1 Solve the equation $8\sin^2\theta + 6\cos\theta + 1 = 0$ for $0° < \theta < 180°$. [3]

5 Solve the equation

$$\frac{\tan\theta + 3\sin\theta + 2}{\tan\theta - 3\sin\theta + 1} = 2$$

for $0° \leqslant \theta \leqslant 90°$. [5]

1 Solve the equation $2\cos\theta = 7 - \dfrac{3}{\cos\theta}$ for $-90° < \theta < 90°$. [4]

1 Solve the equation $4\sin\theta + \tan\theta = 0$ for $0° < \theta < 180°$. [3]

6 **(a)** Show that the equation

$$\frac{1}{\sin\theta + \cos\theta} + \frac{1}{\sin\theta - \cos\theta} = 1$$

may be expressed in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are constants to be found. [3]

(b) Hence solve the equation $\dfrac{1}{\sin\theta + \cos\theta} + \dfrac{1}{\sin\theta - \cos\theta} = 1$ for $0° \leqslant \theta \leqslant 360°$. [3]

6 **(a)** Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$. [4]

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(b) Hence solve the equation $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

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7 **(a)** Show that $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} \equiv 2\tan^2\theta$. [3]

(b) Hence solve the equation $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} = 8$, for $0° < \theta < 180°$. [3]

10 **(a)** Prove the identity $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} \equiv \dfrac{4\tan x}{\cos x}$. [4]

(b) Hence solve the equation $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} = 8\tan x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [3]

7 **(a)** Prove the identity $\dfrac{1 - 2\sin^2\theta}{1 - \sin^2\theta} \equiv 1 - \tan^2\theta$. [2]

(b) Hence solve the equation $\dfrac{1 - 2\sin^2\theta}{1 - \sin^2\theta} = 2\tan^4\theta$ for $0° \leqslant \theta \leqslant 180°$. [3]

7 **(a)** Show that the equation $\dfrac{\tan x + \cos x}{\tan x - \cos x} = k$, where k is a constant, can be expressed as

$$(k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0.$$ [4]

(b) Hence solve the equation $\dfrac{\tan x + \cos x}{\tan x - \cos x} = 4$ for $0° \leqslant x \leqslant 360°$. [4]

4 **(a)** Show that the equation

$$3\tan^2 x - 3\sin^2 x - 4 = 0$$

may be expressed in the form $a\cos^4 x + b\cos^2 x + c = 0$, where a, b and c are constants to be found. [3]

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(b) Hence solve the equation $3\tan^2 x - 3\sin^2 x - 4 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

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4 **(a)** Prove the identity $\dfrac{\sin^3\theta}{\sin\theta - 1} - \dfrac{\sin^2\theta}{1 + \sin\theta} \equiv -\tan^2\theta(1 + \sin^2\theta)$. [4]

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(b) Hence solve the equation

$$\frac{\sin^3\theta}{\sin\theta - 1} - \frac{\sin^2\theta}{1 + \sin\theta} = \tan^2\theta(1 - \sin^2\theta)$$

for $0 < \theta < 2\pi$. [2]

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(b) Find the exact solutions of the equation $4\sin\left(\frac{1}{2}x - 30°\right) = 2\sqrt{2}$ for $0° \leqslant x \leqslant 360°$. [3]

7 **(a)** **(i)** By first expanding $(\cos\theta + \sin\theta)^2$, find the three solutions of the equation

$$(\cos\theta + \sin\theta)^2 = 1$$

for $0 \leqslant \theta \leqslant \pi$. [3]

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(ii) Hence verify that the only solutions of the equation $\cos\theta + \sin\theta = 1$ for $0 \leqslant \theta \leqslant \pi$ are 0 and $\frac{1}{2}\pi$. [2]

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(b) Prove the identity $\dfrac{\sin\theta}{\cos\theta+\sin\theta}+\dfrac{1-\cos\theta}{\cos\theta-\sin\theta}\equiv\dfrac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}$. [3]

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(c) Using the results of **(a)(ii)** and **(b)**, solve the equation

$$\frac{\sin\theta}{\cos\theta+\sin\theta}+\frac{1-\cos\theta}{\cos\theta-\sin\theta}=2(\cos\theta+\sin\theta-1)$$

for $0\leqslant\theta\leqslant\pi$. [3]

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