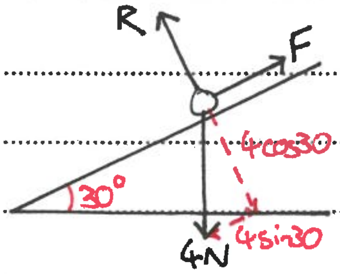
2 A particle P of mass 0.4 kg is in limiting equilibrium on a plane inclined at 30° to the horizontal.

(a) Show that the coefficient of friction between the particle and the plane is $\frac{1}{3}\sqrt{3}$. [3]



$R(\nwarrow): \; R - 4\cos 30 = 0$

$R = 2\sqrt{3}\text{ N}$

$R(\swarrow): \; 4\sin 30 - F = 0$

$2 - \mu R = 0$

$2 - 2\sqrt{3}\mu = 0$

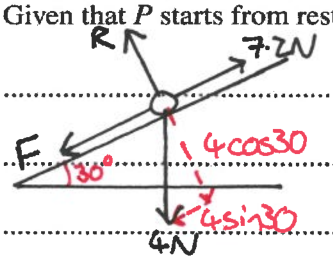
$2\sqrt{3}\mu = 2$

$\mu = \frac{2}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$ QED

A force of magnitude 7.2 N is now applied to P directly up a line of greatest slope of the plane.

(b) Given that P starts from rest, find the time that it takes for P to move 1 m up the plane. [4]



$R(\nearrow): \; 7.2 - 4\sin 30 - \mu R = ma$

$7.2 - 2 - \frac{\sqrt{3}}{3} \times 2\sqrt{3} = 0.4a$

$5.2 - 2 = 0.4a$

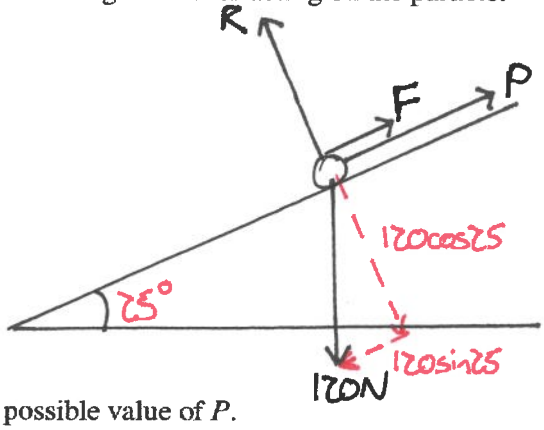
$3.2 = 0.4a$

$a = 8\text{ ms}^{-2}$

$s = 1$	$s = ut + \frac{1}{2}at^2$
$u = 0$	$1 = 0 + \frac{1}{2}(8) \times t^2$
$v =$	$1 = 4t^2$
$a = 8$	$t^2 = \frac{1}{4}$
$t =$	$t = 0.5\text{ s}$

4 A particle of mass 12 kg is stationary on a rough plane inclined at an angle of 25° to the horizontal. A force of magnitude P N acting parallel to a line of greatest slope of the plane is used to prevent the particle sliding down the plane. The coefficient of friction between the particle and the plane is 0.35.

(a) Draw a sketch showing the forces acting on the particle. [1]



(b) Find the least possible value of P. [5]

Least possible value of P is when particle is in limiting equilibrium, about to slide down the plane, so friction is up the plane.

$R(\nwarrow)$: $R - 120\cos 25 = 0$

$\underline{R = 120\cos 25 \text{ N}}$

$R(\swarrow)$: $120\sin 25 - F - P = 0$

$120\sin 25 - \mu R - P = 0$

$120\sin 25 - 0.35 \times 120\cos 25 - P = 0$

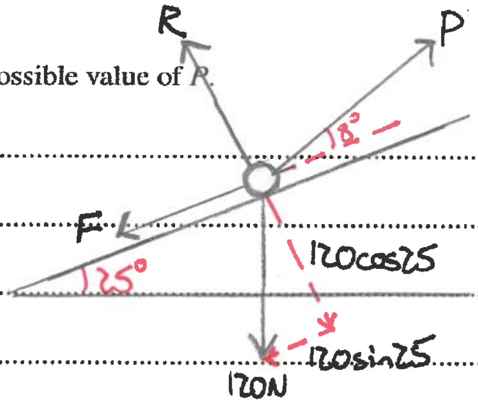
$120\sin 25 - 42\cos 25 - P = 0$

$P = 120\sin 25 - 42\cos 25$

$= \underline{12.6\text{ N}}$

4 A particle of mass 12 kg is stationary on a rough plane inclined at an angle of 25° to the horizontal. A pulling force of magnitude P N acts at an angle of 8° above a line of greatest slope of the plane. This force is used to keep the particle in equilibrium. The coefficient of friction between the particle and the plane is 0.3.

Find the greatest possible value of P. [6]



Greatest value of P is when particle is in limiting equilibrium, about to move up the plane, so friction is acting down the plane.

$R(\nwarrow)$: $R + P\sin 8 - 120\cos 25$

$R = 120\cos 25 - P\sin 8$

$R(\nearrow)$: $P\cos 8 - 120\sin 25 - F = 0$

$P\cos 8 - 120\sin 25 - \mu R = 0$

$P\cos 8 - 120\sin 25 - 0.3(120\cos 25 - P\sin 8) = 0$

$P\cos 8 - 120\sin 25 - 36\cos 25 + 0.3P\sin 8 = 0$

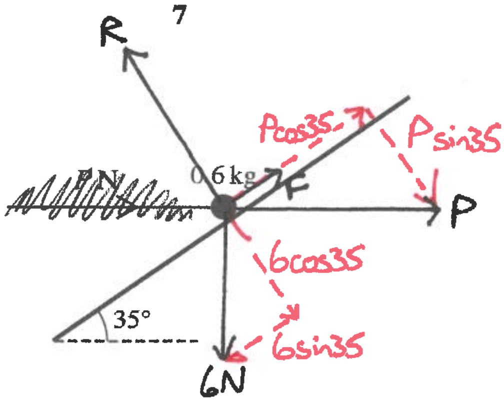
$P\cos 8 + 0.3P\sin 8 = 120\sin 25 + 36\cos 25$

$P(\cos 8 + 0.3\sin 8) = 120\sin 25 + 36\cos 25$

$$P = \frac{120\sin 25 + 36\cos 25}{\cos 8 + 0.3\sin 8}$$

$P = \underline{80.8\text{N}}$

5



A particle of mass 0.6 kg is placed on a rough plane which is inclined at an angle of 35° to the horizontal. The particle is kept in equilibrium by a horizontal force of magnitude P N acting in a vertical plane containing a line of greatest slope (see diagram). The coefficient of friction between the particle and plane is 0.4.

Find the least possible value of P. [6]

For the least possible value of P, the particle is on the point of moving down the plane and friction is acting up the plane.

$R(\nwarrow)$: $R - 6\cos35 - P\sin35 = 0$

$R = 6\cos35 + P\sin35$

$R(\swarrow)$: $6\sin35 - P\cos35 - F = 0$

$6\sin35 - P\cos35 - \mu R = 0$

$6\sin35 - P\cos35 - 0.4(6\cos35 + P\sin35) = 0$

$6\sin35 - P\cos35 - 2.4\cos35 - 0.4P\sin35 = 0$

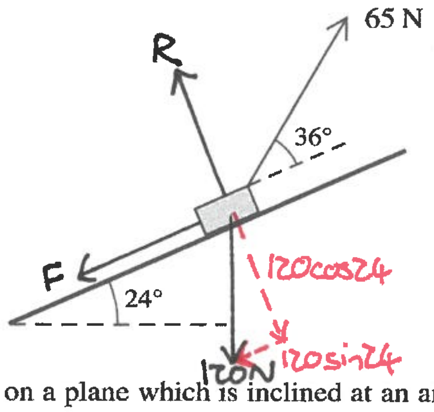
$6\sin35 - 2.4\cos35 = P\cos35 + 0.4P\sin35$

$6\sin35 - 2.4\cos35 = P(\cos35 + 0.4\sin35)$

$$P = \frac{6\sin35 - 2.4\cos35}{\cos35 + 0.4\sin35}$$

$P = \underline{1.41\text{ N}}$

5



A block of mass 12 kg is placed on a plane which is inclined at an angle of 24° to the horizontal. A light string, making an angle of 36° above a line of greatest slope, is attached to the block. The tension in the string is 65 N (see diagram). The coefficient of friction between the block and plane is μ. The block is in limiting equilibrium and is on the point of sliding up the plane.

Find μ. [6]

$R(\nwarrow): R + 65\sin 36 - 120\cos 24 = 0$

$R = 120\cos 24 - 65\sin 36$

$= 71.419\text{ N}$ STO

$R(\nearrow): 65\cos 36 - 120\sin 24 - F = 0$

$65\cos 36 - 120\sin 24 - \mu R = 0$

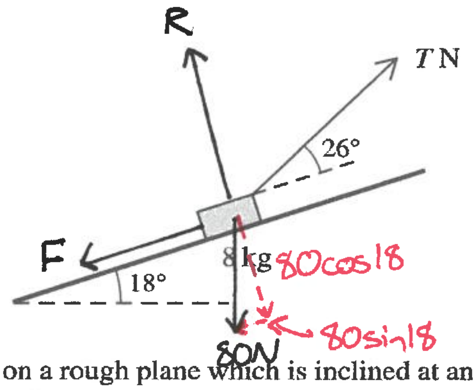
$65\cos 36 - 120\sin 24 - 71.419\mu = 0$

$71.419\mu = 65\cos 36 - 120\sin 24$

$\mu = \dfrac{65\cos 36 - 120\sin 24}{71.419}$

$\mu = 0.0529$

4



A block of mass 8 kg is placed on a rough plane which is inclined at an angle of 18° to the horizontal. The block is pulled up the plane by a light string that makes an angle of 26° above a line of greatest slope. The tension in the string is T N (see diagram). The coefficient of friction between the block and plane is 0.65.

(a) The acceleration of the block is $0.2\,\text{m}\,\text{s}^{-2}$.

Find T. [7]

$R(\nwarrow)$: $R + T\sin 26 - 80\cos 18 = 0$

$R = 80\cos 18 - T\sin 26$

$R(\nearrow)$: $T\cos 26 - 80\sin 18 - F = ma$

$T\cos 26 - 80\sin 18 - \mu R = 8 \times 0.2$

$T\cos 26 - 80\sin 18 - 0.65(80\cos 18 - T\sin 26) = 1.6$

$T\cos 26 - 80\sin 18 - 52\cos 18 + 0.65T\sin 26 = 1.6$

$T\cos 26 + 0.65T\sin 26 = 1.6 + 80\sin 18 + 52\cos 18$

$T(\cos 26 + 0.65\sin 26) = 1.6 + 80\sin 18 + 52\cos 18$

$T = \dfrac{1.6 + 80\sin 18 + 52\cos 18}{\cos 26 + 0.65\sin 26}$

$T = \underline{64.0\text{N}}$

(b) The block is initially at rest.

Find the distance travelled by the block during the fourth second of motion. [2]

$t=3$:

$s=$
$u=0$
$v=$
$a=0.2$
$t=3$

$s=ut+\frac{1}{2}at^2$
$=0+\frac{1}{2}(0.2)\times3^2$
$s=0.9\text{m}$

$t=4$:

$s=$
$u=0$
$v=$
$a=0.2$
$t=4$

$s=ut+\frac{1}{2}at^2$
$=0+\frac{1}{2}(0.2)\times4^2$
$s=1.6\text{m}$

so distance travelled in fourth second $=1.6-0.9$
$=0.7\text{m}$

6 A block of mass 5 kg is placed on a plane inclined at 30° to the horizontal. The coefficient of friction between the block and the plane is μ.

(a)

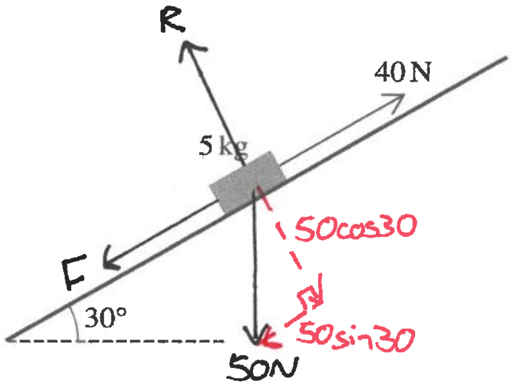


Fig. 6.1

When a force of magnitude 40 N is applied to the block, acting up the plane parallel to a line of greatest slope, the block begins to slide up the plane (see Fig. 6.1).

Show that $\mu < \frac{1}{5}\sqrt{3}$. [4]

$R(\nwarrow): \quad R - 50\cos 30 = 0$

$R = 50\cos 30$

$R = 25\sqrt{3}\ N$

$R(\nearrow): \quad 40 - 50\sin 30 - F > 0$

$40 - 25 - \mu R > 0$ because particle is accelerating up the plane

$15 - \mu \times 25\sqrt{3} > 0$

$15 > 25\sqrt{3}\mu$

$25\sqrt{3}\mu < 15$

$\mu < \frac{15}{25\sqrt{3}}$

$\mu < \frac{3}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\mu < \frac{3\sqrt{3}}{15}$

$\mu < \frac{\sqrt{3}}{5}$ QED

(b)

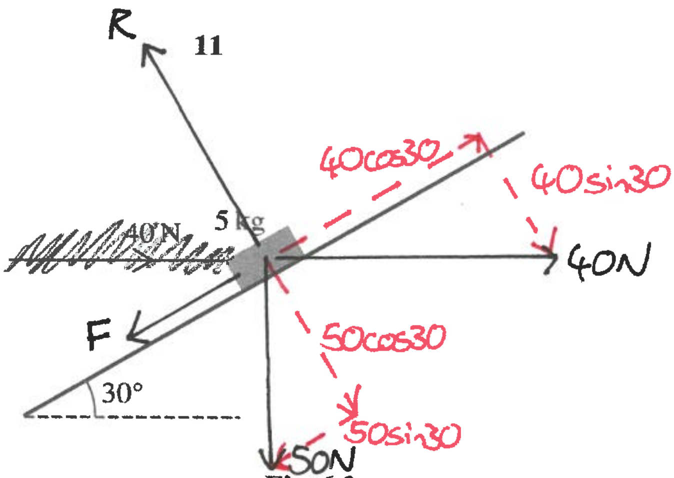


Fig. 6.2

When a force of magnitude 40 N is applied horizontally, in a vertical plane containing a line of greatest slope, the block does not move (see Fig. 6.2).

Show that, correct to 3 decimal places, the least possible value of μ is 0.152. [4]

$40\cos 30 > 50\sin 30$, so without friction it would move up the plane, so friction is down the plane.

Least possible value of μ is when block is in limiting equilibrium ($F = \mu R$)

$R(\nwarrow)$: $R - 40\sin 30 - 50\cos 30 = 0$

$R = 40\sin 30 + 50\cos 30$

$R = 20 + 25\sqrt{3}$

$R(\nearrow)$: $40\cos 30 - 50\sin 30 - F = 0$

$20\sqrt{3} - 25 - \mu R = 0$

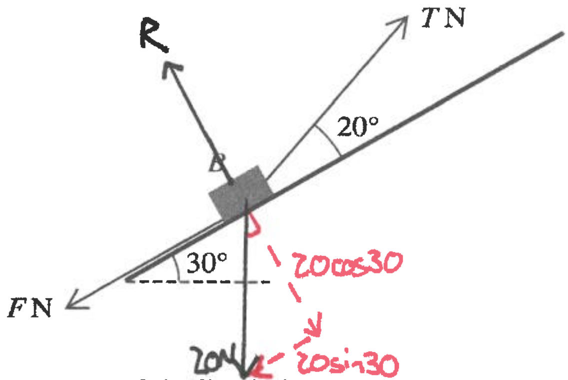
$20\sqrt{3} - 25 - \mu(20 + 25\sqrt{3}) = 0$

$\mu(20 + 25\sqrt{3}) = 20\sqrt{3} - 25$

$\mu = \dfrac{20\sqrt{3} - 25}{20 + 25\sqrt{3}}$

$\mu = 0.152$ QED

6



A block B, of mass 2 kg, lies on a rough inclined plane sloping at 30° to the horizontal. A light rope, inclined at an angle of 20° above a line of greatest slope, is attached to B. The tension in the rope is T N. There is a friction force of F N acting on B (see diagram). The coefficient of friction between B and the plane is μ.

(a) It is given that $F = 5$ and that the acceleration of B up the plane is $1.2\,\mathrm{m\,s^{-2}}$.

(i) Find the value of T. [3]

$R(\nearrow)$: $T\cos 20 - 20\sin 30 - F = ma$

$T\cos 20 - 10 - 5 = 2 \times 1.2$

$T\cos 20 - 15 = 2.4$

$T\cos 20 = 17.4$

$T = \dfrac{17.4}{\cos 20}$

$T = \underline{18.5\,\mathrm{N}}$ STO

(ii) Find the value of μ. [3]

$R(\nwarrow)$: $R + T\sin 20 - 20\cos 30 = 0$

$R + 18.5\sin 20 - 20\cos 30 = 0$

$R = 20\cos 30 - 18.5\sin 20$

$R = \underline{10.987\,\mathrm{N}}$ STO

Block is in motion so $F = \mu R$:

$5 = \mu \times 10.987$

$\mu = \underline{0.455}$

(b) It is given instead that $\mu = 0.8$ and $T = 15$.

Determine whether B will move up the plane. [3]

$R(\nwarrow): \quad R + 15\sin 20 - 20\cos 30 = 0$

$R = 20\cos 30 - 15\sin 20$

$\underline{R = 12.19 \text{ N}}$ STO

Max possible value of friction, $F = \mu R$:

$F = 0.8 \times 12.19$

$= \underline{9.75 \text{ N}}$ STO

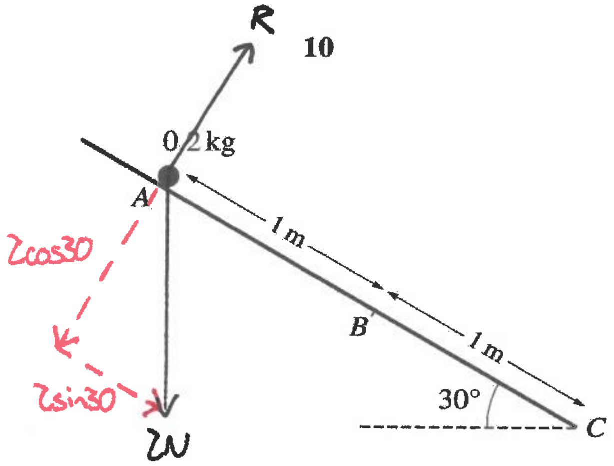
Resolve up the plane (without friction)

$R(\nearrow): \quad 15\cos 20 - 20\sin 30$

$= 4.095 \text{N}$

→ Resultant force up the plane (without friction) is 4.095 N, but the maximum possible value of friction is greater than this (9.75N) so the block won't move.

7



Three points A, B and C lie on a line of greatest slope of a plane inclined at an angle of 30° to the horizontal, with $AB = 1$ m and $BC = 1$ m, as shown in the diagram. A particle of mass 0.2 kg is released from rest at A and slides down the plane. The part of the plane from A to B is smooth. The part of the plane from B to C is rough, with coefficient of friction μ between the plane and the particle.

(a) Given that $\mu = \frac{1}{2}\sqrt{3}$, find the speed of the particle at C. [8]

$R(\nearrow)$: $R - 2\cos30 = 0$

$R = 2\cos30$

$\underline{R = \sqrt{3}\,N}$

Smooth section:

$R(\searrow)$: $2\sin30 = ma$

$1 = 0.2a$

$\underline{a = 5\,ms^{-2}}$

$s = 1$	$v^2 = u^2 + 2as$
$u = 0$	$v^2 = 0^2 + 2\times5\times1$
$v =$	$v^2 = 10$
$a = 5$	$\underline{v = \sqrt{10}\,ms^{-1}}$
$t =$	

Rough section:

$R(\searrow)$: $2\sin30 - \mu R = ma$

$1 - \frac{1}{2}\sqrt{3}\times\sqrt{3} = 0.2a$

$1 - 1.5 = 0.2a$

$-0.5 = 0.2a$

$\underline{a = -2.5\,ms^{-2}}$

$s = 1$	$v^2 = u^2 + 2as$
$u = \sqrt{10}$	$= (\sqrt{10})^2 + 2(-2.5) \times 1$
$v =$	$= 10 - 5$
$a = -2.5$	$v^2 = 5$
$t =$	$v = \sqrt{5} \longrightarrow \underline{\underline{v = 2.24\,ms^{-1}}}$

(b) Given instead that the particle comes to rest at C, find the exact value of μ. **[4]**

Find required acceleration for $v=0$:

$s = 1$	$v^2 = u^2 + 2as$
$u = \sqrt{10}$	$0^2 = (\sqrt{10})^2 + 2 \times a \times 1$
$v = 0$	$0 = 10 + 2a$
$a =$	$2a = -10$
$t =$	$\underline{a = -5\,ms^{-2}}$

Now resolve forces for rough section again:

$R(\searrow)$: $2\sin 30 - \mu R = ma$

$1 - \mu \times \sqrt{3} = 0.2 \times -5$

$1 - \sqrt{3}\mu = -1$

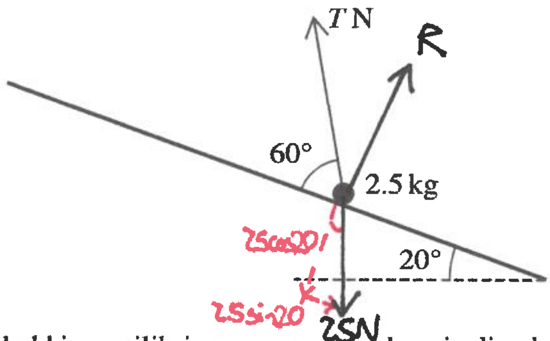
$-\sqrt{3}\mu = -2$

$\sqrt{3}\mu = 2$

$\mu = \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$\underline{\underline{\mu = \frac{2\sqrt{3}}{3}}}$

3



A particle of mass 2.5 kg is held in equilibrium on a rough plane inclined at 20° to the horizontal by a force of magnitude T N making an angle of 60° with a line of greatest slope of the plane (see diagram). The coefficient of friction between the particle and the plane is 0.3.

Find the greatest and least possible values of T. [8]

For greatest possible value of T, the particle is about to move up the plane and friction is acting down the plane:

$R(\uparrow)$: $R + T\sin 60 - 25\cos 20 = 0$

$R = 25\cos 20 - T\sin 60$

$R(\nwarrow)$: $T\cos 60 - 25\sin 20 - F = 0$

$T\cos 60 - 25\sin 20 - \mu R = 0$

$T\cos 60 - 25\sin 20 - 0.3(25\cos 20 - T\sin 60) = 0$

$T\cos 60 - 25\sin 20 - 7.5\cos 20 + 0.3T\sin 60 = 0$

$T\cos 60 + 0.3T\sin 60 = 7.5\cos 20 + 25\sin 20$

$T(\cos 60 + 0.3\sin 60) = 7.5\cos 20 + 25\sin 20$

$$T = \frac{7.5\cos 20 + 25\sin 20}{\cos 60 + 0.3\sin 60}$$

$T = 20.5\text{N}$

Continued →

For least possible value of T, the particle is about to move <u>down</u> the plane and friction is acting <u>up</u> the plane.

R is unchanged: $R = 25\cos 20 - T\sin 60$

$R(\searrow)$: $25\sin 20 - T\cos 60 - F = 0$

$25\sin 20 - T\cos 60 - \mu R = 0$

$25\sin 20 - T\cos 60 - 0.3(25\cos 20 - T\sin 60) = 0$

$25\sin 20 - T\cos 60 - 7.5\cos 20 + 0.3T\sin 60 = 0$

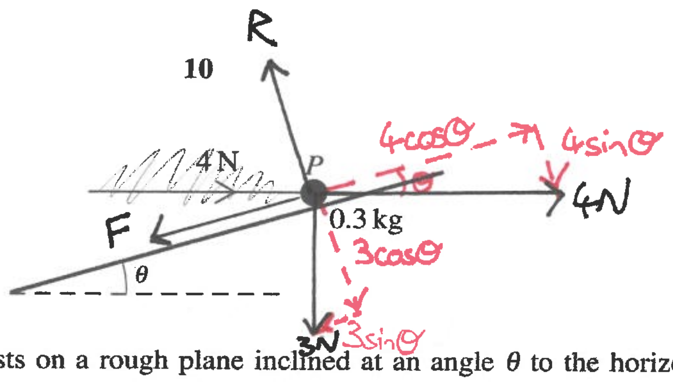
$0.3T\sin 60 - T\cos 60 = 7.5\cos 20 - 25\sin 20$

$T(0.3\sin 60 - \cos 60) = 7.5\cos 20 - 25\sin 20$

$$T = \frac{7.5\cos 20 - 25\sin 20}{0.3\sin 60 - \cos 60}$$

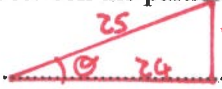
$T = \underline{6.26\text{N}}$

7



A particle P of mass 0.3 kg rests on a rough plane inclined at an angle θ to the horizontal, where $\sin\theta = \frac{7}{25}$. A horizontal force of magnitude 4 N, acting in the vertical plane containing a line of greatest slope of the plane, is applied to P (see diagram). The particle is on the point of sliding up the plane.

(a) Show that the coefficient of friction between the particle and the plane is $\frac{3}{4}$. [4]

$\sin\Theta = \frac{7}{25} \rightarrow$ 

$\cos\Theta = \frac{24}{25}$ $\tan\Theta = \frac{7}{24}$

$R(\nwarrow)$: $R - 3\cos\Theta - 4\sin\Theta = 0$

$R - 3 \times \frac{24}{25} - 4 \times \frac{7}{25} = 0$

$R - 4 = 0$

$\underline{R = 4\text{ N}}$

$R(\nearrow)$: $4\cos\Theta - 3\sin\Theta - F = 0$

$4 \times \frac{24}{25} - 3 \times \frac{7}{25} - \mu R = 0$

$3 - \mu R = 0$

$3 - 4\mu = 0$

$4\mu = 3$

$\underline{\mu = 3/4}$ QED

The force acting horizontally is replaced by a force of magnitude 4 N acting up the plane parallel to a line of greatest slope.

(b) Find the acceleration of P. [3]

$R(\nwarrow)$: $R - 3\cos\Theta = 0$

$R = 3 \times \frac{24}{25}$

$R = \underline{2.88\text{ N}}$

$R(\nearrow)$: $4 - 3\sin\Theta - F = ma$

$4 - 3 \times \frac{7}{25} - \mu R = 0.3a$

$4 - 0.84 - \frac{3}{4} \times 2.88 = 0.3a$

$4 - 0.84 - 2.16 = 0.3a$

$1 = 0.3a$

$a = 3.33 ms^{-2}$

STO

(c) Starting with P at rest, the force of 4 N parallel to the plane acts for 3 seconds and is then removed.

Find the total distance travelled until P comes to instantaneous rest. [3]

$s =$

$u = 0$

$v =$

$a = \frac{10}{3}$ (3.33..)

$t = 3$

$s = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times \left(\frac{10}{3}\right) \times 3^2$

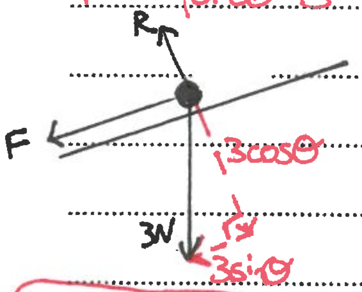
$s = 15m$

$v = u + at$

$= 0 + \frac{10}{3} \times 3$

$v = 10 ms^{-1}$

After 3s, P has travelled 15m and is moving at 10ms⁻¹. 4N force is removed. Particle is still moving UP, so friction is acting down.



$R(\nearrow): -3\sin\theta - F = ma$

$-3 \times \frac{7}{25} - \mu R = 0.3a$

$-0.84 - \frac{3}{4} \times 2.88 = 0.3a$

$-3 = 0.3a$

$a = -10$

$s =$

$u = 10$

$v = 0$

$a = -10$

$t =$

$v^2 = u^2 + 2as$

$0^2 = 10^2 + 2(-10)s$

$0 = 100 - 20s$

$20s = 100$

$s = 5m$ → P moves $15 + 5 = 20m$