6 It is given that $\alpha = \cos^{-1}\left(\frac{8}{17}\right)$.

Find, without using the trigonometric functions on your calculator, the exact value of $\dfrac{1}{\sin\alpha} + \dfrac{1}{\tan\alpha}$. [5]

2 **(a)** Express the equation $3\cos\theta = 8\tan\theta$ as a quadratic equation in $\sin\theta$. [3]

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(b) Hence find the acute angle, in degrees, for which $3\cos\theta = 8\tan\theta$. [2]

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3 Solve the equation $\dfrac{\tan\theta + 2\sin\theta}{\tan\theta - 2\sin\theta} = 3$ for $0° < \theta < 180°$. [4]

3 Solve the equation $3\tan^2\theta + 1 = \dfrac{2}{\tan^2\theta}$ for $0° < \theta < 180°$. [5]

3 **(a)** Find the set of values of k for which the equation $8x^2 + kx + 2 = 0$ has no real roots. [2]

(b) Solve the equation $8\cos^2\theta - 10\cos\theta + 2 = 0$ for $0° \leqslant \theta \leqslant 180°$. [3]

7 **(a)** Show that $\dfrac{\tan\theta}{1+\cos\theta}+\dfrac{\tan\theta}{1-\cos\theta}\equiv\dfrac{2}{\sin\theta\cos\theta}$. [4]

(b) Hence solve the equation $\frac{\tan\theta}{1+\cos\theta} + \frac{\tan\theta}{1-\cos\theta} = \frac{6}{\tan\theta}$ for $0° < \theta < 180°$. [4]

7 **(a)** Prove the identity $\dfrac{1 + \sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1 + \sin\theta} \equiv \dfrac{2}{\cos\theta}$. [3]

(b) Hence solve the equation $\frac{1 + \sin\theta}{\cos\theta} + \frac{\cos\theta}{1 + \sin\theta} = \frac{3}{\sin\theta}$, for $0 \leqslant \theta \leqslant 2\pi$. [3]

7 **(a)** Prove the identity $\dfrac{\sin\theta}{\sin\theta + \cos\theta} + \dfrac{\cos\theta}{\sin\theta - \cos\theta} \equiv \dfrac{\tan^2\theta + 1}{\tan^2\theta - 1}$. [3]

(b) Hence find the exact solutions of the equation $\dfrac{\sin\theta}{\sin\theta+\cos\theta}+\dfrac{\cos\theta}{\sin\theta-\cos\theta}=2$ for $0 \leqslant \theta \leqslant \pi$. [4]

7 **(a)** Show that $\dfrac{\sin\theta + 2\cos\theta}{\cos\theta - 2\sin\theta} - \dfrac{\sin\theta - 2\cos\theta}{\cos\theta + 2\sin\theta} \equiv \dfrac{4}{5\cos^2\theta - 4}$. [4]

(b) Hence solve the equation $\dfrac{\sin\theta + 2\cos\theta}{\cos\theta - 2\sin\theta} - \dfrac{\sin\theta - 2\cos\theta}{\cos\theta + 2\sin\theta} = 5$ for $0° < \theta < 180°$. [3]

3 Solve, by factorising, the equation

$$6\cos\theta\tan\theta - 3\cos\theta + 4\tan\theta - 2 = 0,$$

for $0° \leqslant \theta \leqslant 180°$. [4]

7 **(a)** By first obtaining a quadratic equation in $\cos\theta$, solve the equation

$$\tan\theta\sin\theta = 1$$

for $0° < \theta < 360°$. [5]

(b) Show that $\frac{\tan\theta}{\sin\theta} - \frac{\sin\theta}{\tan\theta} \equiv \tan\theta\sin\theta$. [3]

4 **(a)** Show that the equation

$$\frac{\tan x + \sin x}{\tan x - \sin x} = k,$$

where k is a constant, may be expressed as

$$\frac{1 + \cos x}{1 - \cos x} = k.$$ [2]

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(b) Hence express $\cos x$ in terms of k. [2]

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(c) Hence solve the equation $\dfrac{\tan x + \sin x}{\tan x - \sin x} = 4$ for $-\pi < x < \pi$. [2]

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5 **(a)** Solve the equation $6\sqrt{y} + \dfrac{2}{\sqrt{y}} - 7 = 0$. [4]

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(b) Hence solve the equation $6\sqrt{\tan x} + \dfrac{2}{\sqrt{\tan x}} - 7 = 0$ for $0° \leqslant x \leqslant 360°$. [3]

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11 The function f is given by $f(x) = 4\cos^4 x + \cos^2 x - k$ for $0 \leqslant x \leqslant 2\pi$, where k is a constant.

(a) Given that $k = 3$, find the exact solutions of the equation $f(x) = 0$. [5]

(b) Use the quadratic formula to show that, when $k > 5$, the equation $f(x) = 0$ has no solutions. [5]

11 **(a)** Solve the equation $3\tan^2 x - 5\tan x - 2 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

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(b) Find the set of values of k for which the equation $3\tan^2 x - 5\tan x + k = 0$ has no solutions. [2]

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(c) For the equation $3\tan^2 x - 5\tan x + k = 0$, state the value of k for which there are three solutions in the interval $0° \leqslant x \leqslant 180°$, and find these solutions. [3]