0.2V_Q$$
$$0.6 = 0.2V_Q$$
$$V_Q = \underline{3\,\mathrm{ms^{-1}}}$$

Q now collides with R. Immediately after the collision with Q, R begins to move with speed $V\,\mathrm{m\,s^{-1}}$.

(b) Given that there is no subsequent collision between P and Q, find the greatest possible value of V. [3]



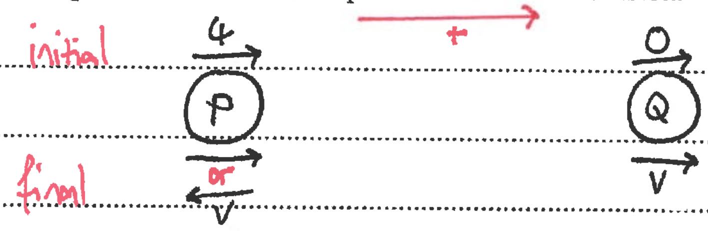
$$m_Q u_Q + m_R u_R = m_Q V_Q + m_R V_R$$
$$0.2 \times 3 + 0 = 0.2V_Q + 0.5V_R$$
$$0.6 = 0.2V_Q + 0.5V_R$$

If P and Q do not collide, the minimum speed for Q is the same speed as P, ie. $V_Q = -1$:

$$0.6 = 0.2 \times -1 + 0.5V_R$$
$$0.6 = -0.2 + 0.5V_R$$
$$0.8 = 0.5V_R$$
$$V_R = \underline{1.6\,\mathrm{ms^{-1}}}$$

1 Two particles P and Q, of masses 0.1 kg and 0.4 kg respectively, are free to move on a smooth horizontal plane. Particle P is projected with speed $4\,\mathrm{m\,s^{-1}}$ towards Q which is stationary. After P and Q collide, the speeds of P and Q are equal.

Find the two possible values of the speed of P after the collision. [3]



Scenario 1: P continues moving right after collision:

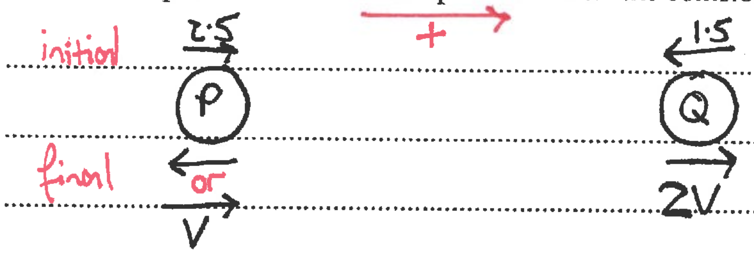
$$m_P u_P + m_Q u_Q = m_P v_P + m_Q v_Q$$
$$0.1 \times 4 + 0 = 0.1V + 0.4V$$
$$0.4 = 0.5V$$
$$V = \underline{0.8\,\mathrm{ms^{-1}}}$$

Scenario 2: P moves left after the collision:

$$m_P u_P + m_Q u_Q = m_P v_P + m_Q v_Q$$
$$0.1 \times 4 + 0 = 0.1 \times -V + 0.4 \times V$$
$$0.4 = -0.1V + 0.4V$$
$$0.4 = 0.3V$$
$$V = \underline{\tfrac{4}{3}\,\mathrm{ms^{-1}}}$$

1 Particles P of mass 0.4 kg and Q of mass 0.5 kg are free to move on a smooth horizontal plane. P and Q are moving directly towards each other with speeds $2.5\,\mathrm{m\,s^{-1}}$ and $1.5\,\mathrm{m\,s^{-1}}$ respectively. After P and Q collide, the speed of Q is twice the speed of P.

Find the two possible values of the speed of P after the collision. [4]



(Q must be moving to the right, otherwise P and Q would coalesce)

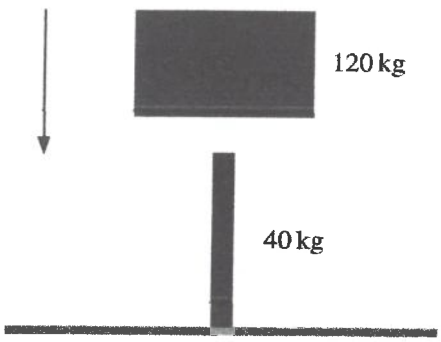
• Scenario 1: P goes left:

$$M_P U_P + M_Q U_Q = M_P V_P + M_Q V_Q$$
$$0.4 \times 2.5 + 0.5 \times -1.5 = 0.4 \times -V + 0.5 \times 2V$$
$$1 - 0.75 = -0.4V + 1V$$
$$0.25 = 0.6V$$
$$V = \underline{\underline{0.417\,\mathrm{ms^{-1}}}}$$

• Scenario 2: P goes right:

$$M_P U_P + M_Q U_Q = M_P V_P + M_Q V_Q$$
$$0.4 \times 2.5 + 0.5 \times -1.5 = 0.4 \times V + 0.5 \times 2V$$
$$1 - 0.75 = 0.4V + 1V$$
$$0.25 = 1.4V$$
$$V = \underline{\underline{0.179\,\mathrm{ms^{-1}}}}$$

1



A metal post is driven vertically into the ground by dropping a heavy object onto it from above. The mass of the object is 120 kg and the mass of the post is 40 kg (see diagram). The object hits the post with speed $8\,\mathrm{m\,s^{-1}}$ and remains in contact with it after the impact.

(a) Calculate the speed with which the combined post and object moves immediately after the impact. [2]

initial $\quad$ final $\quad +\downarrow$

$8\downarrow O \qquad Q \downarrow V_Q \qquad 0\downarrow P$

$M_o u_o + M_p u_p = M_Q V_Q$

$120 \times 8 + 0 = 160\, V_Q$

$960 = 160\, V_Q$

$V_Q = \underline{6\,\mathrm{ms^{-1}}}$

(b) There is a constant force resisting the motion of magnitude 4800 N.

Calculate the distance the post is driven into the ground. [3]

4800 N $\uparrow$ $\quad$ 160 kg $\quad$ 1600 N $\downarrow$

$R(\downarrow): F = ma$

$1600 - 4800 = 160a$

$-3200 = 160a$

$a = \underline{-20\,\mathrm{ms^{-2}}}$

$+\downarrow$

$s =$ $\quad$ $v^2 = u^2 + 2as$

$u = 6$ $\quad$ $0^2 = 6^2 + 2 \times -20 \times s$

$v = 0$ $\quad$ $0 = 36 - 40s$

$a = -20$ $\quad$ $40s = 36$

$t =$ $\quad$ $s = \underline{0.9\,\mathrm{m}}$

6 Three particles A, B and C of masses 0.3 kg, 0.4 kg and m kg respectively lie at rest in a straight line on a smooth horizontal plane. The distance between B and C is 2.1 m. A is projected directly towards B with speed $2\,\text{m s}^{-1}$. After A collides with B the speed of A is reduced to $0.6\,\text{m s}^{-1}$, still moving in the same direction.

(a) Show that the speed of B after the collision is $1.05\,\text{m s}^{-1}$. [2]

$$M_A U_A + M_B U_B = M_A V_A + M_B V_B$$

$$0.3 \times 2 + 0 = 0.3 \times 0.6 + 0.4 V_B$$

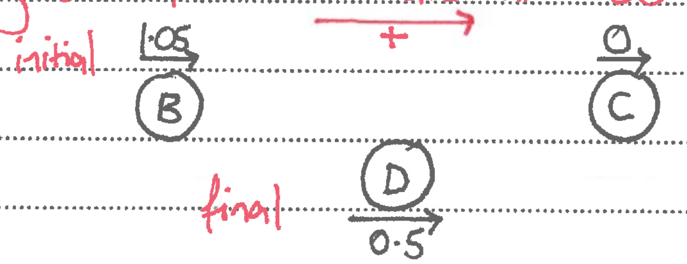
$$0.6 = 0.18 + 0.4 V_B$$

$$0.42 = 0.4 V_B \rightarrow \underline{V_B = 1.05\,\text{ms}^{-1}} \text{ QED}$$

After the collision between A and B, B moves directly towards C. Particle B now collides with C. After this collision, the two particles coalesce and have a combined speed of $0.5\,\text{m s}^{-1}$.

(b) Find m. [2]

The plane is smooth and there are no other horizontal forces acting on B, so it remains at $1.05\,\text{ms}^{-1}$ until it hits C:



$$M_B U_B + M_C U_C = M_D V_D$$

$$0.4 \times 1.05 + 0 = (0.4 + m) \times 0.5$$

$$0.42 = 0.2 + 0.5m$$

$$0.22 = 0.5m$$

$$m = \underline{0.44\text{ kg}}$$

(c) Find the time that it takes, from the instant when B and C collide, until A collides with the combined particle. [5]

How much time passes until B and C collide?

B has travelled 2.1m at a constant speed of $1.05 ms^{-1}$:

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{2.1}{1.05}$$

$$\text{time} = 2s$$

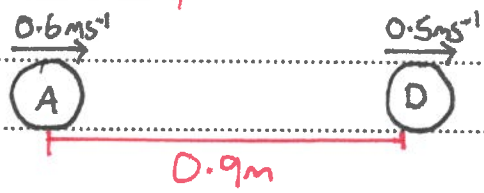
How far has A travelled in this time?

$$\text{distance} = \text{speed} \times \text{time}$$

$$= 0.6 \times 2$$

$$= 1.2m$$

This means that when B and C collide, A is 0.9m away $(2.1 - 1.2 = 0.9m)$



When A and D collide, A will have travelled 0.9m further than D. So distance travelled by D $= x$, and distance travelled by A $= x + 0.9$.

A: distance = speed × time D: distance = speed × time

$x + 0.9 = 0.6t$ (1) $x = 0.5t$ (2)

sub. (2) → (1):

$$0.5t + 0.9 = 0.6t$$

$$0.9 = 0.1t$$

$$t = 9s$$

6 A particle A is projected vertically upwards from level ground with an initial speed of $30\,\text{m s}^{-1}$. At the same instant a particle B is released from rest 15 m vertically above A. The mass of one of the particles is twice the mass of the other particle. During the subsequent motion A and B collide and coalesce to form particle C.

Find the difference between the two possible times at which C hits the ground. [8]

Need to find the position and velocities of A and B when they collide:

$+\uparrow A$: $s = s_A$, $u = 30$, $v =$, $a = -10$, $t =$

$s_A = ut + \frac{1}{2}at^2$

$s_A = 30t - 5t^2$

$+\downarrow B$: $s = s_B$, $u = 0$, $v =$, $a = 10$, $t =$

$s_B = ut + \frac{1}{2}at^2$

$s_B = 0 + 5t^2$

$s_B = 5t^2$

$$s_A + s_B = 15$$
$$30t - 5t^2 + 5t^2 = 15$$
$$30t = 15$$
$$t = 0.5\,\text{s}$$

Find height above ground by substituting $t = 0.5$ into s_A:

$$s_A = 30(0.5) - 5(0.5)^2$$
$$= 15 - 1.25$$
$$= \underline{13.75\,\text{m}} \text{ above ground}$$

Find velocities of A and B:

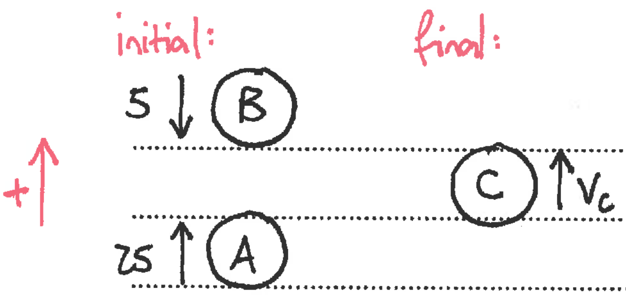
$+\uparrow A$: $s =$, $u = 30$, $v =$, $a = -10$, $t = 0.5$

$v = u + at$
$= 30 + (-10) \times 0.5$
$= 30 - 5$
$= \underline{25\,\text{ms}^{-1}}$

$+\downarrow B$: $s =$, $u = 0$, $v =$, $a = 10$, $t = 0.5$

$v = u + at$
$= 0 + 10(0.5)$
$= \underline{5\,\text{ms}^{-1}}$

continued...



Scenario 1: $A = 2m,\ B = m$:

$$m_A u_A + m_B u_B = m_C V_C$$
$$2m \times 25 + m \times -5 = 3m V_C$$
$$50m - 5m = 3m V_C$$
$$45\cancel{m} = 3\cancel{m} V_C$$
$$45 = 3V_C$$
$$\underline{V_C = 15\,\mathrm{ms^{-1}}} \quad (1)$$

Scenario 2: $A = m,\ B = 2m$:

$$m_A u_A + m_B u_B = m_C V_C$$
$$m \times 25 + 2m \times -5 = 3m V_C$$
$$25m - 10m = 3m V_C$$
$$15\cancel{m} = 3\cancel{m} V_C$$
$$15 = 3V_C$$
$$\underline{V_C = 5\,\mathrm{ms^{-1}}} \quad (2)$$

Scenario 1:		Scenario 2:	
$+\uparrow$ $s = -13.75$	$s = ut + \frac{1}{2}at^2$	$+\uparrow$ $s = -13.75$	$s = ut + \frac{1}{2}at^2$
$u = 15$	$-13.75 = 15t - 5t^2$	$u = 5$	$-13.75 = 5t - 5t^2$
$v =$	$5t^2 - 15t - 13.75 = 0$	$v =$	$5t^2 - 5t - 13.75 = 0$
$a = -10$	$t = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(5)(-13.75)}}{2(5)}$	$a = -10$	$t = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(5)(-13.75)}}{2(5)}$
$t =$		$t =$	

$\underline{t = 3.74}$ ✓ or $t = -0.736$ ✗ STO

$\underline{t = 2.23}$ ✓ or $t = -1.23$ ✗ STO

difference $= 3.74 - 2.23$

$= \underline{1.50\,\mathrm{s}}$

7 Two particles A and B, of masses 0.4 kg and 0.2 kg respectively, are moving down the same line of greatest slope of a smooth plane. The plane is inclined at 30° to the horizontal, and A is higher up the plane than B. When the particles collide, the speeds of A and B are $3\,\mathrm{m\,s^{-1}}$ and $2\,\mathrm{m\,s^{-1}}$ respectively. In the collision between the particles, the speed of A is reduced to $2.5\,\mathrm{m\,s^{-1}}$.

(a) Find the speed of B immediately after the collision. [2]

initial $\xrightarrow{3}$ $\xrightarrow{+}$ $\xrightarrow{2}$

A B

final $\xrightarrow{2.5}$ $\xrightarrow{V_B}$

$M_A U_A + M_B U_B = M_A V_A + M_B V_B$

$0.4 \times 3 + 0.2 \times 2 = 0.4 \times 2.5 + 0.2 V_B$

$1.2 + 0.4 = 1 + 0.2 V_B$

$1.6 = 1 + 0.2 V_B$

$0.6 = 0.2 V_B$ $V_B = 3\,\mathrm{ms^{-1}}$

After the collision, when B has moved 1.6 m down the plane from the point of collision, it hits a barrier and returns back up the same line of greatest slope. B hits the barrier 0.4 s after the collision, and when it hits the barrier, its speed is reduced by 90%. The two particles collide again 0.44 s after their previous collision, and they then coalesce on impact.

(b) Show that the speed of B immediately after it hits the barrier is $0.5\,\mathrm{m\,s^{-1}}$. Hence find the speed of the combined particle immediately after the second collision between A and B. [7]

B: $\downarrow +$ $S = 1.6$ | $S = \frac{1}{2}(u+v) \times t$

$u = 3$ | $1.6 = \frac{1}{2}(3+v) \times 0.4$

$v =$ | $3.2 = (3+v) \times 0.4$

$a =$ | $8 = 3 + v$

$t = 0.4$ | $v = 5\,\mathrm{ms^{-1}}$

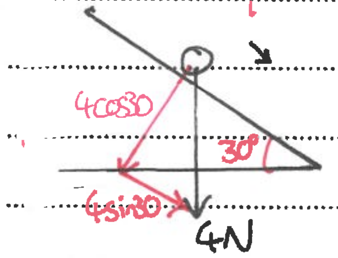
speed of B is reduced by 90%:

$0.9 \times 5 = 4.5$ $5 - 4.5 = 0.5\,\mathrm{ms^{-1}}$ QED

continued...

Need to find velocity of particle A at point of impact:

A:



$R(\searrow)$: $F = ma$

$4\sin 30 = 0.4a$

$2 = 0.4a$

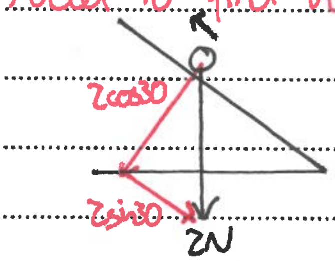
$a = 5\text{ms}^{-2}$

A: $\searrow +$

$s =$	$v = u + at$
$u = 2.5$	$= 2.5 + 5 \times 0.44$
$v =$	$= 2.5 + 2.2$
$a = 5$	$= 4.7\text{ms}^{-1}$
$t = 0.44$	

Need to find velocity of particle B 0.04s after hitting barrier:

B:



$R(\nwarrow)$: $F = ma$

$-2\sin 30 = 0.2a$

$-1 = 0.2a$

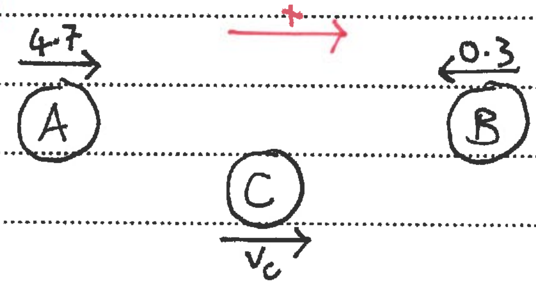
$a = -5\text{ms}^{-2}$

B: $\nwarrow +$

$s =$	$v = u + at$
$u = 0.5$	$= 0.5 + (-5) \times 0.04$
$v =$	$= 0.5 - 0.2$
$a = -5$	$= 0.3\text{ms}^{-1}$
$t = 0.04$	

initial

final



$m_A u_A + m_B u_B = m_C V_C$

$0.4 \times 4.7 + 0.2 \times -0.3 = 0.6V_c$

$1.88 - 0.06 = 0.6V_c$

$1.82 = 0.6V_c$

$V_c = 3.03\text{ms}^{-1}$

long!!