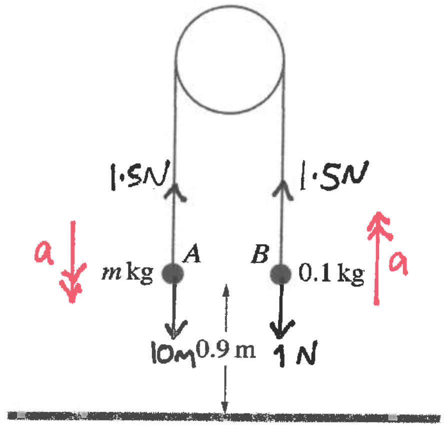
2



Two particles A and B have masses m kg and 0.1 kg respectively, where $m > 0.1$. The particles are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley and the particles hang vertically below it. Both particles are at a height of 0.9 m above horizontal ground (see diagram). The system is released from rest, and while both particles are in motion the tension in the string is 1.5 N. Particle B does not reach the pulley.

(a) Find m. [4]

A:

$R(\downarrow): 10m - 1.5 = ma$ ①

B:

$R(\uparrow): 1.5 - 1 = 0.1a$

$0.5 = 0.1a$

$a = 5\text{ms}^{-2}$

Sub into ①:

$10m - 1.5 = m \times 5$

$10m - 1.5 = 5m$

$5m = 1.5$

$m = 0.3\text{kg}$

(b) Find the speed at which A reaches the ground. [2]

$s = 0.9$

$u = 0$

$v =$

$a = 5$

$t =$

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2(5)\times 0.9$

$v^2 = 9$

$v = 3\text{ms}^{-1}$

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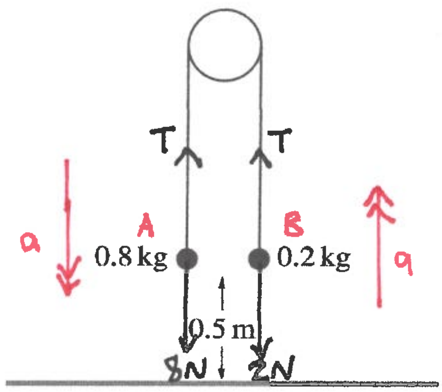

Two particles of masses 0.8 kg and 0.2 kg are connected by a light inextensible string that passes over a fixed smooth pulley. The system is released from rest with both particles 0.5 m above a horizontal floor (see diagram). In the subsequent motion the 0.2 kg particle does not reach the pulley.

(a) Show that the magnitude of the acceleration of the particles is $6\,\text{m s}^{-2}$ and find the tension in the string. [4]

A:

$R(\downarrow)$: $8 - T = ma$

$8 - T = 0.8a$ (1)

B:

$R(\uparrow)$: $T - 2 = ma$

$T - 2 = 0.2a$ (2)

(1) + (2):

$8 - 2 = 1a$

$6\,\text{ms}^{-2} = a$ QED

Sub into (2):

$T - 2 = 0.2 \times 6$

$T - 2 = 1.2$

$T = 3.2\,\text{N}$

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(b) When the 0.8 kg particle reaches the floor it comes to rest.

Find the greatest height of the 0.2 kg particle above the floor. [3]

Find speed of B when A hits floor:

$s = 0.5$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2(6) \times 0.5$
$v =$	$v^2 = 6$
$a = 6$	$\underline{v = \sqrt{6}\ ms^{-1}}$
$t =$	

Once A hits the floor, B moves freely under gravity:

↑+

$s =$	$v^2 = u^2 + 2as$
$u = \sqrt{6}$	$0^2 = (\sqrt{6})^2 + 2(-10) \times s$
$v = 0$	$0 = 6 - 20s$
$a = -10$	$20s = 6$
$t =$	$\underline{s = 0.3m}$

initial height — moved while connected — under freefall

Greatest height reached $= 0.5 + 0.5 + 0.3$

$= \underline{1.3m}$

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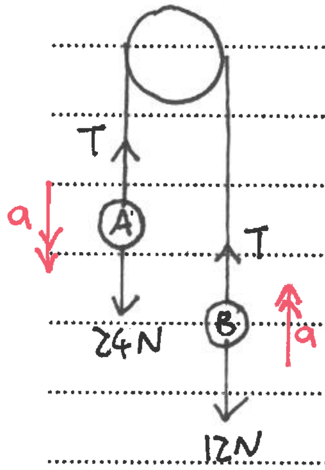
3 Two particles A and B, of masses 2.4 kg and 1.2 kg respectively, are connected by a light inextensible string which passes over a fixed smooth pulley. A is held at a distance of 2.1 m above a horizontal plane and B is 1.5 m above the plane. The particles hang vertically and are released from rest. In the subsequent motion A reaches the plane and does not rebound and B does not reach the pulley.

(a) Show that the tension in the string before A reaches the plane is 16 N and find the magnitude of the acceleration of the particles before A reaches the plane. [4]



A:

$R(\downarrow): 24 - T = 2.4a$ (1)

B:

$R(\uparrow): T - 12 = 1.2a$ (2)

(1)+(2): $12 = 3.6a$

$a = \frac{10}{3}\ ms^{-2}$

sub. into (2): $T - 12 = 1.2 \times \frac{10}{3}$

$T - 12 = 4$

$T = 16\ N$ QED

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(b) Find the greatest height of B above the plane. [3]

Before A hits ground:

$\uparrow + \; s = 2.1$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2\left(\frac{10}{3}\right) \times 2.1$
$v =$	$v^2 = 14$
$a = \frac{10}{3}$	$v = \sqrt{14}$
$t =$	

Once A has hit the ground, B moves under gravity.

$\uparrow + \; s =$	$v^2 = u^2 + 2as$
$u = \sqrt{14}$	$0^2 = (\sqrt{14})^2 + 2(-10) \times s$
$v = 0$	$0 = 14 - 20s$
$a = -10$	$20s = 14$
$t =$	$s = 0.7\text{m}$

Height above plane $= 1.5 + 2.1 + 0.7 = 4.3\text{m}$

initial height

moved before A hits ground

after A hit ground

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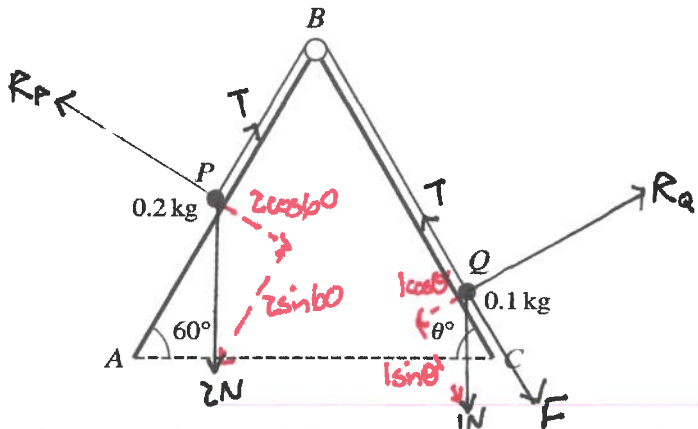

Two particles P and Q, of masses 0.2 kg and 0.1 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley at B which is attached to two inclined planes. Particle P lies on a smooth plane AB which is inclined at 60° to the horizontal. Particle Q lies on a plane BC which is inclined at an angle of $\theta°$ to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes (see diagram).

(a) It is given that $\theta = 60$, the plane BC is rough and the coefficient of friction between Q and the plane BC is 0.7. The particles are released from rest.

Determine whether the particles move. [4]

P is heavier than Q and $\theta = 60°$, so P wants to move down, Q wants to move up.

P:

$R(\swarrow)$: $2\sin 60 - T = ma$

$\sqrt{3} - T = 0.2a$

$T = \sqrt{3} - 0.2a$

Q:

$R(\nearrow)$: $R_Q - 1\cos 60 = 0$

$R_Q = \frac{1}{2}$

$R(\nwarrow)$: $T - 1\sin 60 - F = ma$

$\sqrt{3} - 0.2a - \frac{\sqrt{3}}{2} - \mu R = 0.1a$

$\sqrt{3} - \frac{\sqrt{3}}{2} - \mu R = 0.3a$

$\frac{\sqrt{3}}{2} - 0.7 \times \frac{1}{2} = 0.3a$

$0.516 = 0.3a$

$a = 1.72 \text{ ms}^{-2}$

$a > 0$, so the particles do move.

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(b) It is given instead that the plane BC is smooth. The particles are released from rest and in the subsequent motion the tension in the string is $(\sqrt{3}-1)$ N.

Find the magnitude of the acceleration of P as it moves on the plane, and find the value of θ. [4]

P:

$R(\swarrow)$: $2\sin 60 - T = ma$

$\sqrt{3} - (\sqrt{3}-1) = 0.2a$

$1 = 0.2a$

$a = 5\,\text{ms}^{-2}$

Q:

$R(\nwarrow)$: $T - 1\sin\theta = ma$

$(\sqrt{3}-1) - \sin\theta = 0.1 \times 5$

$\sqrt{3} - 1 - \sin\theta = 0.5$

$\sin\theta = \sqrt{3} - 1 - 0.5$

$\sin\theta = \sqrt{3} - 1.5$

$\theta = 13.4°$

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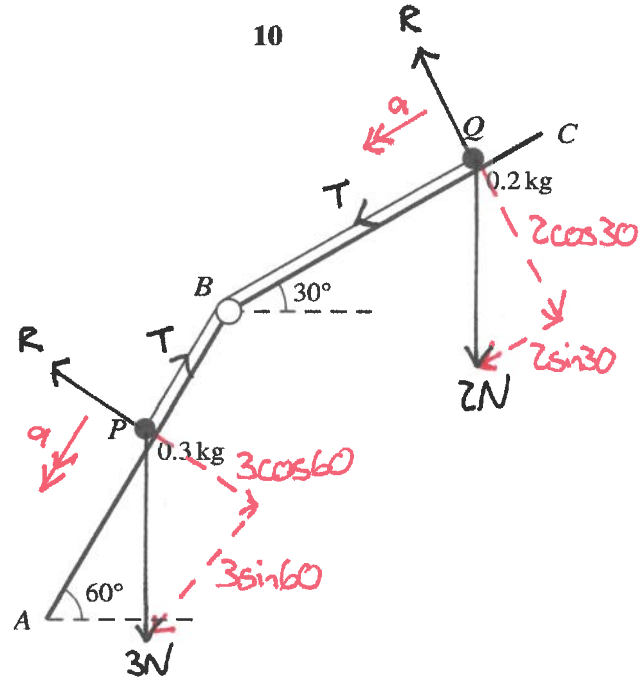

Two particles P and Q, of masses 0.3 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley at B which is attached to two inclined planes. P lies on a smooth plane AB which is inclined at 60° to the horizontal. Q lies on a plane BC which is inclined at 30° to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes (see diagram).

(a) It is given that the plane BC is smooth and that the particles are released from rest.

Find the tension in the string and the magnitude of the acceleration of the particles. [5]

P:

$R(\swarrow)$: $3\sin 60 - T = 0.3a$ (1)

Q:

$R(\swarrow)$: $2\sin 30 + T = 0.2a$ (2)

(1)+(2): $3\sin 60 + 2\sin 30 = 0.5a$

$a = \dfrac{3\sin 60 + 2\sin 30}{0.5}$

$a = 7.20\ \text{ms}^{-2}$ STO

Sub into (2): $2\sin 30 + T = 0.2 \times 7.20$

$1 + T = 1.439$

$T = 0.439\text{N}$

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(b) It is given instead that the plane BC is rough. A force of magnitude 3 N is applied to Q directly up the plane along a line of greatest slope of the plane.

Find the least value of the coefficient of friction between Q and the plane BC for which the particles remain at rest. [5]

P:

$R(\swarrow)$: $3\sin 60 - T = 0$

$T = 3\sin 60$

Q:

Resultant force up the plane = 3N

Resultant force down the plane = $3\sin 60 + 2\sin 30 = 3.60N$

So particle Q wants to move down the plane and friction acts upwards.

$R(\swarrow)$: $T + 2\sin 30 - F - 3 = 0$

$3\sin 60 + 2\sin 30 - \mu R - 3 = 0$

$\frac{3\sqrt{3}}{2} + 1 - \mu \times 2\cos 30 - 3 = 0$

$\mu \times 2\cos 30 = -2 + \frac{3\sqrt{3}}{2}$

$\mu = \frac{-2 + \frac{3\sqrt{3}}{2}}{2\cos 30}$

$\mu = 0.345$

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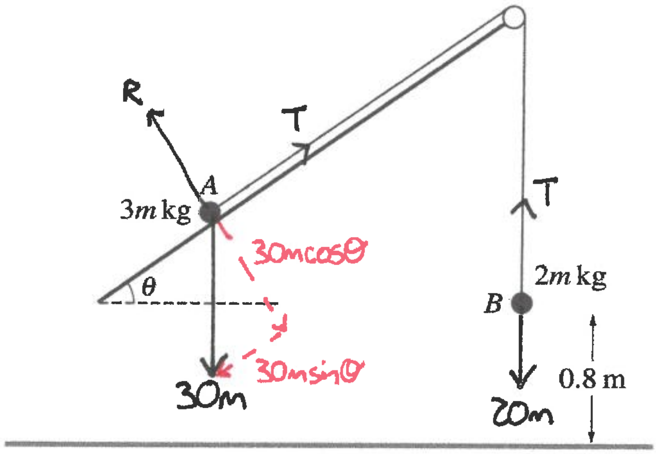

Two particles A and B, of masses $3m$ kg and $2m$ kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley which is attached to the edge of a plane. The plane is inclined at an angle θ to the horizontal. A lies on the plane and B hangs vertically, 0.8 m above the floor, which is horizontal. The string between A and the pulley is parallel to a line of greatest slope of the plane (see diagram). Initially A and B are at rest.

(a) Given that the plane is smooth, find the value of θ for which A remains at rest. [3]

B: A:

$R(\downarrow): 20m - T = 0$ ① $R(\nearrow): T - 30m\sin\theta = 0$ ②

①+②:

$20m - 30m\sin\theta = 0$

$30m\sin\theta = 20m$

$\sin\theta = \frac{20m}{30m}$

$\sin\theta = \frac{2}{3}$

$\theta = 41.8°$

It is given instead that the plane is rough, $\theta = 30°$ and the acceleration of A up the plane is $0.1\ \text{m s}^{-2}$.

(b) Show that the coefficient of friction between A and the plane is $\frac{1}{10}\sqrt{3}$. [5]

B: A:

$R(\downarrow): 20m - T = 2ma$ $R(\nwarrow): R - 30m\cos 30 = 0$

$20m - T = 2m \times 0.1$ $R = 30m\cos 30$

$20m - T = 0.2m$ $R = 15\sqrt{3}m$

$T = 19.8m$ ①

CONT →

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A:

$R(\nearrow)$: $T - 30m\sin 30 - F = 3ma$

$T - 15m - \mu R = 3m \times 0.1$

$T - 15m - \mu \times 15\sqrt{3}m = 0.3m$

$T - 15m - 15\sqrt{3}m\mu = 0.3m$

$T = 15.3m + 15\sqrt{3}m\mu$ (2)

Sub. (1) into (2): $19.8m = 15.3m + 15\sqrt{3}m\mu$

$19.8 = 15.3 + 15\sqrt{3}\mu$

$15\sqrt{3}\mu = 4.5$

$\mu = \frac{4.5}{15\sqrt{3}}$

$\mu = \frac{3}{10\sqrt{3}} \frac{\times\sqrt{3}}{\times\sqrt{3}}$

$\mu = \frac{3\sqrt{3}}{30}$

$\mu = \frac{\sqrt{3}}{10}$ QED

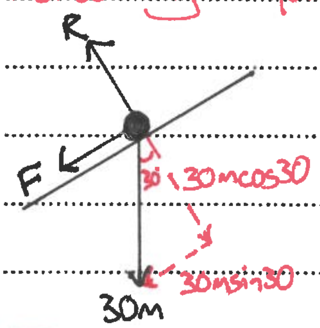
(c) When *B* reaches the floor it comes to rest.

Find the length of time after *B* reaches the floor for which *A* is moving up the plane. [You may assume that *A* does not reach the pulley.] [4]

Before B hits floor:

$s = 0.8$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2(0.1) \times 0.8$
$v =$	$v^2 = 0.16$
$a = 0.1$	$v = 0.4\,ms^{-1}$
$t =$	

Force diagram for A once string goes slack:



$R(\nearrow)$: $-30m\sin 30 - F = 3ma$

$-15m - \mu R = 3ma$

$-15m - \frac{\sqrt{3}}{10} \times 15\sqrt{3}m = 3ma$

$-15m - 4.5m = 3ma$

$-19.5m = 3ma$

$a = \frac{-19.5m}{3m}$

$= -6.5\,ms^{-2}$

$s =$	$v = u + at$
$u = 0.4$	$0 = 0.4 + -6.5 \times t$
$v = 0$	$6.5t = 0.4$
$a = -6.5$	$t = 0.0615\,s$
$t =$	

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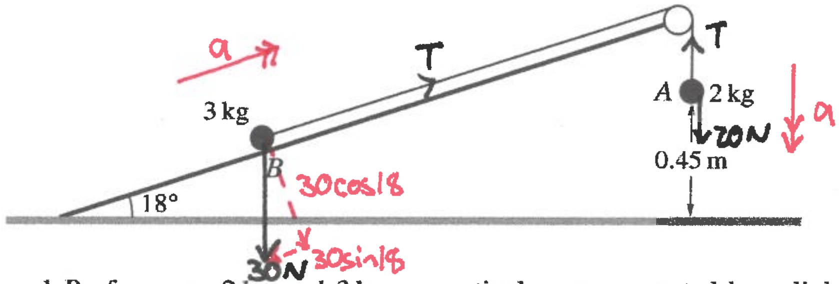

Two particles A and B of masses 2 kg and 3 kg respectively are connected by a light inextensible string. Particle B is on a smooth fixed plane which is at an angle of 18° to horizontal ground. The string passes over a fixed smooth pulley at the top of the plane. Particle A hangs vertically below the pulley and is 0.45 m above the ground (see diagram). The system is released from rest with the string taut. When A reaches the ground, the string breaks.

Find the total distance travelled by B before coming to instantaneous rest. You may assume that B does not reach the pulley. [8]

A:

$R(\downarrow)$: $20 - T = ma$

$20 - T = 2a$ ①

B:

$R(\nearrow)$: $T - 30\sin 18 = ma$

$T - 30\sin 18 = 3a$ ②

① + ②:

$20 - 30\sin 18 = 5a$

$a = \dfrac{20 - 30\sin 18}{5}$

$a = 2.146 \text{ ms}^{-2}$ STO

Find speed of B when A hits ground:

$s = 0.45$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2(2.146) \times 0.45$
$v =$	$v^2 = 1.931$
$a = 2.146$	$v = 1.39 \text{ ms}^{-1}$
$t =$	STO

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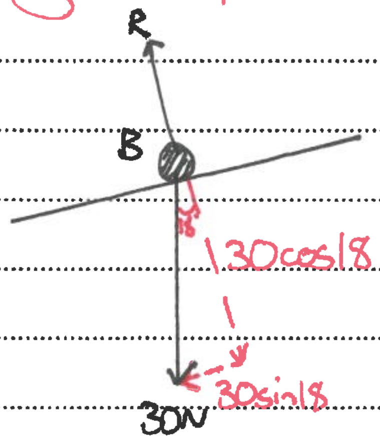
String breaks, so no more tension. Re-draw force diagram:



$R(\nearrow): -30\sin 18 = ma$

$-30\sin 18 = 3a$

$a = -3.09\ ms^{-2}$ STO

Find distance travelled by B after string has snapped:

$+\rightarrow$ $s =$	$v^2 = u^2 + 2as$
$u = 1.39$	$0^2 = 1.39^2 + 2(-3.09) \times s$
$v = 0$	$0 = 1.931 - 6.18s$
$a = -3.09$	$6.18s = 1.931$
$t =$	$s = 0.312\ m$

when attached

after string broke

Total distance travelled by B $= 0.45 + 0.312$

$= 0.762\ m$

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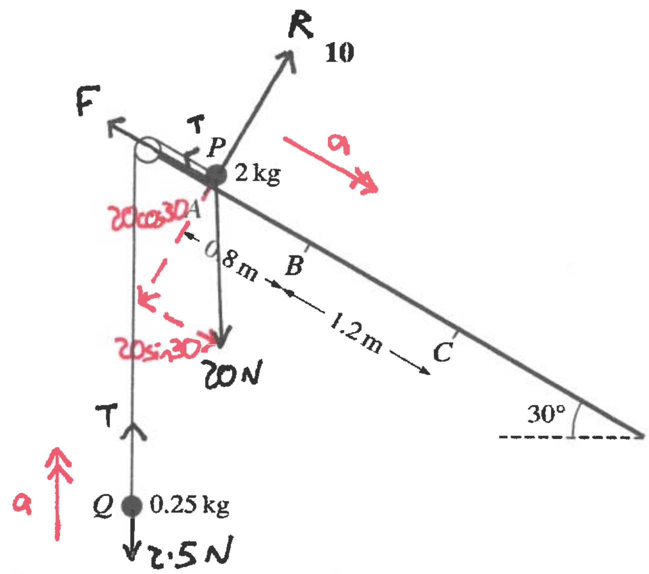

Two particles P and Q, of masses 2 kg and 0.25 kg respectively, are connected by a light inextensible string that passes over a fixed smooth pulley. Particle P is on an inclined plane at an angle of 30° to the horizontal. Particle Q hangs below the pulley. Three points A, B and C lie on a line of greatest slope of the plane with $AB = 0.8$ m and $BC = 1.2$ m (see diagram).

Particle P is released from rest at A with the string taut and slides down the plane. During the motion of P from A to C, Q does not reach the pulley. The part of the plane from A to B is rough, with coefficient of friction 0.3 between the plane and P. The part of the plane from B to C is smooth.

(a) (i) Find the acceleration of P between A and B. [4]

Q:

$R(\uparrow): T - 2.5 = ma$

$T - 2.5 = 0.25a$ (1)

P:

$R(\nearrow): R - 20\cos 30 = 0$

$R = 10\sqrt{3}$ N

$R(\searrow): 20\sin 30 - T - F = ma$

$10 - T - \mu R = 2a$

$10 - T - 0.3 \times 10\sqrt{3} = 2a$

$10 - T - 3\sqrt{3} = 2a$ (2)

(1)+(2): $10 - 2.5 - 3\sqrt{3} = 2.25a$

$7.5 - 3\sqrt{3} = 2.25a$

$a = \dfrac{7.5 - 3\sqrt{3}}{2.25}$

$a = 1.02\text{ms}^{-2}$

STO

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(ii) Hence, find the speed of P at C. [5]

Find speed at B:

A→B

$s = 0.8$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2(1.02) \times 0.8$
$v =$	$v^2 = 1.638$
$a = 1.02$	$v = \underline{1.28 \text{ ms}^{-1}}$ STO
$t =$	

Find acceleration from B to C by resolving forces:

Q:

$R(\uparrow)$: $T - 2.5 = 0.25a$ ①

P:

$R(\searrow)$: $20\sin 30 - T = 2a$ ②

①+②: $20\sin 30 - 2.5 = 2.25a$

$7.5 = 2.25a$

$a = \underline{\frac{10}{3} \text{ ms}^{-2}}$

B→C

$s = 1.2$	$v^2 = u^2 + 2as$
$u = 1.28$	$v^2 = 1.28^2 + 2\left(\frac{10}{3}\right) \times 1.2$
$v =$	$v^2 = 9.638$
$a = \frac{10}{3}$	$v = \underline{\underline{3.10 \text{ ms}^{-1}}}$ STO
$t =$	

(b) Find the time taken for P to travel from A to C. [4]

A→B

$s = 0.8$	$v = u + at$
$u = 0$	$1.28 = 0 + 1.02t$
$v = 1.28$	$t = \underline{1.25 \text{ s}}$ STO
$a = 1.02$	
$t =$	

B→C

$s = 1.2$	$v = u + at$
$u = 1.28$	$3.10 = 1.28 + \frac{10}{3}t$
$v = 3.10$	$\frac{10}{3}t = 1.8246$
$a = \frac{10}{3}$	$t = \underline{0.547}$ STO
$t =$	

Time taken $= 1.25 + 0.547$

$= \underline{1.80 \text{ s}}$

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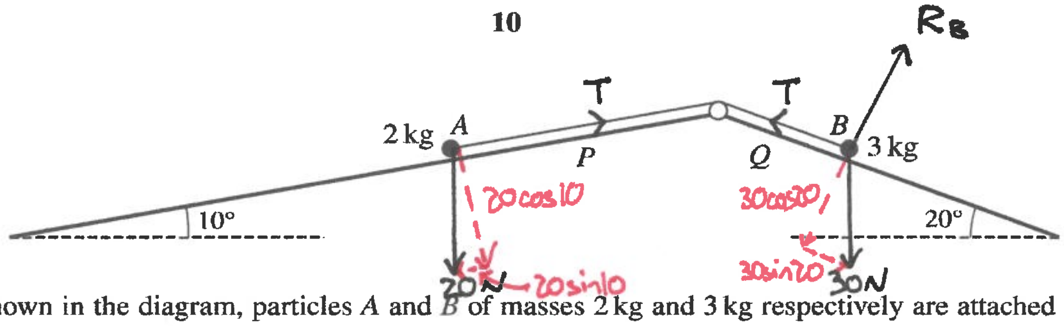

As shown in the diagram, particles A and B of masses 2 kg and 3 kg respectively are attached to the ends of a light inextensible string. The string passes over a small fixed smooth pulley which is attached to the top of two inclined planes. Particle A is on plane P, which is inclined at an angle of 10° to the horizontal. Particle B is on plane Q, which is inclined at an angle of 20° to the horizontal. The string is taut, and the two parts of the string are parallel to lines of greatest slope of their respective planes.

(a) It is given that plane P is smooth, plane Q is rough, and the particles are in limiting equilibrium.

Find the coefficient of friction between particle B and plane Q. [5]

Without friction, B would move down the plane, so friction is acting up the plane.

P:

$R(\rightarrow): T - 20\sin 10 = 0$

$T = 20\sin 10$

Q:

$R(\uparrow): R_B - 30\cos 20 = 0$

$R_B = 30\cos 20$

$R(\rightarrow): 30\sin 20 - T - F = 0$

$30\sin 20 - 20\sin 10 - \mu R = 0$

$30\sin 20 - 20\sin 10 - \mu \times 30\cos 20 = 0$

$\mu \times 30\cos 20 = 30\sin 20 - 20\sin 10$

$\mu = \dfrac{30\sin 20 - 20\sin 10}{30\cos 20}$

$\mu = 0.241$

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(b) It is given instead that both planes are smooth and that the particles are released from rest at the same horizontal level.

Find the time taken until the difference in the vertical height of the particles is 1 m. [You should assume that this occurs before A reaches the pulley or B reaches the bottom of plane Q.] [6]

P:

$R(\nearrow): T - 20\sin 10 = 2a$ (1)

Q:

$R(\searrow): 30\sin 20 - T = 3a$ (2)

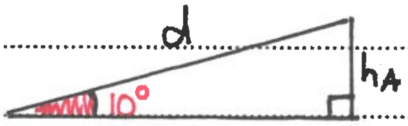
(1) + (2):

$30\sin 20 - 20\sin 10 = 5a$

$a = \dfrac{30\sin 20 - 20\sin 10}{5}$

$a = 1.3575 \text{ ms}^{-2}$ STO

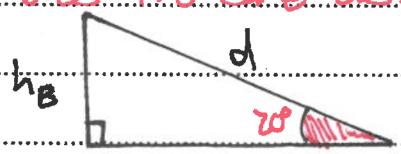
If A moves a distance "d" up the plane:



$\sin 10 = \dfrac{h_A}{d}$

$\rightarrow h_A = d\sin 10$

If B moves the same distance, "d" down the plane:



$\sin 20 = \dfrac{h_B}{d}$

$\rightarrow h_B = d\sin 20$

Since A moves up and B moves down, we want $h_A + h_B = 1$:

$d\sin 10 + d\sin 20 = 1$

$d(\sin 10 + \sin 20) = 1$

$d = \dfrac{1}{\sin 10 + \sin 20}$

$d = 1.939 \text{ m}$ STO

$s = 1.939$
$u = 0$
$v =$
$a = 1.3575$
$t =$

$s = ut + \frac{1}{2}at^2$

$1.939 = 0 + \frac{1}{2} \times 1.3575 t^2$

$1.939 = 0.6788 t^2$

$t^2 = 2.857$

$t = 1.69 \text{ s}$

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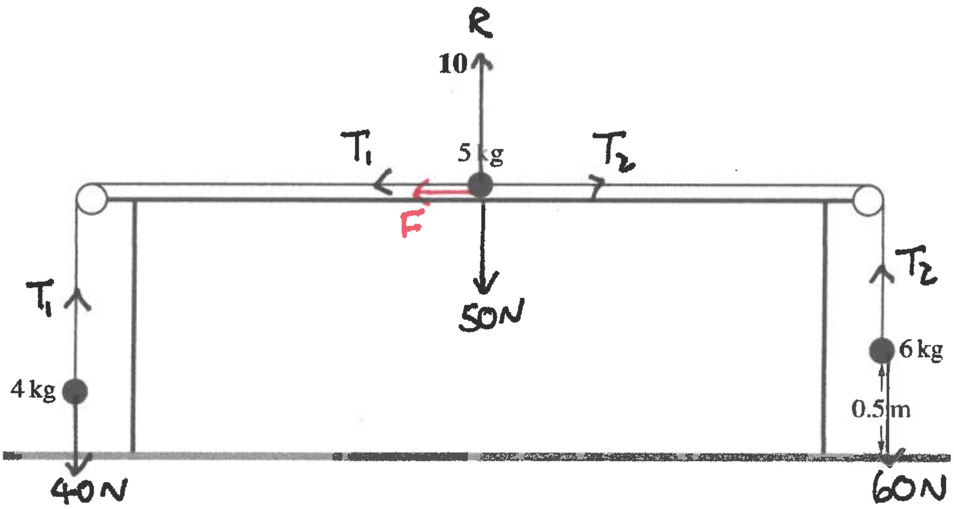

The diagram shows a particle of mass 5 kg on a rough horizontal table, and two light inextensible strings attached to it passing over smooth pulleys fixed at the edges of the table. Particles of masses 4 kg and 6 kg hang freely at the ends of the strings. The particle of mass 6 kg is 0.5 m above the ground. The system is in limiting equilibrium.

(a) Show that the coefficient of friction between the 5 kg particle and the table is 0.4. [2]

4kg:

$R(\uparrow): T_1 - 40 = 0$

$T_1 = 40N$

6kg:

$R(\downarrow): 60 - T_2 = 0$

$T_2 = 60N$

So Friction must be acting to the left and must be 20N.

$F = \mu R$

$20 = \mu \times 50$

$\mu = \frac{20}{50}$

$\mu = 0.4$ QED

The 6 kg particle is now replaced by a particle of mass 8 kg and the system is released from rest.

(b) Find the acceleration of the 4 kg particle and the tensions in the strings. [5]

4kg will move up, 5kg will move right, 8kg will move down.

4kg:

$R(\uparrow): T_1 - 40 = 4a$

$T_1 = 40 + 4a$ ①

8kg:

$R(\downarrow): 80 - T_2 = 8a$

$T_2 = 80 - 8a$ ②

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5kg:

$R(\rightarrow)$: $T_2 - T_1 - F = 5a$

$T_2 - T_1 - \mu R = 5a$

$T_2 - T_1 - 0.4 \times 50 = 5a$

Sub. ① and ② into this⑤

$(80 - 8a) - (40 + 4a) - 20 = 5a$

$80 - 8a - 40 - 4a - 20 = 5a$

$20 - 12a = 5a$

$20 = 17a$

$a = 1.18 \text{ ms}^{-2}$ STO

Sub into ①:

$T_1 = 40 + 4(1.18)$

$= 44.7 \text{ N}$

Sub into ②:

$T_2 = 80 - 8(1.18)$

$= 70.6 \text{ N}$

(c) In the subsequent motion the 8 kg particle hits the ground and does not rebound.

Find the time that elapses after the 8 kg particle hits the ground before the other two particles come to instantaneous rest. (You may assume this occurs before either particle reaches a pulley.) [5]

Find speed of 4kg and 5kg particles when 8kg hits ground:

$s = 0.5$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2(1.18) \times 0.5$
$v =$	$v^2 = 1.18$
$a = 1.18$	$v = 1.085 \text{ ms}^{-1}$ STO
$t =$	

Resolve forces for 4kg and 5kg now string has gone slack:

4kg:

$R(\uparrow)$: $T_1 - 40 = 4a$ ①

5kg:

$R(\rightarrow)$: $-T_1 - F = 5a$

$-T_1 - \mu R = 5a$

$-T_1 - 0.4 \times 50 = 5a$

continued →

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$-T_1 - 20 = 5a$ (2)

(1) + (2):

$-40 - 20 = 9a$

$-60 = 9a$

$a = \frac{-20}{3}\ ms^{-2}$

Find time that elapses before 4kg + 5kg come to rest:

$s =$	$v = u + at$
$u = 1.085$	$0 = 1.085 + \left(\frac{-20}{3}\right)t$
$v = 0$	$\frac{20}{3}t = 1.085$
$a = \frac{-20}{3}$	
$t =$	$t = 0.163s$

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