4 A cyclist starts from rest at a point A and travels along a straight road AB, coming to rest at B. The displacement of the cyclist from A at time t s after the start is s m, where

$$s = 0.004(75t^2 - t^3).$$

(a) Show that the distance AB is 250 m. $\hookrightarrow$ $S = 0.3t^2 - 0.004t^3$ [4]

find when cyclist is at rest:

$V = \frac{ds}{dt} = 0.6t - 0.012t^2$

$V=0$: $0.6t - 0.012t^2 = 0$ $\div 0.012$

$50t - t^2 = 0$

$t(50-t) = 0$

$t=0$, $t=50$

Sub. $t=50$ into s:

$S = 0.3(50)^2 - 0.004(50)^3$

$= 750 - 500$

$= 250\text{m}$ QED

(b) Find the maximum velocity of the cyclist. [3]

$V = 0.6t - 0.012t^2$ is a negative quadratic, so max. V is at $\frac{dv}{dt} = 0$:

$\frac{dv}{dt} = 0.6 - 0.024t$

$0.6 - 0.024t = 0$

$0.024t = 0.6$

$t = 25\text{s}$

Sub. $t = 25$ into v:

$V = 0.6(25) - 0.012(25)^2$

$= 15 - 7.5$

$= 7.5\text{ms}^{-1}$

6 A particle moves in a straight line AB. The velocity $v\,\mathrm{m\,s^{-1}}$ of the particle t s after leaving A is given by $v = k(t^2 - 10t + 21)$, where k is a constant. The displacement of the particle from A, in the direction towards B, is 2.85 m when $t = 3$ and is 2.4 m when $t = 6$.

(a) Find the value of k. Hence find an expression, in terms of t, for the displacement of the particle from A. [7]

$V = kt^2 - 10kt + 21k$

$s = \int (kt^2 - 10kt + 21k)\,dt$

$s = \frac{1}{3}kt^3 - 5kt^2 + 21kt + C$

$s = 2.85$ when $t = 3$:

$2.85 = \frac{1}{3}k(3)^3 - 5k(3)^2 + 21k(3) + C$

$2.85 = 9k - 45k + 63k + C$

$2.85 = 27k + C$ (1)

$s = 2.4$ when $t = 6$:

$2.4 = \frac{1}{3}k(6)^3 - 5k(6)^2 + 21k(6) + C$

$2.4 = 72k - 180k + 126k + C$

$2.4 = 18k + C$ (2)

(1) − (2): $0.45 = 9k$

$k = 0.05$

sub. into (2): $2.4 = 18(0.05) + C$

$2.4 = 0.9 + C$

$C = 1.5$

→ $s = \frac{1}{3}(0.05)t^3 - 5(0.05)t^2 + 21(0.05)t + 1.5$

$s = \frac{0.05}{3}t^3 - 0.25t^2 + 1.05t + 1.5$

(b) Find the displacement of the particle from A when its velocity is a minimum. [4]

$V = 0{\cdot}05t^2 - 0{\cdot}5t + 1{\cdot}05$

$\frac{dV}{dt} = 0{\cdot}1t - 0{\cdot}5$

SP, so $\frac{dV}{dt} = 0$:

$0{\cdot}1t - 0{\cdot}5 = 0$

$0{\cdot}1t = 0{\cdot}5$

$\underline{t = 5s}$

Sub. $t=5$ into s:

$s = \frac{0{\cdot}05}{3}(5)^3 - 0{\cdot}25(5)^2 + 1{\cdot}05(5) + 1{\cdot}5$

$s = \frac{25}{12} - 6{\cdot}25 + 5{\cdot}25 + 1{\cdot}5$

$= \underline{2{\cdot}58m}$

6 A particle travels in a straight line PQ. The velocity of the particle t s after leaving P is $v\,\text{m s}^{-1}$, where

$$v = 4.5 + 4t - 0.5t^2.$$

(a) Find the velocity of the particle at the instant when its acceleration is zero. [3]

$a = \frac{dv}{dt} = 4 - t$

$a = 0$:

$4 - t = 0$

$t = 4\text{s}$

Sub. $t = 4$ into v:

$v = 4.5 + 4(4) - 0.5(4)^2$

$= 4.5 + 16 - 8$

$= 12.5\,\text{ms}^{-1}$

The particle comes to instantaneous rest at Q.

(b) Find the distance PQ. [6]

$V = 0$:

$4.5 + 4t - 0.5t^2 = 0$ $\times 2$

$9 + 8t - t^2 = 0$ $\times -1$

$t^2 - 8t - 9 = 0$

$(t-9)(t+1) = 0$

$t = 9$ ✓ or $t = -1$ ✗

$S = \int (4.5 + 4t - \frac{1}{2}t^2)\,dt$

$S = 4.5t + 2t^2 - \frac{1}{6}t^3 + C$

at P, $t = 0$:

$S = 0 + 0 - 0 + C$

$\underline{S = C}$

at Q, $t = 9$:

$S = 4.5(9) + 2(9)^2 - \frac{1}{6}(9)^3 + C$

$S = 40.5 + 162 - 121.5 + C$

$\underline{S = 81 + C}$

distance between P and Q:

distance $= 81 + C - C$

$= \underline{81\text{m}}$

5 A particle moving in a straight line starts from rest at a point A and comes instantaneously to rest at a point B. The acceleration of the particle at time t s after leaving A is a m s^{-2}, where

$$a = 6t^{\frac{1}{2}} - 2t.$$

(a) Find the value of t at point B. [3]

$V = \int (6t^{\frac{1}{2}} - 2t)\,dt$

$= \frac{2}{3} \times 6t^{3/2} - t^2 + C$

$= 4t^{3/2} - t^2 + C$

$V = 0$ when $t = 0$:

$0 = 0 - 0 + C$

$C = 0$

$\rightarrow V = 4t^{3/2} - t^2$

$V = 0$ at point B:

$0 = 4t^{3/2} - t^2$

$0 = t^{3/2}(4 - t^{\frac{1}{2}})$

$t^{3/2} = 0$ or $4 - t^{\frac{1}{2}} = 0$

$t = 0$ ✗ $\quad t^{\frac{1}{2}} = 4$

$\underline{t = 16\text{s}}$ ✓

(b) Find the distance travelled from A to the point at which the acceleration of the particle is again zero. [5]

$a = 0$:

$6t^{\frac{1}{2}} - 2t = 0$

$2t^{\frac{1}{2}}(3 - t^{\frac{1}{2}}) = 0$

$2t^{\frac{1}{2}} = 0$ or $3 - t^{\frac{1}{2}} = 0$

$t = 0$ ✗ $t^{\frac{1}{2}} = 3$

$\underline{t = 9}$ ✓

$S = \int (4t^{3/2} - t^2)\,dt$

$= \frac{2}{5} \times 4t^{5/2} - \frac{1}{3}t^3 + C$

$= \frac{8}{5}t^{5/2} - \frac{1}{3}t^3 + C$

At point A, $t = 0$, $s = 0$:

$0 = 0 - 0 + C$

$C = 0$

→ $S = \frac{8}{5}t^{5/2} - \frac{1}{3}t^3$

Sub. $t = 9$ into S:

$S = \frac{8}{5}(9)^{5/2} - \frac{1}{3}(9)^3$

$= 388.8 - 243$

$= \underline{\underline{145.8\text{m}}}$

7 A particle P travels in a straight line, starting at rest from a point O. The acceleration of P at time t s after leaving O is denoted by $a\,\mathrm{m\,s^{-2}}$, where

$$a = 0.3t^{\frac{1}{2}} \quad \text{for } 0 \leqslant t \leqslant 4,$$
$$a = -kt^{-\frac{3}{2}} \quad \text{for } 4 < t \leqslant T,$$

where k and T are constants.

(a) Find the velocity of P at $t = 4$. [2]

0-4: $V = \int 0.3t^{\frac{1}{2}}\,dt$

$= \frac{2}{3} \times 0.3t^{3/2} + C$

$V = 0.2t^{3/2} + C$

when $t=0$, $V=0$:

$0 = 0 + C$

$\rightarrow V = 0.2t^{3/2}$

$t=4$:

$V = 0.2 \times 4^{3/2}$

$= 0.2 \times 8$

$= \underline{\underline{1.6\,\mathrm{ms^{-1}}}}$

(b) It is given that there is no change in the velocity of P at $t = 4$ and that the velocity of P at $t = 16$ is $0.3\,\mathrm{m\,s^{-1}}$.

Show that $k = 2.6$ and find an expression, in terms of t, for the velocity of P for $4 \leqslant t \leqslant T$. [4]

4-T: $V = \int -kt^{-3/2}\,dt$

$V = 2kt^{-\frac{1}{2}} + C$

when $t=4$, $V=1.6$:

$1.6 = 2k(4)^{-\frac{1}{2}} + C$

$1.6 = k + C$ ①

when $t=16$, $V=0.3$:

$0.3 = 2k(16)^{-\frac{1}{2}} + C$

$0.3 = \frac{1}{2}k + C$ ②

① − ②:

$1.3 = \frac{1}{2}k$

$\underline{\underline{k = 2.6}}$ QED

Sub. into ①:

$1.6 = 2.6 + C$

$\underline{C = -1}$

$\rightarrow V = 2 \times 2.6 \times t^{-\frac{1}{2}} - 1$

$\underline{\underline{V = 5.2t^{-\frac{1}{2}} - 1}}$

(c) Given that P comes to instantaneous rest at $t = T$, find the exact value of T. [2]

$V = 0$:

$5.2t^{-\frac{1}{2}} - 1 = 0$

$5.2t^{-\frac{1}{2}} = 1$

$\frac{5.2}{\sqrt{t}} = 1$

$5.2 = \sqrt{t}$

$t = \underline{27.04s}$

(d) Find the total distance travelled between $t = 0$ and $t = T$. [4]

Turning points are at $t=0$, $t=4$ and $t=27.04$, so need to find distance between 0-4 and 4-27.04:

0-4s: $s = \int_0^4 0.2t^{3/2}\,dt$

$= \left[0.08t^{5/2}\right]_0^4$

$= \left[0.08 \times 4^{5/2}\right] - [0]$

$= \underline{2.56m}$

4-27.04s: $s = \int_4^{27.04} (5.2t^{-\frac{1}{2}} - 1)\,dt$

$= \left[10.4t^{\frac{1}{2}} - t\right]_4^{27.04}$

$= \left[10.4(27.04)^{\frac{1}{2}} - 27.04\right] - \left[10.4(4)^{\frac{1}{2}} - 4\right]$

$= [27.04] - [16.8]$

$= \underline{10.24m}$

Total distance $= 2.56 + 10.24$

$= \underline{12.8m}$

3 A particle moves in a straight line starting from rest from a point O. The acceleration of the particle at time t s after leaving O is $a\,\mathrm{m\,s^{-2}}$, where $a = 4t^{\frac{1}{2}}$.

(a) Find the speed of the particle when $t = 9$. [2]

$V = \int 4t^{\frac{1}{2}}\,dt$

$V = \frac{2}{3} \times 4t^{3/2} + C$

$V = \frac{8}{3}t^{3/2} + C$

$V = 0$ at $t = 0$:

$0 = 0 + C$

$C = 0$

$\rightarrow V = \frac{8}{3}t^{3/2}$

sub. $t = 9$:

$V = \frac{8}{3} \times 9^{3/2}$

$V = \frac{8}{3} \times 27$

$V = 72\,\mathrm{ms^{-1}}$

(b) Find the time after leaving O at which the speed (in metres per second) and the distance travelled (in metres) are numerically equal. [3]

$S = \int \frac{8}{3}t^{3/2}\,dt$

$S = \frac{2}{5} \times \frac{8}{3}t^{5/2} + C$

$S = \frac{16}{15}t^{5/2} + C$

$S = 0$ at $t = 0$:

$0 = 0 + C$

$C = 0$

$\rightarrow S = \frac{16}{15}t^{5/2}$

$S = V$:

$\frac{16}{15}t^{5/2} = \frac{8}{3}t^{3/2}$

$48t^{5/2} = 120t^{3/2}$

$2t^{5/2} = 5t^{3/2}$

$2t^{5/2} - 5t^{3/2} = 0$

$t^{3/2}(2t - 5) = 0$

$t^{3/2} = 0$ or $2t - 5 = 0$

$t = 0$ ✗ $\quad 2t = 5$

$t = 2.5\,\mathrm{s}$ ✓

5 A particle P moves in a straight line. It starts at a point O on the line and at time t s after leaving O it has velocity $v\,\mathrm{m\,s^{-1}}$, where $v = 4t^2 - 20t + 21$.

(a) Find the values of t for which P is at instantaneous rest. [2]

$v = 0$:

$4t^2 - 20t + 21 = 0$

factorise:

$4t^2 - 6t - 14t + 21 = 0$

$2t(2t-3) - 7(2t-3) = 0$

$(2t-7)(2t-3) = 0$

$2t - 7 = 0$ or $2t - 3 = 0$

$2t = 7$ $\quad 2t = 3$

$t = 3.5\,\mathrm{s}$ or $t = 1.5\,\mathrm{s}$

(b) Find the initial acceleration of P. [2]

$a = \frac{dv}{dt} = 8t - 20$

initial acceleration: $t = 0$:

$a = 8(0) - 20$

$= -20\,\mathrm{ms^{-2}}$

(c) Find the minimum velocity of P. [2]

minimum point of v is when $\frac{dv}{dt} = 0$:

$8t - 20 = 0$

$8t = 20$

$t = 2.5$

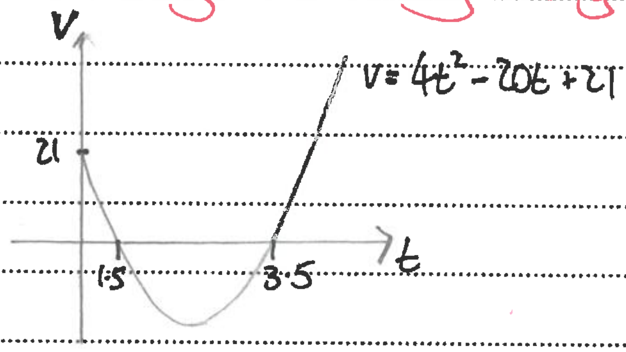
sub into v:

$v = 4(2.5)^2 - 20(2.5) + 21$

$= -4\,\mathrm{ms^{-1}}$

(d) Find the distance travelled by P during the time when its velocity is negative. [4]

It helps to envisage the velocity-time graph:



→ velocity is negative between 1.5s and 3.5s

$S = \int (4t^2 - 20t + 21)\,dt$

$S = \frac{4}{3}t^3 - 10t^2 + 21t + C$

S=0 when t=0:

$0 = 0 - 0 + 0 + C$

$\rightarrow S = \frac{4}{3}t^3 - 10t^2 + 21t$

The turning points of s are at 1.5s and 3.5s, so we don't need to worry about them:

t=1.5:

$S = \frac{4}{3}(1.5)^3 - 10(1.5)^2 + 21(1.5)$

$= 4.5 - 22.5 + 31.5$

$= \underline{13.5\,m}$

t=3.5:

$S = \frac{4}{3}(3.5)^3 - 10(3.5)^2 + 21(3.5)$

$= \frac{343}{6} - 122.5 + 73.5$

$= \underline{\frac{49}{6}\,m}$

→ distance travelled $= 13.5 - \frac{49}{6}$

$= \underline{5.33\,m}$

6 A particle starts from a point O and moves in a straight line. The velocity $v\,\mathrm{m\,s^{-1}}$ of the particle at time t s after leaving O is given by

$$v = k(3t^2 - 2t^3),$$

where k is a constant.

(a) Verify that the particle returns to O when $t = 2$. [4]

$V = 3kt^2 - 2kt^3$

$S = \int (3kt^2 - 2kt^3)\,dt$

$S = kt^3 - \frac{1}{2}kt^4 + C$

$S = 0$ when $t = 0$:

$0 = 0 - 0 + C$

$C = 0$

$\rightarrow S = kt^3 - \frac{1}{2}kt^4$

when particle returns to O, $S = 0$:

$0 = kt^3 - \frac{1}{2}kt^4$

$0 = kt^3(1 - \frac{1}{2}t)$

$t^3 = 0$ or $1 - \frac{1}{2}t = 0$

$t = 0$ $\quad \frac{t}{2} = 1$

$t = 2$

so particle returns to O when $t = 2$.

(b) It is given that the acceleration of the particle is $-13.5\,\mathrm{m\,s^{-2}}$ for the positive value of t at which $v = 0$.

Find k and hence find the total distance travelled in the first two seconds of motion. [6]

$V=0$:

$3kt^2 - 2kt^3 = 0$

$kt^2(3 - 2t) = 0$

$t^2 = 0$ or $3 - 2t = 0$

$t = 0$ ✗ $\quad 2t = 3$

$t = 1.5$ ✓

$a = \frac{dv}{dt} = 6kt - 6kt^2$

$a = -13.5$ when $t = 1.5$:

$6k(1.5) - 6k(1.5)^2 = -13.5$

$9k - 13.5k = -13.5$

$-4.5k = -13.5$

$k = 3$

→ $S = 3t^3 - \frac{3}{2}t^4$

$V=0$ when $t=1.5$, so turning point is at $t = 1.5$, meaning particle travels away from point O for 1.5s, then returns to starting point in the next 0.5s.

→ Find distance travelled in first 1.5s and multiply by 2:

sub. $t = 1.5$:

$S = 3 \times 1.5^3 - \frac{3}{2} \times 1.5^4$

$= 2.53125$

Total distance $= 2 \times 2.53125$

$= 5.0625\,\mathrm{m}$

6 A cyclist starts from rest at a fixed point O and moves in a straight line, before coming to rest k seconds later. The acceleration of the cyclist at time t s after leaving O is $a\,\mathrm{m\,s^{-2}}$, where $a = 2t^{-\frac{1}{2}} - \frac{3}{5}t^{\frac{1}{2}}$ for $0 < t \leqslant k$.

(a) Find the value of k. [4]

$V=0$ when $t=k$

$V = \int\left(2t^{-\frac{1}{2}} - \frac{3}{5}t^{\frac{1}{2}}\right)dt$

$V = 2\times 2t^{\frac{1}{2}} - \frac{2}{3}\times\frac{3}{5}t^{\frac{3}{2}} + C$

$V = 4t^{\frac{1}{2}} - \frac{2}{5}t^{\frac{3}{2}} + C$

$V=0$ at $t=0$:

$0 = 0 - 0 + C$

$C = 0$

$\rightarrow V = 4t^{\frac{1}{2}} - \frac{2}{5}t^{\frac{3}{2}}$

$V=0$:

$0 = 4t^{\frac{1}{2}} - \frac{2}{5}t^{\frac{3}{2}}$

$0 = t^{\frac{1}{2}}\left(4 - \frac{2}{5}t\right)$

$t^{\frac{1}{2}} = 0$ or $4 - \frac{2}{5}t = 0$

$t = 0$ $\quad \frac{2}{5}t = 4$

$2t = 20$

$t = 10$

so $k = 10$

(b) Find the maximum speed of the cyclist. [3]

max. speed is when $a = \frac{dv}{dt} = 0$:

$2t^{-\frac{1}{2}} - \frac{3}{5}t^{\frac{1}{2}} = 0$

$\frac{2}{\sqrt{t}} - \frac{3}{5}\sqrt{t} = 0$

$\frac{2}{\sqrt{t}} = \frac{3}{5}\sqrt{t}$

$2 = \frac{3}{5}t$

$3t = 10$

$t = \frac{10}{3}$

Sub into V:

$V = 4\left(\frac{10}{3}\right)^{\frac{1}{2}} - \frac{2}{5}\left(\frac{10}{3}\right)^{\frac{3}{2}}$

$V = 4.87\,\mathrm{ms^{-1}}$

(c) Find an expression for the displacement from O in terms of t. Hence find the total distance travelled by the cyclist from the time at which she reaches her maximum speed until she comes to rest. [4]

$S = \int \left(4t^{\frac{1}{2}} - \frac{2}{5}t^{\frac{3}{2}}\right) dt$

$S = \frac{2}{3} \times 4t^{\frac{3}{2}} - \frac{2}{5} \times \frac{2}{5}t^{\frac{5}{2}} + C$

$S = \frac{8}{3}t^{\frac{3}{2}} - \frac{4}{25}t^{\frac{5}{2}} + C$

$S = 0$ at $t = 0$, so:

$0 = 0 - 0 + C$

$C = 0$

$\rightarrow S = \frac{8}{3}t^{\frac{3}{2}} - \frac{4}{25}t^{\frac{5}{2}}$

displacement at maximum speed: $t = \frac{10}{3}$

$S = \frac{8}{3}\left(\frac{10}{3}\right)^{\frac{3}{2}} - \frac{4}{25}\left(\frac{10}{3}\right)^{\frac{5}{2}}$

$S = 12.983\text{m}$

displacement when she comes to rest: $t = 10$

$S = \frac{8}{3}(10)^{\frac{3}{2}} - \frac{4}{25}(10)^{\frac{5}{2}}$

$S = 33.731\text{m}$

distance travelled $= 33.731 - 12.983$

$= \underline{20.7\text{m}}$

7 A particle moves in a straight line through the point O. The displacement of the particle from O at time t s is s m, where

$$s = t^2 - 3t + 2 \qquad \text{for } 0 \leqslant t \leqslant 6,$$

$$s = \frac{24}{t} - \frac{t^2}{4} + 25 \qquad \text{for } t \geqslant 6.$$

(a) Find the value of t when the particle is instantaneously at rest during the first 6 seconds of its motion. [2]

$V = \frac{ds}{dt} = 2t - 3$

$V = 0$:

$2t - 3 = 0$

$2t = 3$

$t = 1.5$ s

At $t = 6$, the particle hits a barrier at a point P and rebounds.

(b) Find the velocity with which the particle arrives at P and also the velocity with which the particle leaves P. [3]

Sub. $t=6$ into first expression for v:

$V = 2t - 3$

$= 2(6) - 3$

$= 12 - 3$

$= 9\,\text{ms}^{-1}$ ← arrives at P

find v for $t \geq 6$:

$S = 24t^{-1} - \frac{1}{4}t^2 + 25$

$V = \frac{ds}{dt} = -24t^{-2} - \frac{1}{2}t$

Sub. $t = 6$:

$V = -24(6)^{-2} - \frac{1}{2}(6)$

$= \frac{-24}{36} - 3$

$= -3.67\,\text{ms}^{-1}$ ← leaves P

(c) Find the total distance travelled by the particle in the first 10 seconds of its motion. [5]

Careful: turning point of $s = t^2 - 3t + 2$ is at $t = 1.5$, so particle travels in one direction, then turns at $t = 1.5$ and comes back.

$t = 0$:

$s = 0^2 - 3(0) + 2$

$s = 2$ (initial displacement)

$t = 1.5$:

$s = 1.5^2 - 3(1.5) + 2$

$s = -0.25$

$t = 6$:

$s = 6^2 - 3(6) + 2$

$s = 20$

$t = 10$:

$s = \frac{24}{10} - \frac{10^2}{4} + 25$

$s = 2.4$

so particle goes from: $2m \rightarrow -0.25m \rightarrow 20m \rightarrow 2.4m$

distance ~~travelled~~: $2.25m$ $20.25m$ $17.6m$

Total distance $= 2.25 + 20.25 + 17.6$

$= 40.1m$

6 A particle P moves in a straight line starting from a point O and comes to rest 14 s later. At time t s after leaving O, the velocity v m s^{-1} of P is given by

$$\text{A} \quad v = pt^2 - qt \qquad 0 \leqslant t \leqslant 6,$$
$$\text{B} \quad v = 63 - 4.5t \qquad 6 \leqslant t \leqslant 14,$$

where p and q are positive constants.

The acceleration of P is zero when $t = 2$.

(a) Given that there are no instantaneous changes in velocity, find p and q. [3]

$a = \frac{dv}{dt} = 2pt - q$

$a = 0$ when $t = 2$:

$0 = 2p \times 2 - q$

$0 = 4p - q$ 1

No instantaneous change in velocity so velocity at $t = 6$s must be same in both equations:

A: $V = p \times 6^2 - q \times 6$

$V = 36p - 6q$

B: $V = 63 - 4.5 \times 6$

$V = 36$

$36p - 6q = 36$ $\div 6$

$6p - q = 6$ 2

2 − 1: $2p = 6$

$p = 3$

1: $0 = 4(3) - q$

$0 = 12 - q$

$q = 12$

(b) Sketch the velocity-time graph. [3]

A: $V = 3t^2 - 12t$

quadratic with roots at: $3t^2 - 12t = 0$

$t^2 - 4t = 0$

$t(t-4) = 0$

$t = 0$, $t = 4$

turning point: $V = 3(t^2 - 4t)$

$= 3[(t-2)^2 - 4]$

$= 3(t-2)^2 - 12$

$(2, -12)$

B: $V = 63 - 4.5t$

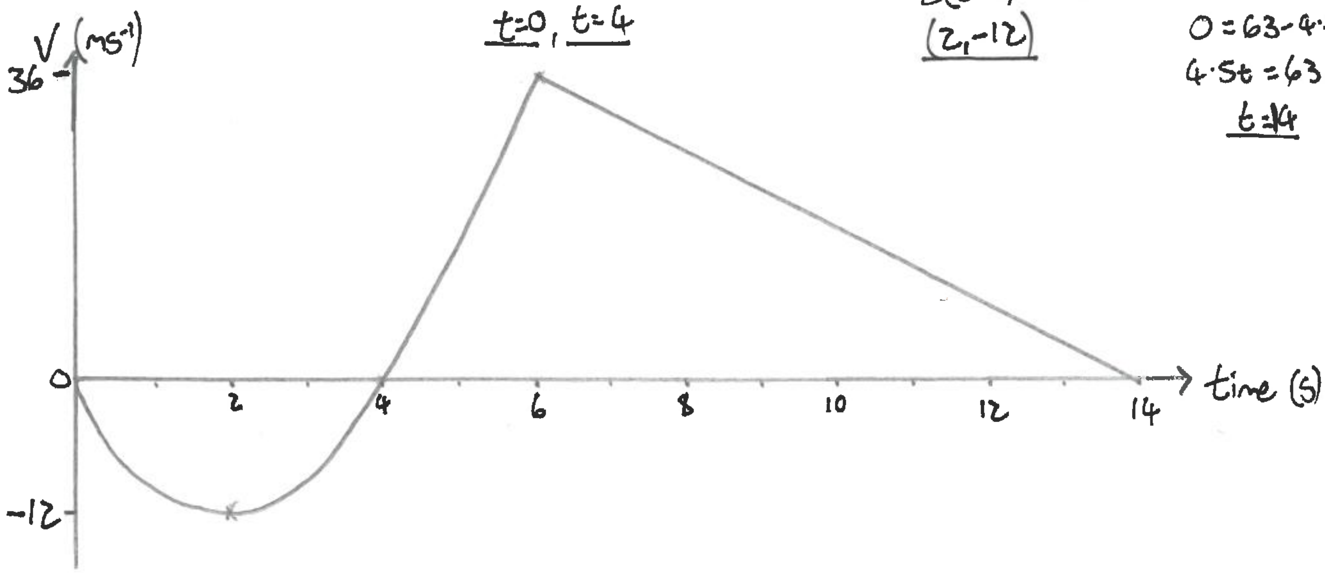
Straight line

$V = 0$:

$0 = 63 - 4.5t$

$4.5t = 63$

$t = 14$



(c) Find the total distance travelled by P during the 14 s. **[5]**

$0 \to 6$s: Need to find displacements between turning points of s. These occur when $v=0$, so at $t=0$, $t=4$, $t=6$:

$0 \to 4$s: $s = \int_0^4 (3t^2 - 12t)\,dt$

$= \left[t^3 - 6t^2\right]_0^4$

$= \left[4^3 - 6(4)^2\right] - \left[0\right] = -32$m

$4 \to 6$s: $s = \int_4^6 (3t^2 - 12t)\,dt$

$= \left[t^3 - 6t^2\right]_4^6$

$= \left[6^3 - 6(6)^2\right] - \left[4^3 - 6(4)^2\right]$

$= \left[0\right] - \left[-32\right] = 32$m

distance travelled between 0 and 6s: $d = 32 + 32 = \underline{64\text{m}}$

$6 \to 14$s: Find area of triangle on v-t graph:

Area $= \frac{1}{2} \times 8 \times 36$

$= 144$m

Total distance $= 64 + 144$

$= \underline{208\text{m}}$

7 A particle P moving in a straight line starts from rest at a point O and comes to rest 16 s later. At time t s after leaving O, the acceleration a m s^{-2} of P is given by

$$a = 6 + 4t \qquad 0 \leqslant t < 2,$$
$$a = 14 \qquad 2 \leqslant t < 4,$$
$$a = 16 - 2t \qquad 4 \leqslant t \leqslant 16.$$

There is no sudden change in velocity at any instant.

(a) Find the values of t when the velocity of P is 55 m s^{-1}. [5]

① v for $0 \leqslant t < 2$:

$v = \int (6+4t)\,dt$

$v = 6t + 2t^2 + C$

$v = 0$ at $t = 0$:

$0 = 0 + 0 + C$

$C = 0$

→ $v = 6t + 2t^2$ ①

Sub. $t = 2$ to find v:

$v = 6(2) + 2(2)^2$

$= 20\text{ms}^{-1}$

② v for $2 \leqslant t < 4$:

$v = \int 14\,dt$

$v = 14t + C$

$v = 20$ when $t = 2$:

$20 = 14(2) + C$

$20 = 28 + C$

$C = -8$

→ $v = 14t - 8$ ②

Sub. $t = 4$ to find v:

$v = 14(4) - 8$

$= 48\text{ms}^{-1}$

③ v for $4 \leqslant t \leqslant 16$

$v = \int (16 - 2t)\,dt$

$v = 16t - t^2 + C$

$v = 48$ when $t = 4$:

$48 = 16(4) - 4^2 + C$

$48 = 48 + C$

$C = 0$

→ $v = 16t - t^2$ ③

Solve ① $= 55\text{ms}^{-1}$:

$2t^2 + 6t = 55$

$2t^2 + 6t - 55 = 0$

$t = \dfrac{-6 \pm \sqrt{6^2 - 4(2)(-55)}}{2(2)}$

$t = 3.95\text{s}$ ✗, $t = -6.95\text{s}$ ✗

outside $0 \leqslant t < 2$

Solve ② $= 55\text{ms}^{-1}$:

$14t - 8 = 55$

$14t = 63$

$t = 4.5\text{s}$ ✗ outside $2 \leqslant t < 4$

Continued →

Solve ③ $= 55 ms^{-1}$:

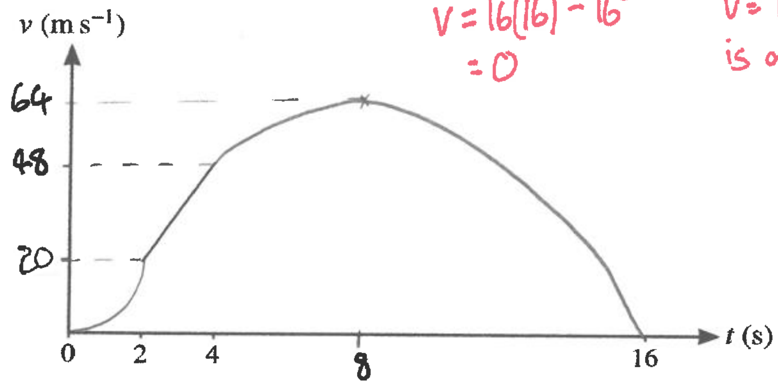
$16t - t^2 = 55$

$t^2 - 16t + 55 = 0$

$(t-5)(t-11) = 0$

$t = 5s$ or $t = 11s$

(b) Complete the sketch of the velocity-time diagram. [2]



$t=16$:
$v = 16(16) - 16^2$
$= 0$

turning point of
$v = 16t - t^2$
is at $(8, 64)$

(c) Find the distance travelled by P when it is decelerating. [3]

P is decelerating between $t=8$ and $t=16$.

$$s = \int_8^{16} (16t - t^2)\,dt$$

$$= \left[8t^2 - \frac{1}{3}t^3\right]_8^{16}$$

$$= \left[8(16)^2 - \frac{1}{3}(16)^3\right] - \left[8(8)^2 - \frac{1}{3}(8)^3\right]$$

$$= \frac{2048}{3} - \frac{1024}{3}$$

$$= \frac{1024}{3} \text{ m}$$