1 A particle P is projected vertically upwards with speed $v\,\mathrm{m\,s^{-1}}$ from a point on the ground. P reaches its greatest height after 3 s.

(a) Find v. [1]

↑+ $s =$ | $v = u + at$

$u =$ | $0 = u + (-10) \times 3$

$v = 0$ | $0 = u - 30$

$a = -10$ | $u = \underline{30\,\mathrm{ms^{-1}}}$

$t = 3$

(b) Find the greatest height of P above the ground. [2]

↑+ $s =$ | $s = \frac{1}{2}(u + v) \times t$

$u = 30$ | $= \frac{1}{2}(30 + 0) \times 3$

$v = 0$ | $= \frac{1}{2} \times 30 \times 3$

$a = -10$ | $= \underline{45\,\mathrm{m}}$

$t = 3$

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$a = -10$ | $= 45\,\mathrm{m}$
$t = 3$

2 A particle P is projected vertically upwards from horizontal ground with speed $u\,\mathrm{m\,s^{-1}}$. P reaches a maximum height of 20 m above the ground.

(a) Find the value of u. [2]

$\uparrow +$ $s = 20$ | $v^2 = u^2 + 2as$

$u =$ | $0^2 = u^2 + 2(-10)(20)$

$v = 0$ | $0 = u^2 - 400$

$a = -10$ | $u^2 = 400$

$t =$ | $u = \underline{\underline{20\ \mathrm{ms^{-1}}}}$

(b) Find the total time for which P is at least 15 m above the ground. [3]

$\uparrow +$ $s = 15$ | $s = ut + \frac{1}{2}at^2$

$u = 20$ | $15 = 20t + \frac{1}{2}(-10)\times t^2$

$v =$ | $15 = 20t - 5t^2$

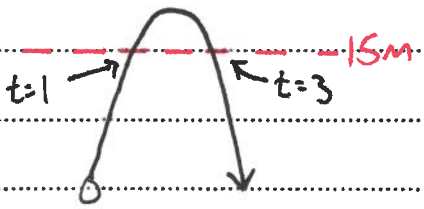
$a = -10$ | $5t^2 - 20t + 15 = 0$

$t =$ | $t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 3$ or $t = 1$

so P is 15m above ground at $t=1$ and $t=3$



so it is above 15m for :

$3 - 1 = \underline{\underline{2\ \text{seconds}}}$

2 A particle P is projected vertically upwards from horizontal ground with speed $15\,\text{m}\,\text{s}^{-1}$.

(a) Find the speed of P when it is 10 m above the ground. [2]

$\uparrow +$ $s = 10$, $u = 15$, $v =$, $a = -10$, $t =$

$v^2 = u^2 + 2as$

$v^2 = 15^2 + 2(-10)(10)$

$v^2 = 225 - 200$

$v^2 = 25$

$v = \underline{5\,\text{ms}^{-1}}$

At the same instant that P is projected, a second particle Q is dropped from a height of 18 m above the ground in the same vertical line as P.

(b) Find the height above the ground at which the two particles collide. [3]

P: $\uparrow +$ $s =$, $u = 15$, $v =$, $a = -10$, $t =$

$S_P = ut + \frac{1}{2}at^2$

$= 15t + \frac{1}{2}(-10) \times t^2$

$S_P = 15t - 5t^2$

Q: $\downarrow +$ $s =$, $u = 0$, $v =$, $a = 10$, $t =$

$S_Q = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2}(10) \times t^2$

$S_Q = 5t^2$

$S_P + S_Q = 18$:

$15t - 5t^2 + 5t^2 = 18$

$15t = 18$

$t = 18/15$

$= 1.2\text{s}$

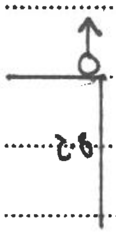
Sub into S_P :

$S_P = 15(1.2) - 5(1.2)^2$

$= \underline{10.8\text{m}}$

3 A particle P is projected vertically upwards with speed $5\,\text{m}\,\text{s}^{-1}$ from a point A which is 2.8 m above horizontal ground.

(a) Find the greatest height above the ground reached by P. [3]



$\uparrow +$ $s =$ | $v^2 = u^2 + 2as$

$u = 5$ | $0^2 = 5^2 + 2(-10) \times s$

$v = 0$ | $0 = 25 - 20s$

$a = -10$ | $20s = 25$

$t =$ | $s = 1.25\,\text{m}$

height above ground $= 2.8 + 1.25$

$= \underline{4.05\,\text{m}}$

(b) Find the length of time for which P is at a height of more than 3.6 m above the ground. [4]

3.6m above ground = 0.8m above starting point:

$\uparrow +$ $s = 0.8$ | $s = ut + \frac{1}{2}at^2$

$u = 5$ | $0.8 = 5t + \frac{1}{2} \times (-10) \times t^2$

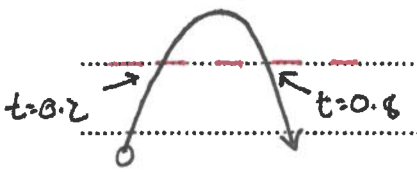
$v =$ | $0.8 = 5t - 5t^2$

$a = -10$ | $5t^2 - 5t + 0.8 = 0$

$t =$ | $t = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(5)(0.8)}}{2(5)}$

$t = 0.8$ or $t = 0.2$

so P is 3.6m above ground at $t = 0.2$ and $t = 0.8$



so it is above 3.6m for:

$0.8 - 0.2 = \underline{0.6\,\text{s}}$

4 A particle is projected vertically upwards with speed $u\,\mathrm{m\,s^{-1}}$ from a point on horizontal ground. After 2 seconds, the height of the particle above the ground is 24 m.

(a) Show that $u = 22$. [2]

$\uparrow +$ $s = 24$ | $s = ut + \frac{1}{2}at^2$

$u =$ | $24 = 2u + \frac{1}{2}(-10) \times 2^2$

$v =$ | $24 = 2u - 20$

$a = -10$ | $44 = 2u$

$t = 2$ | $u = 22\,\mathrm{ms^{-1}}$ QED

(b) The height of the particle above the ground is more than h m for a period of 3.6 s.

Find h. [4]

Find maximum height:

$\uparrow +$ $s =$ | $v^2 = u^2 + 2as$

$u = 22$ | $0^2 = 22^2 + 2(-10) \times s$

$v = 0$ | $0 = 484 - 20s$

$a = -10$ | $20s = 484$

$t =$ | $s = 24.2\,\mathrm{m}$

Now find height 1.8s later, on the way down:

$3.6 \div 2$

$\downarrow +$ $s =$

$u = 0$ | $s = ut + \frac{1}{2}at^2$

$v =$ | $= 0 + \frac{1}{2} \times 10 \times 1.8^2$

$a = 10$ | $= 16.2\,\mathrm{m}$

$t = 1.8$

$\rightarrow$ Height above ground $= 24.2 - 16.2$

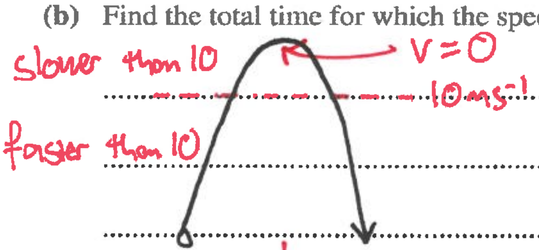
$= 8\,\mathrm{m}$

2 A particle P is projected vertically upwards from horizontal ground. P reaches a maximum height of 45 m. After reaching the ground, P comes to rest without rebounding.

(a) Find the speed at which P was projected. [2]

$\uparrow +$ $s = 45$	$v^2 = u^2 + 2as$
$u =$	$0^2 = u^2 + 2(-10)\times 45$
$v = 0$	$0 = u^2 - 900$
$a = -10$	$u^2 = 900$
$t =$	$u = 30\text{ms}^{-1}$

(b) Find the total time for which the speed of P is at least $10\,\text{m s}^{-1}$. [3]



$s =$	$v = u + at$	$s =$	$v = u + at$
$u = 30$	$10 = 30 + (-10)\times t$	$u = 30$	$-10 = 30 + (-10)t$
$v = 10$	$10 = 30 - 10t$	$v = -10$	$-10 = 30 - 10t$
$a = -10$	$10t = 20$	$a = -10$	$10t = 40$
$t =$	$t = 2$	$t =$	$t = 4$

so $v = 10\text{ms}^{-1}$ at $t = 2\text{s}$ and $t = 4\text{s}$

So it must be faster than 10ms^{-1} between 0 - 2s on the way up and 4 - 6s on the way down, so 4s in total.

5 A particle is projected vertically upwards with speed $40\,\mathrm{m\,s^{-1}}$ alongside a building of height h m.

(a) Given that the particle is above the level of the top of the building for 4 s, find h. [4]

Find maximum height of particle:

↑+ $s =$ | $v^2 = u^2 + 2as$
$u = 40$ | $0^2 = 40^2 + 2(-10) \times s$
$v = 0$ | $0 = 1600 - 20s$
$a = -10$ | $20s = 1600$
$t =$ | $s = 80\,\mathrm{m}$

Find height 2s later, on the way down:
← 4 ÷ 2

↓+ $s =$ | $s = ut + \frac{1}{2}at^2$
$u = 0$ | $s = 0 + \frac{1}{2}(10) \times 2^2$
$v =$ | $s = 5 \times 4$
$a = 10$ | $= 20$
$t = 2$

height above ground $= 80 - 20$
$= \underline{\underline{60\,\mathrm{m}}}$

(b) One second after the first particle is projected, a second particle is projected vertically upwards from the top of the building with speed $20\,\text{m}\,\text{s}^{-1}$.

Denoting the time after projection of the first particle by t s, find the value of t for which the two particles are at the same height above the ground. [4]

First particle (from ground):

$\uparrow +$ $s =$ | $s = ut + \frac{1}{2}at^2$

$u = 40$ | $= 40t + \frac{1}{2}(-10) \times t^2$

$v =$ | $= 40t - 5t^2$

$a = -10$

$t =$ | $\underline{s_1 = 40t - 5t^2}$

Second particle (from top of building):

$\uparrow +$ $s =$ | $s = ut + \frac{1}{2}at^2$

$u = 20$ | $= 20(t-1) + \frac{1}{2}(-10)(t-1)^2$

$v =$ | $= 20t - 20 - 5(t^2 - 2t + 1)$

$a = -10$ | $= 20t - 20 - 5t^2 + 10t - 5$

$t = t-1$ | $\underline{s = -5t^2 + 30t - 25}$

↑ has been travelling for 1s less than first particle

height of particle 2 above ground $= s + 60$:

$$\underline{s_2 = -5t^2 + 30t + 35}$$

$s_1 = s_2$:

$$40t - 5t^2 = -5t^2 + 30t + 35$$

$$40t = 30t + 35$$

$$10t = 35$$

$$\underline{\underline{t = 3.5\text{s}}}$$