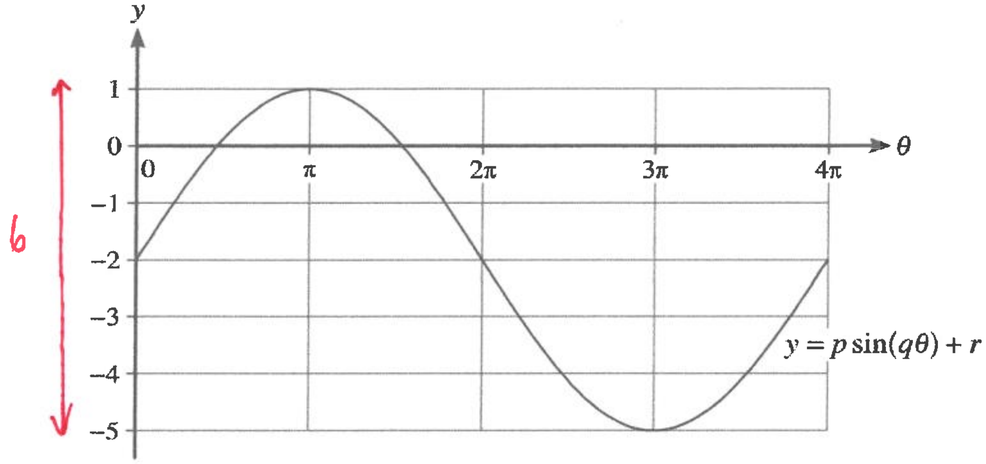
2



The diagram shows part of the curve with equation $y = p\sin(q\theta) + r$, where p, q and r are constants.

(a) State the value of p. [1]

Amplitude is 6, where it is usually 2, so $p = 3$

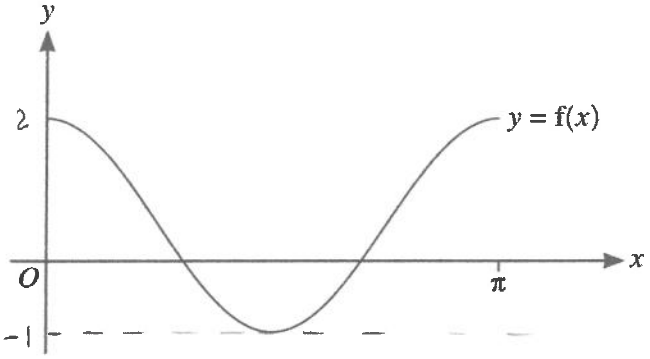
(b) State the value of q. [1]

Usually, one period is 2π, but here it is 4π, so $q = \frac{1}{2}$

(c) State the value of r. [1]

Graph starts at $(0,-2)$ instead of $(0,0)$, so $r = -2$

4



The diagram shows the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$.

(a) State the range of f. [2]

Amplitude has been multiplied by $\frac{3}{2}$, so $-\frac{3}{2} \leq y \leq \frac{3}{2}$, but then we've added $\frac{1}{2}$, so $-1 \leq y \leq 2$

A function g is such that $\mathrm{g}(x) = \mathrm{f}(x) + k$, where k is a positive constant. The x-axis is a tangent to the curve $y = \mathrm{g}(x)$.

(b) State the value of k and hence describe fully the transformation that maps the curve $y = \mathrm{f}(x)$ on to $y = \mathrm{g}(x)$. [2]

If the x-axis is a tangent, the curve must have been shifted up by 1, so $k = 1$ and translation by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

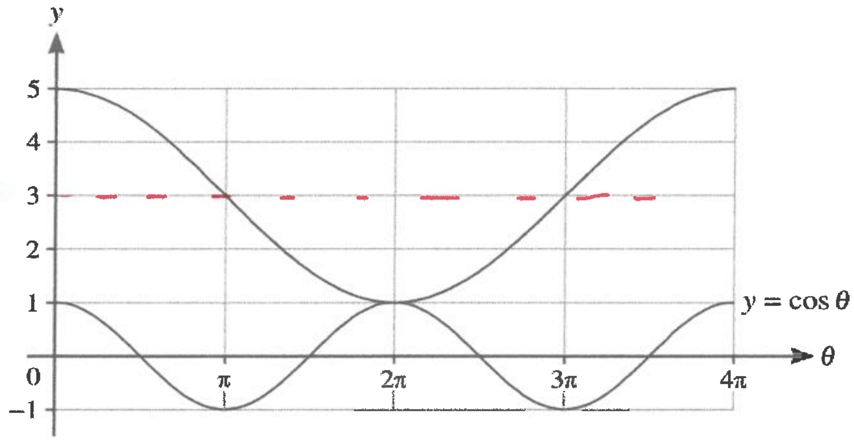
(c) State the equation of the curve which is the reflection of $y = \mathrm{f}(x)$ in the x-axis. Give your answer in the form $y = a\cos 2x + b$, where a and b are constants. [1]

Reflection in x-axis: $f(x) \to -f(x)$:

$y = -\left[\frac{3}{2}\cos 2x + \frac{1}{2}\right]$

$y = -\frac{3}{2}\cos 2x - \frac{1}{2}$

4



In the diagram, the lower curve has equation $y = \cos\theta$. The upper curve shows the result of applying a combination of transformations to $y = \cos\theta$.

Find, in terms of a cosine function, the equation of the upper curve. [3]

① period has doubled, so $f(x) \to f(\frac{1}{2}x)$

② amplitude has also doubled so $f(\frac{1}{2}x) \to 2f(\frac{1}{2}x)$

③ the "middle" of the graph (vertically) has shifted from 0 to 3, so $2f(\frac{1}{2}x) + 3$

$\to f(\theta) = \cos\theta$

$f(\frac{1}{2}\theta) = \cos(\frac{1}{2}\theta)$

$2f(\frac{1}{2}\theta) = 2\cos(\frac{1}{2}\theta)$

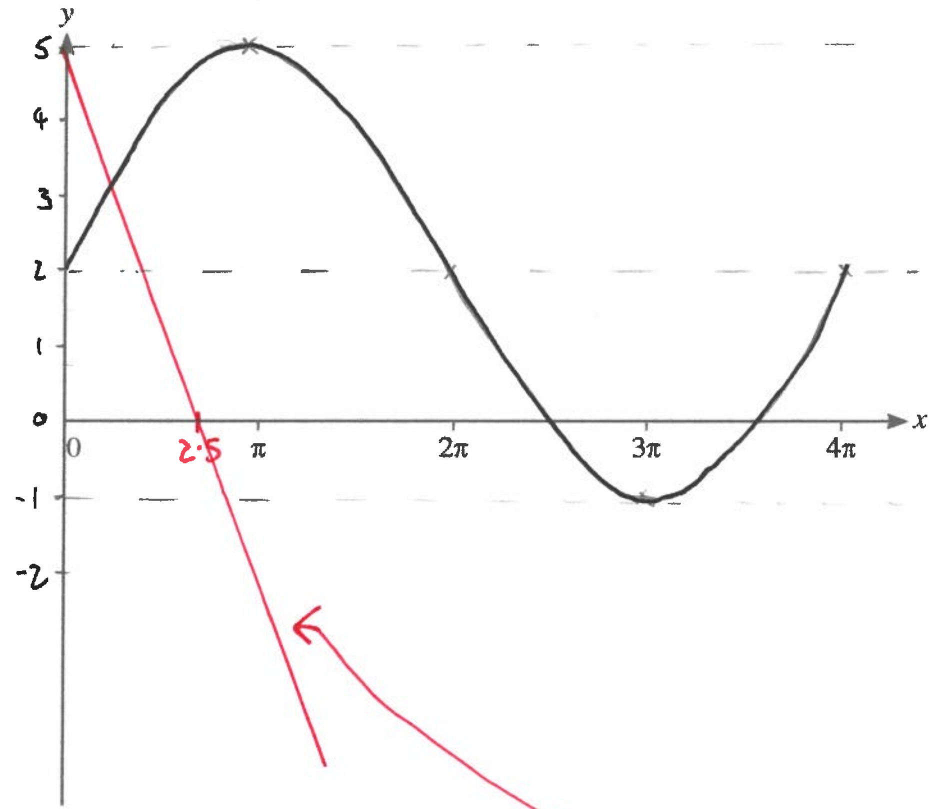
$2f(\frac{1}{2}\theta) + 3 = 2\cos(\frac{1}{2}\theta) + 3$

7 A curve has equation $y = 2 + 3\sin\frac{1}{2}x$ for $0 \leqslant x \leqslant 4\pi$.

(a) State greatest and least values of y. [2]

Amplitude has been multiplied by 3, so $-3 \leq y \leq 3$, but then we've added 2, so $-1 \leq y \leq 5$

(b) Sketch the curve. [2]



(c) State the number of solutions of the equation

$$2 + 3\sin\tfrac{1}{2}x = 5 - 2x$$

for $0 \leqslant x \leqslant 4\pi$. [1]

$y = 5 - 2x$: y intercept: $(0, 5)$

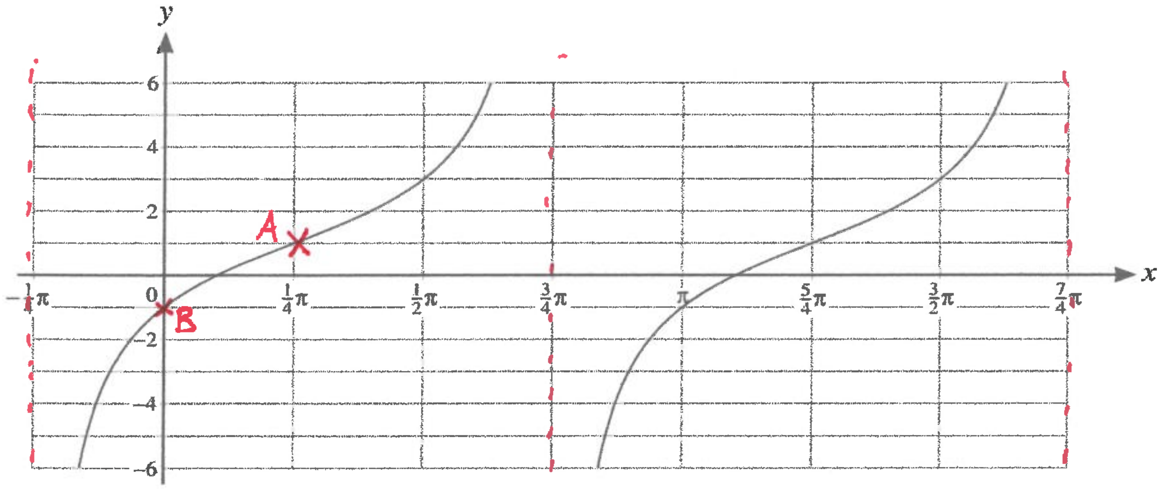
x intercept: $0 = 5 - 2x$

$2x = 5$

$x = \frac{5}{2}$

$\therefore$ Only one solution.

4



The diagram shows part of the graph of $y = a\tan(x - b) + c$.

Given that $0 < b < \pi$, state the values of the constants a, b and c. [3]

Using the position of the asymptotes, we can see the graph has been shifted right by $\frac{\pi}{4}$, so $b = \frac{\pi}{4}$

Then, using point A on the graph, we can see that this is the middle of one period and should be on the x-axis, so it has been shifted up by 1. This means $c = 1$

Now we can use a coordinate to find the final value, e.g. B: $(0, -1)$:

$$y = a\tan\left(x - \frac{\pi}{4}\right) + 1$$
$$-1 = a\tan\left(0 - \frac{\pi}{4}\right) + 1$$
$$-1 = a \times (-1) + 1$$
$$-2 = -a$$
$$a = 2$$

8 **(a)** The curve $y = \sin x$ is transformed to the curve $y = 4\sin(\frac{1}{2}x - 30°)$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied. [5]

!!! Two transformations to x-coords, so the order is not what you might expect:

① Translation by $\begin{pmatrix} 30 \\ 0 \end{pmatrix}$

② Stretch parallel to x-axis, scale factor 2

③ Stretch parallel to y-axis, scale factor 4

9 Functions f and g are such that

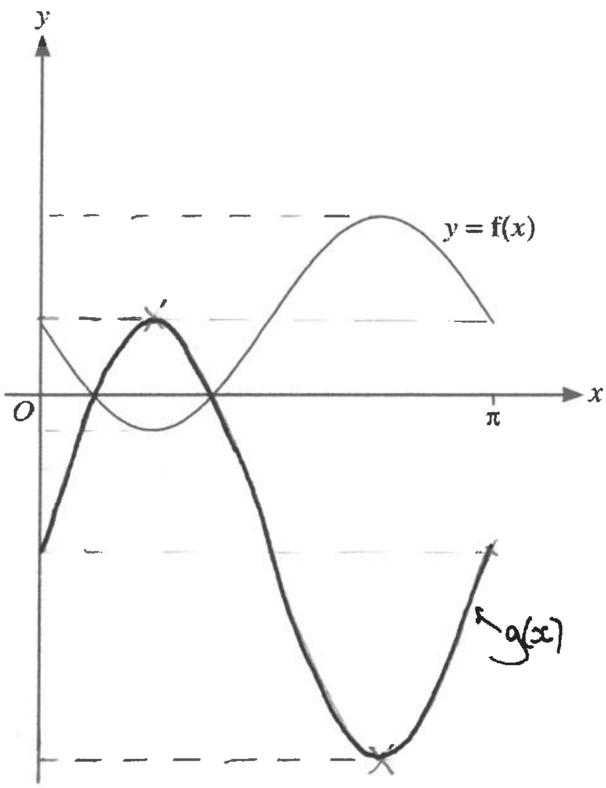
$$f(x) = 2 - 3\sin 2x \quad \text{for } 0 \leqslant x \leqslant \pi,$$
$$g(x) = -2f(x) \quad \text{for } 0 \leqslant x \leqslant \pi.$$

(a) State the ranges of f and g. [3]

f(x): Amplitude has been multiplied by -3, so $-3 \leq y \leq 3$, but then we've added 2, so: $-1 \leq f(x) \leq 5$

g(x): f(x) has been multiplied by -2, so: $-10 \leq g(x) \leq 2$

The diagram below shows the graph of $y = f(x)$.



(b) Sketch, on this diagram, the graph of $y = g(x)$. [2]

The function h is such that

$$h(x) = g(x + \pi) \quad \text{for } -\pi \leqslant x \leqslant 0.$$

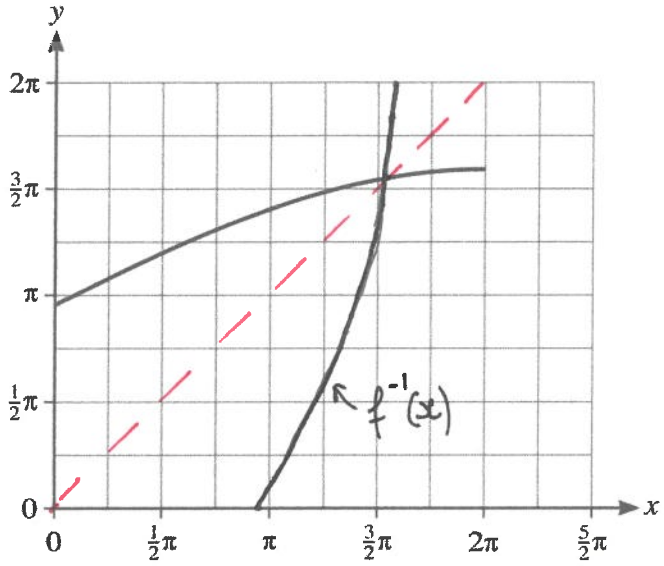
(c) Describe fully a sequence of transformations that maps the curve $y = f(x)$ on to $y = h(x)$. [3]

$f(x) \rightarrow g(x) \rightarrow h(x)$: $f(x) \rightarrow -2f(x) \rightarrow -2f(x+\pi)$

① Translation by $\begin{pmatrix} -\pi \\ 0 \end{pmatrix}$ ② Stretch parallel to y-axis, scale factor 2. ③ Reflection in x-axis.

NB: error in mark-scheme

8



The diagram shows the graph of $y = \mathrm{f}(x)$ where the function f is defined by

$$\mathrm{f}(x) = 3 + 2\sin\tfrac{1}{4}x \text{ for } 0 \leqslant x \leqslant 2\pi.$$

(a) On the diagram above, sketch the graph of $y = \mathrm{f}^{-1}(x)$. **[2]**

reflect in $y=x$

(b) Find an expression for $\mathrm{f}^{-1}(x)$. **[2]**

$y = 3 + 2\sin\left(\frac{x}{4}\right)$

$y - 3 = 2\sin\left(\frac{x}{4}\right)$

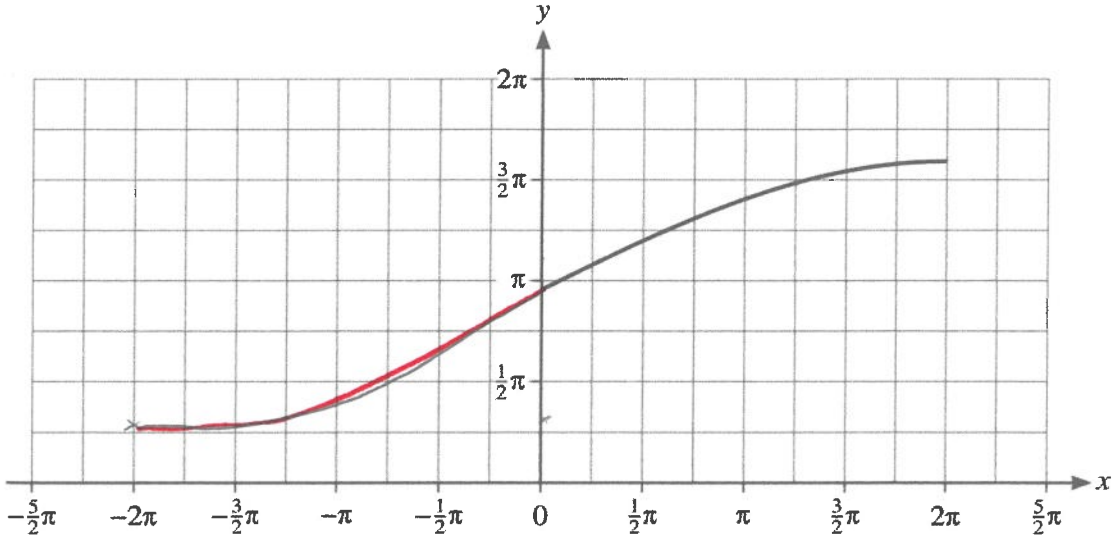
$\frac{y-3}{2} = \sin\left(\frac{x}{4}\right)$

$\sin^{-1}\left(\frac{y-3}{2}\right) = \frac{x}{4}$

$x = 4\sin^{-1}\left(\frac{y-3}{2}\right)$

$f^{-1}(x) = 4\sin^{-1}\left(\frac{x-3}{2}\right)$

(c)



The diagram above shows part of the graph of the function $g(x) = 3 + 2\sin\frac{1}{4}x$ for $-2\pi \leqslant x \leqslant 2\pi$.

Complete the sketch of the graph of $g(x)$ on the diagram above and hence explain whether the function g has an inverse. [2]

Yes, it has an inverse because it is one-to-one.

(d) Describe fully a sequence of three transformations which can be combined to transform the graph of $y = \sin x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ to the graph of $y = f(x)$, making clear the order in which the transformations are applied. [6]

$\sin x \rightarrow 3 + 2\sin(\frac{1}{4}x)$:

$f(x) \rightarrow f(\frac{1}{4}x) \rightarrow 2f(\frac{1}{4}x) \rightarrow 2f(\frac{1}{4}x) + 3$

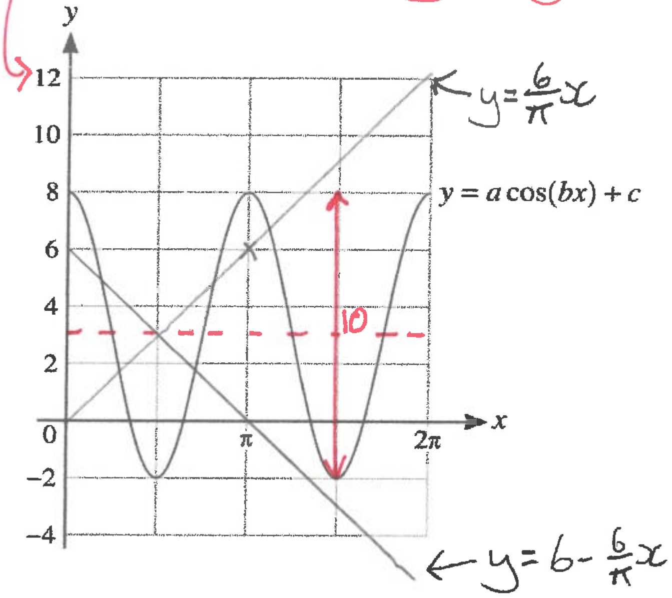
1. Stretch parallel to x-axis, scale factor 4.

2. Stretch parallel to y-axis, scale factor 2.

3. Translation by $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

5

careful with numbering on y-axis!



The diagram shows part of the graph of $y = a\cos(bx) + c$.

(a) Find the values of the positive integers a, b and c. [3]

Amplitude is 10, where it is usually 2, so $a = 5$

Period is π, where it is usually 2π, so $b = 2$

"middle" of graph vertically is 3, where it is usually 0, so $c = 3$

(b) For these values of a, b and c, use the given diagram to determine the number of solutions in the interval $0 \leqslant x \leqslant 2\pi$ for each of the following equations.

(i) $a\cos(bx) + c = \frac{6}{\pi}x$ → $y = \frac{6}{\pi}x$ [1]

$y = \frac{6}{\pi}x$ passes through the points $(0,0)$ and $(\pi, 6)$ → see diagram. Crosses in 3 places, so 3 solutions.

(ii) $a\cos(bx) + c = 6 - \frac{6}{\pi}x$ → $y = 6 - \frac{6}{\pi}x$ [1]

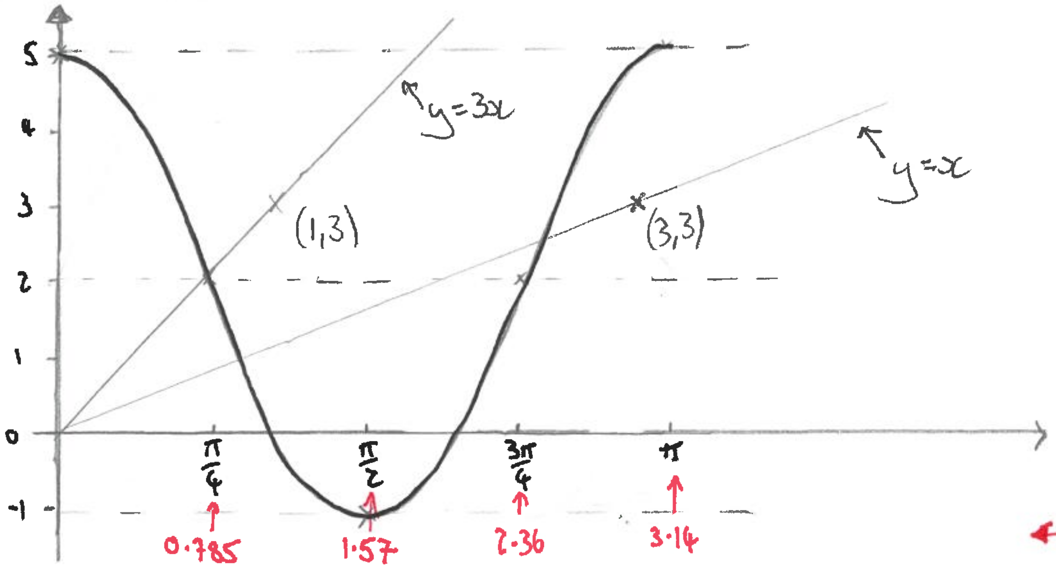
$y = 6 - \frac{6}{\pi}x$ passes through $(0,6)$ and $(\pi, 0)$ → see diagram. Crosses in 2 places, so 2 solutions.

11 A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

(a) State the greatest and least values of y. [2]

Amplitude has been multiplied by 3, so $-3 \leqslant y \leqslant 3$, but then we've added 2, so : $-1 \leqslant y \leqslant 5$

(b) Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]



(c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

(i) $k = -3$ [1]

No solutions (goes through (0,0) with negative gradient and misses graph.

helpful to write numerical values on x-axis for this

(ii) $k = 1$ [1]

2 solutions : See $y=x$ on diagram

(iii) $k = 3$ [1]

1 solution : See $y=3x$ on diagram

Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$f(x) = 3\cos 2x + 2,$$
$$g(x) = f(2x) + 4,$$
$$h(x) = 2f(x + \tfrac{1}{2}\pi).$$

(d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]

$g(x) = f(2x) + 4$

$\rightarrow \quad f(x) \rightarrow f(2x) \rightarrow f(2x) + 4$

① Stretch parallel to x-axis, scale factor $\frac{1}{2}$

② Translation by $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

(e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

$h(x) = 2f(x + \frac{\pi}{2})$

$\rightarrow \quad f(x) \rightarrow f(x + \frac{\pi}{2}) \rightarrow 2f(x + \frac{\pi}{2})$

① Translation by $\begin{pmatrix} -\frac{\pi}{2} \\ 0 \end{pmatrix}$

② Stretch parallel to y-axis, scale factor 2