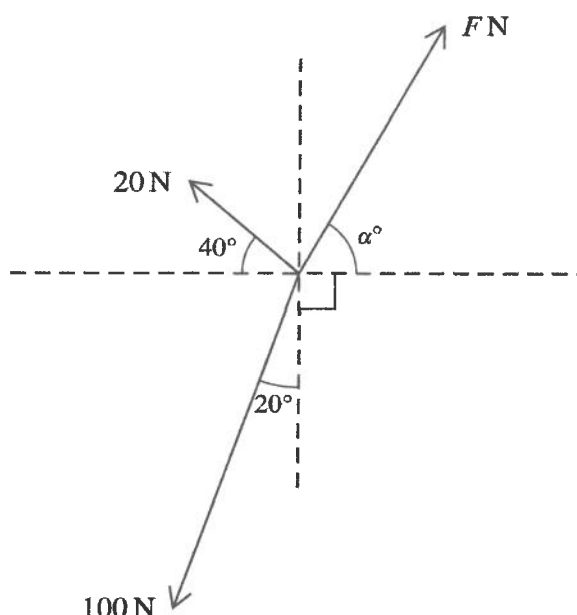




Three coplanar forces of magnitudes 20 N, 100 N and F N act at a point. The directions of these forces are shown in the diagram.

Given that the three forces are in equilibrium, find F and α .

[6]

$$R(\uparrow): F \sin \alpha + 20 \sin 40 - 100 \cos 20 = 0$$

$$F \sin \alpha = 100 \cos 20 - 20 \sin 40$$

$$F \sin \alpha = 81.114 \text{ STO } \textcircled{1}$$

$$R(\rightarrow): F \cos \alpha - 20 \cos 40 - 100 \sin 20 = 0$$

$$F \cos \alpha = 100 \sin 20 + 20 \cos 40$$

$$F \cos \alpha = 49.523 \text{ STO } \textcircled{2}$$

$$\textcircled{1} \div \textcircled{2}: \frac{F \sin \alpha}{F \cos \alpha} = \frac{81.114}{49.523}$$

$$\tan \alpha = 1.638$$

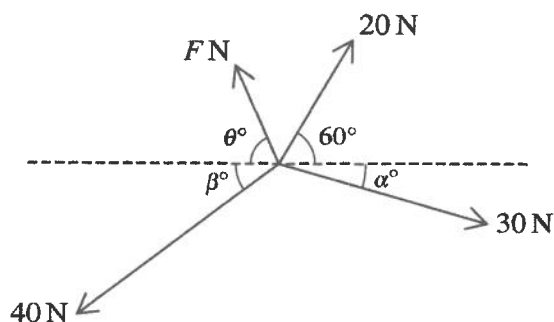
$$\alpha = 58.6^\circ$$

$$\text{sub into } \textcircled{1}: F \sin(58.6) = 81.114$$

$$F = \frac{81.114}{\sin(58.6)}$$

$$= 95.0 \text{ N}$$

3



Four coplanar forces act at a point. The magnitudes of the forces are 20 N, 30 N, 40 N and F N. The directions of the forces are as shown in the diagram, where $\sin \alpha^\circ = 0.28$ and $\sin \beta^\circ = 0.6$.

Given that the forces are in equilibrium, find F and θ .

$$\alpha = 16.26^\circ \text{ STO} \quad \beta = 36.87^\circ \text{ STO} \quad [6]$$

$$R(\uparrow): F \sin \theta + 20 \sin 60 - 40 \sin \beta - 30 \sin \alpha = 0$$

$$F \sin \theta + 20 \sin 60 - 40 \times 0.6 - 30 \times 0.28 = 0$$

$$F \sin \theta + 20 \sin 60 - 24 - 8.4 = 0$$

$$F \sin \theta = 32.4 - 20 \sin 60 \quad (1)$$

$$R(\rightarrow): 20 \cos 60 + 30 \cos \alpha - F \cos \theta - 40 \cos \beta = 0$$

$$20 \cos 60 + 30 \cos(16.26) - F \cos \theta - 40 \cos(36.87) = 0$$

$$F \cos \theta = 20 \cos 60 + 30 \cos(16.26) - 40 \cos(36.87)$$

$$F \cos \theta = 6.8 \quad (2)$$

$$(1) \div (2): \frac{F \sin \theta}{F \cos \theta} = \frac{32.4 - 20 \sin 60}{6.8}$$

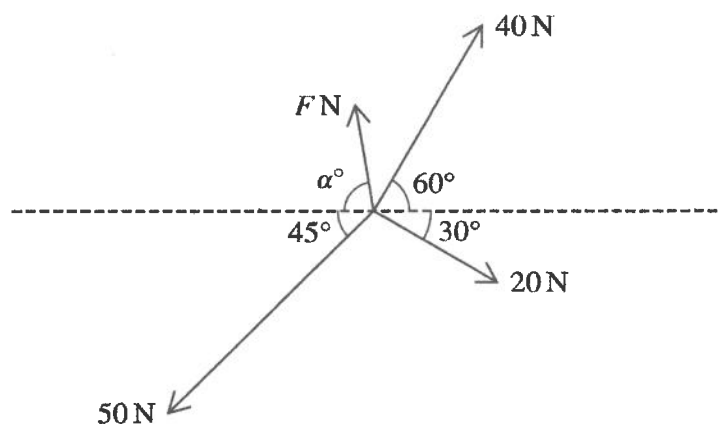
$$\tan \theta = 2.218$$

$$\theta = \underline{\underline{65.7^\circ \text{ STO}}}$$

$$\text{Sub into (2): } F \cos(65.7) = 6.8$$

$$F = \frac{6.8}{\cos(65.7)}$$

$$F = \underline{\underline{16.5 \text{ N}}}$$



Four coplanar forces of magnitudes 40 N, 20 N, 50 N and F N act at a point in the directions shown in the diagram. The four forces are in equilibrium.

Find F and α .

[6]

$$R(\uparrow): 40 \sin 60 + F \sin \alpha - 20 \sin 30 - 50 \sin 45 = 0$$

$$F \sin \alpha = 20 \sin 30 + 50 \sin 45 - 40 \sin 60$$

$$F \sin \alpha = 10.714 \text{ STO } \textcircled{1}$$

$$R(\rightarrow): 40 \cos 60 + 20 \cos 30 - F \cos \alpha - 50 \cos 45 = 0$$

$$F \cos \alpha = 40 \cos 60 + 20 \cos 30 - 50 \cos 45$$

$$F \cos \alpha = 1.965 \text{ STO } \textcircled{2}$$

$$\textcircled{1} \div \textcircled{2}: \frac{F \sin \alpha}{F \cos \alpha} = \frac{10.714}{1.965}$$

$$\tan \alpha = 5.452 \dots$$

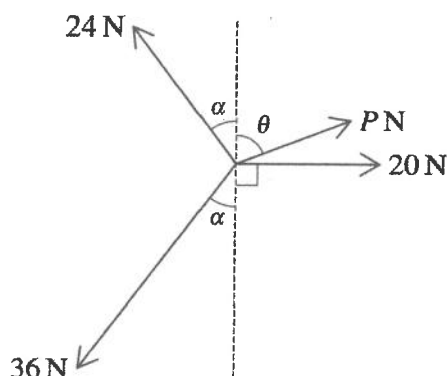
$$\alpha = \underline{79.6^\circ} \text{ STO}$$

$$\text{Sub into } \textcircled{1}: F \sin(79.6) = 10.714$$

$$F = \frac{10.714}{\sin(79.6)}$$

$$= \underline{10.9 \text{ N}}$$

3



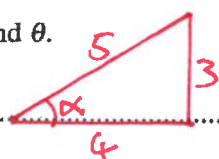
Coplanar forces of magnitudes 24 N, P N, 20 N and 36 N act at a point in the directions shown in the diagram. The system is in equilibrium.

Given that $\sin \alpha = \frac{3}{5}$, find the values of P and θ .

$$\cos \alpha = \frac{4}{5}$$

[6]

$$\sin \alpha = \frac{3}{5} \rightarrow$$



$$\tan \alpha = \frac{3}{4}$$

$$R(\uparrow): 24 \cos \alpha + P \cos \theta - 36 \cos \alpha = 0$$

$$24 \times \frac{4}{5} + P \cos \theta - 36 \times \frac{4}{5} = 0$$

$$19.2 + P \cos \theta - 28.8 = 0$$

$$P \cos \theta - 9.6 = 0$$

$$P \cos \theta = 9.6 \quad (1)$$

$$R(\rightarrow): 20 + P \sin \theta - 24 \sin \alpha - 36 \sin \alpha = 0$$

$$20 + P \sin \theta - 24 \times \frac{3}{5} - 36 \times \frac{3}{5} = 0$$

$$20 + P \sin \theta - 14.4 - 21.6 = 0$$

$$P \sin \theta - 16 = 0$$

$$P \sin \theta = 16 \quad (2)$$

$$(2) \div (1): \frac{P \sin \theta}{P \cos \theta} = \frac{16}{9.6}$$

$$\tan \theta = \frac{5}{3}$$

$$\theta = 59.0^\circ$$

STO

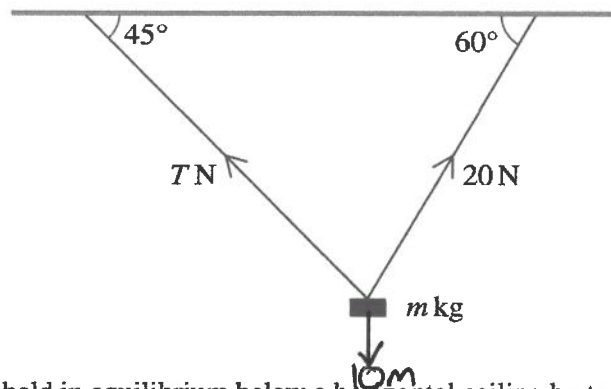
Sub into (1):

$$P \cos(59.0) = 9.6$$

$$P = \frac{9.6}{\cos(59.0)}$$

$$= 18.7 \text{ N}$$

3



A block of mass m kg is held in equilibrium below a horizontal ceiling by two strings, as shown in the diagram. One of the strings is inclined at 45° to the horizontal and the tension in this string is T N. The other string is inclined at 60° to the horizontal and the tension in this string is 20 N.

Find T and m .

[5]

$$R(\rightarrow): 20 \cos 60 - T \cos 45 = 0$$

$$10 - T \cos 45 = 0$$

$$T \cos 45 = 10$$

$$T = \underline{\underline{10\sqrt{2} \text{ N}}}$$

$$R(\uparrow): T \sin 45 + 20 \sin 60 - 10m = 0$$

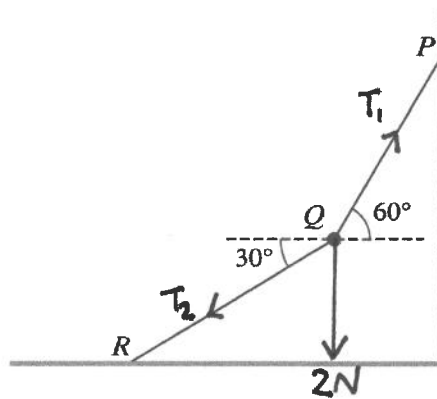
$$10\sqrt{2} \sin 45 + 10\sqrt{3} - 10m = 0$$

$$10 + 10\sqrt{3} - 10m = 0$$

$$10m = 10 + 10\sqrt{3}$$

$$m = 1 + \sqrt{3}$$

$$= \underline{\underline{2.73 \text{ kg}}}$$



A particle Q of mass 0.2 kg is held in equilibrium by two light inextensible strings PQ and QR . P is a fixed point on a vertical wall and R is a fixed point on a horizontal floor. The angles which strings PQ and QR make with the horizontal are 60° and 30° respectively (see diagram).

Find the tensions in the two strings.

[5]

$$R(\rightarrow): T_1 \cos 60 - T_2 \cos 30 = 0$$

$$T_1 \cos 60 = T_2 \cos 30$$

$$T_1 = \frac{T_2 \cos 30}{\cos 60}$$

$$T_1 = \sqrt{3} T_2 \quad (1)$$

$$R(\uparrow): T_1 \sin 60 - T_2 \sin 30 - 2 = 0 \quad (2)$$

Sub. (1) into (2):

$$\sqrt{3} T_2 \sin 60 - T_2 \sin 30 - 2 = 0$$

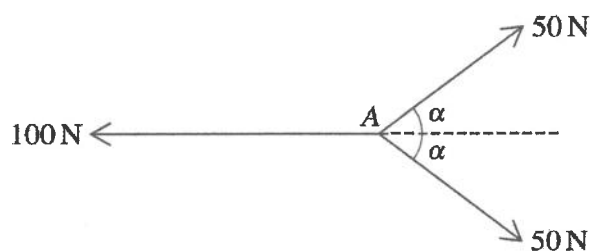
$$\sqrt{3} T_2 \times \frac{\sqrt{3}}{2} - T_2 \times \frac{1}{2} - 2 = 0$$

$$\frac{3}{2} T_2 - \frac{1}{2} T_2 = 2$$

$$\underline{T_2 = 2 \text{ N}}$$

$$\begin{aligned} \text{Sub. into (1): } T_1 &= \sqrt{3} \times 2 \\ &= \underline{2\sqrt{3} \text{ N}} \end{aligned}$$

1



Three coplanar forces of magnitudes 100 N, 50 N and 50 N act at a point A, as shown in the diagram. The value of $\cos \alpha$ is $\frac{4}{5}$.

Find the magnitude of the resultant of the three forces and state its direction.

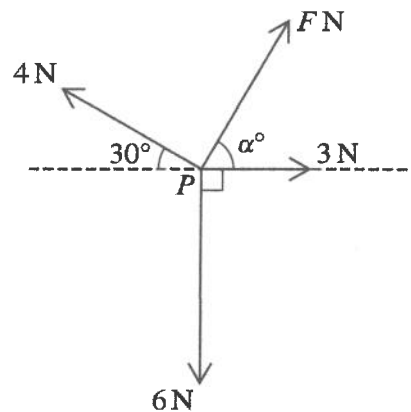
[3]

$$\cos \alpha = \frac{4}{5} \rightarrow \begin{array}{c} 5 \\ \alpha \\ 4 \end{array} \quad \begin{array}{c} 3 \\ \text{right angle} \end{array} \quad \begin{array}{l} \sin \alpha = \frac{3}{5} \\ \tan \alpha = \frac{3}{4} \end{array}$$

$$R(\uparrow): 50 \sin \alpha - 50 \sin \alpha \\ = \underline{0}$$

$$R(\rightarrow): 50 \cos \alpha + 50 \cos \alpha - 100 \\ = 50 \times \frac{4}{5} + 50 \times \frac{4}{5} - 100 \\ = 40 + 40 - 100 \\ = -20 \text{ N}$$

$\rightarrow$ magnitude = 20 N
direction: in direction of 100 N force



Coplanar forces, of magnitudes F N, 3 N, 6 N and 4 N, act at a point P , as shown in the diagram.

- (a) Given that $\alpha = 60$, and that the resultant of the four forces is in the direction of the 3 N force, find F . [3]

Resultant is in direction of 3 N force, so resultant in vertical direction = 0:

$$R(\uparrow): 4 \sin 30 + F \sin 60 - 6 = 0$$

$$2 + F \sin 60 - 6 = 0$$

$$F \sin 60 = 4$$

$$F = \frac{4}{\sin 60}$$

$$F = \underline{\underline{4.62 \text{ N}}}$$

(b) Given instead that the four forces are in equilibrium, find the values of F and α .

[5]

$$R(\uparrow): 4\sin 30 + F\sin\alpha - 6 = 0$$

$$F\sin\alpha = 6 - 4\sin 30$$

$$F\sin\alpha = 6 - 2$$

$$F\sin\alpha = 4 \quad (1)$$

$$R(\rightarrow): 3 + F\cos\alpha - 4\cos 30 = 0$$

$$F\cos\alpha = 4\cos 30 - 3$$

$$F\cos\alpha = 2\sqrt{3} - 3 \quad (2)$$

$$(1) \div (2): \frac{F\sin\alpha}{F\cos\alpha} = \frac{4}{2\sqrt{3} - 3}$$

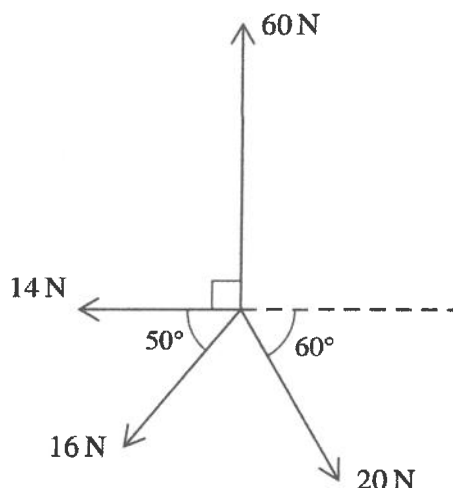
$$\tan\alpha = \frac{4}{2\sqrt{3} - 3}$$

$$\alpha = \underline{83.4^\circ} \text{ STO}$$

$$\text{Sub into (1): } F\sin(83.4) = 4$$

$$F = \frac{4}{\sin(83.4)}$$

$$= \underline{4.03 \text{ N}}$$



Coplanar forces of magnitudes 60 N, 20 N, 16 N and 14 N act at a point in the directions shown in the diagram.

Find the magnitude and direction of the resultant force.

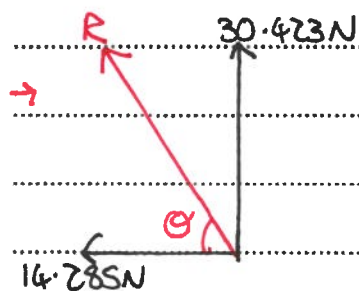
[6]

$$R(\uparrow): 60 - 20\sin 60 - 16\sin 50$$

$$= \underline{30.423 \text{ N}}$$

$$R(\rightarrow): 20\cos 60 - 16\cos 50 - 14$$

$$= \underline{-14.285 \text{ N}}$$



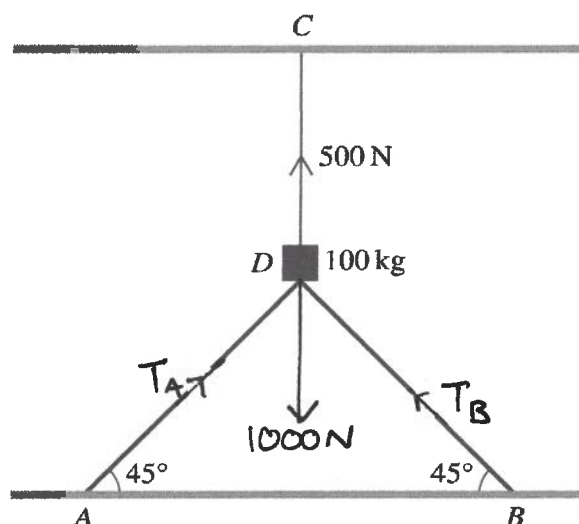
$$R = \sqrt{30.423^2 + 14.285^2}$$

$$= \underline{33.6 \text{ N}}$$

$$\tan \theta = \frac{30.423}{14.285}$$

$$\theta = \underline{64.8^\circ \text{ above negative } x \text{ axis}}$$

(note mistake in mark scheme)



The diagram shows a block D of mass 100 kg supported by two sloping struts AD and BD , each attached at an angle of 45° to fixed points A and B respectively on a horizontal floor. The block is also held in place by a vertical rope CD attached to a fixed point C on a horizontal ceiling. The tension in the rope CD is 500 N and the block rests in equilibrium.

- (a) Find the magnitude of the force in each of the struts AD and BD .

[3]

$$R(\rightarrow): T_B \cos 45 = T_A \cos 45$$

$$T_B = T_A (=T)$$

$$R(\uparrow): 500 - 1000 + T \sin 45 + T \sin 45 = 0$$

$$-500 + \frac{\sqrt{2}}{2} T + \frac{\sqrt{2}}{2} T = 0$$

$$-500 + \sqrt{2} T = 0$$

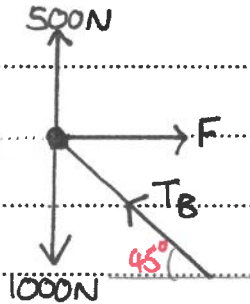
$$\sqrt{2} T = 500$$

$$T = \frac{500}{\sqrt{2}}$$

$$T = \underline{\underline{354\text{ N}}}$$

A horizontal force of magnitude F N is applied to the block in a direction parallel to AB .

- (b) Find the value of F for which the magnitude of the force in the strut AD is zero. [3]



$$R(\uparrow): 500 + T_B \sin 45 - 1000 = 0$$

$$T_B \sin 45 - 500 = 0$$

$$T_B \sin 45 = 500$$

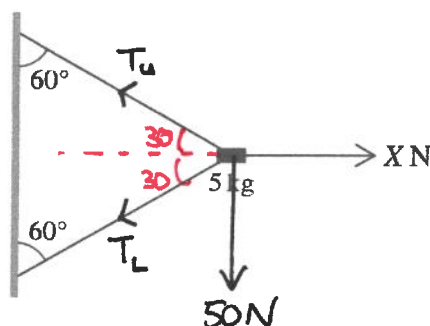
$$T_B = \underline{707.1 \text{ N}} \text{ Sto.}$$

$$R(\rightarrow): F - T_B \cos 45 = 0$$

$$F = T_B \cos 45$$

$$= 707.1 \times \cos 45$$

$$= \underline{500 \text{ N}}$$



A block of mass 5 kg is held in equilibrium near a vertical wall by two light strings and a horizontal force of magnitude X N, as shown in the diagram. The two strings are both inclined at 60° to the vertical.

- (a) Given that $X = 100$, find the tension in the lower string.

[4]

$$\begin{aligned} R(\uparrow): T_u \sin 30 - T_L \sin 30 - 50 &= 0 \\ \frac{1}{2} T_u - \frac{1}{2} T_L &= 50 \\ T_u - T_L &= 100 \quad (1) \end{aligned}$$

$$\begin{aligned} R(\rightarrow): 100 - T_u \cos 30 - T_L \cos 30 &= 0 \\ 100 - \frac{\sqrt{3}}{2} T_u - \frac{\sqrt{3}}{2} T_L &= 0 \\ 200 - \sqrt{3} T_u - \sqrt{3} T_L &= 0 \\ \sqrt{3} T_u + \sqrt{3} T_L &= 200 \\ T_u + T_L &= \frac{200}{\sqrt{3}} \quad (2) \end{aligned}$$

$$\begin{aligned} (1) + (2): 2 T_u &= 100 + \frac{200}{\sqrt{3}} \\ 2 T_u &= 215.470 \\ T_u &= 107.735 \end{aligned}$$

$$\begin{aligned} \text{Sub into (1): } T_u - T_L &= 100 \\ 107.735 - T_L &= 100 \\ -T_L &= -7.735 \\ T_L &= \underline{\underline{7.74 \text{ N}}} \end{aligned}$$

- (b) Find the least value of X for which the block remains in equilibrium in the position shown. [4]

As X decreases, so will T_u and T_L

T_u cannot be zero or there would be nothing to hold the block up, so set $T_L = 0$:

$$R(\uparrow): T_u \sin 30 - 50 = 0$$

$$T_u \sin 30 = 50$$

$$T_u = \underline{100 \text{ N}}$$

$$R(\rightarrow): X - T_u \cos 30 = 0$$

$$X - 100 \cos 30 = 0$$

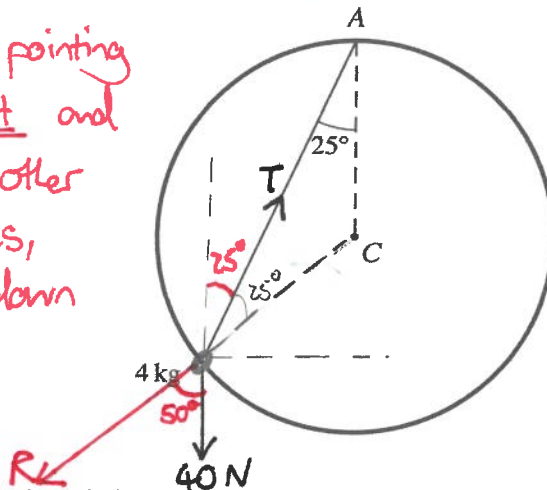
$$X = 100 \cos 30$$

$$= \underline{86.6 \text{ N}}$$

3

R is perpendicular to surface⁵

Since T is pointing up and right and there are no other horizontal forces, R must be down and left.



A ring of mass 4 kg is threaded on a smooth circular rigid wire with centre C. The wire is fixed in a vertical plane and the ring is kept at rest by a light string connected to A, the highest point of the circle. The string makes an angle of 25° to the vertical (see diagram).

Find the tension in the string and the magnitude of the normal reaction of the wire on the ring. [6]

$$R(\rightarrow): T \sin 25 - R \sin 50 = 0$$

$$T \sin 25 = R \sin 50$$

$$T = \frac{R \sin 50}{\sin 25} \quad (1)$$

$$R(\uparrow): T \cos 25 - 40 - R \cos 50 = 0 \quad (2)$$

sub (1) into (2):

$$\frac{R \sin 50 \cos 25}{\sin 25} - 40 - R \cos 50 = 0$$

$$R \left(\frac{\sin 50 \cos 25}{\sin 25} - \cos 50 \right) = 40$$

$$R = 40 \div \left(\frac{\sin 50 \cos 25}{\sin 25} - \cos 50 \right)$$

$$R = \underline{40 \text{ N}}$$

sub into (1): $T = \frac{40 \sin 50}{\sin 25}$

$$T = \underline{72.5 \text{ N}}$$