

- 6** It is given that $\alpha = \cos^{-1}\left(\frac{8}{17}\right)$.

Find, without using the trigonometric functions on your calculator, the exact value of $\frac{1}{\sin \alpha} + \frac{1}{\tan \alpha}$.

[5]

[illegible]

- 2 (a) Express the equation $3 \cos \theta = 8 \tan \theta$ as a quadratic equation in $\sin \theta$. [3]

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- (b) Hence find the acute angle, in degrees, for which $3 \cos \theta = 8 \tan \theta$. [2]

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- 3** Solve the equation $\frac{\tan \theta + 2 \sin \theta}{\tan \theta - 2 \sin \theta} = 3$ for $0^\circ < \theta < 180^\circ$. [4]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- [5]

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- 3 (a) Find the set of values of k for which the equation $8x^2 + kx + 2 = 0$ has no real roots. [2]

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- (b) Solve the equation $8 \cos^2 \theta - 10 \cos \theta + 2 = 0$ for $0^\circ \leq \theta \leq 180^\circ$. [3]

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7 (a) Show that $\frac{\tan \theta}{1 + \cos \theta} + \frac{\tan \theta}{1 - \cos \theta} \equiv \frac{2}{\sin \theta \cos \theta}$. [4]

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[illegible]

- 7 (a)** Prove the identity $\frac{\sin \theta}{\sin \theta + \cos \theta} + \frac{\cos \theta}{\sin \theta - \cos \theta} \equiv \frac{\tan^2 \theta + 1}{\tan^2 \theta - 1}$. [3]

[illegible]

[illegible]

7 (a) Show that $\frac{\sin \theta + 2 \cos \theta}{\cos \theta - 2 \sin \theta} - \frac{\sin \theta - 2 \cos \theta}{\cos \theta + 2 \sin \theta} \equiv \frac{4}{5 \cos^2 \theta - 4}$. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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3 Solve, by factorising, the equation

$$6 \cos \theta \tan \theta - 3 \cos \theta + 4 \tan \theta - 2 = 0,$$

for $0^\circ \leq \theta \leq 180^\circ$.

[4]

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- 7 (a)** By first obtaining a quadratic equation in $\cos \theta$, solve the equation

$$\tan \theta \sin \theta = 1$$

for $0^\circ < \theta < 360^\circ$.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dashed lines, creating a series of uniform gaps for letter height. The lines are evenly spaced across the entire page, providing a guide for consistent letter formation. There is no text or other markings on the page.

- 4 (a) Show that the equation

$$\frac{\tan x + \sin x}{\tan x - \sin x} = k,$$

where k is a constant, may be expressed as

$$\frac{1 + \cos x}{1 - \cos x} = k. \quad [2]$$

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- (b) Hence express $\cos x$ in terms of k . [2]

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- (c) Hence solve the equation $\frac{\tan x + \sin x}{\tan x - \sin x} = 4$ for $-\pi < x < \pi$. [2]

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- 5 (a) Solve the equation $6\sqrt{y} + \frac{2}{\sqrt{y}} - 7 = 0$. [4]

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- (b) Hence solve the equation $6\sqrt{\tan x} + \frac{2}{\sqrt{\tan x}} - 7 = 0$ for $0^\circ \leq x \leq 360^\circ$. [3]

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- 11** The function f is given by $f(x) = 4 \cos^4 x + \cos^2 x - k$ for $0 \leq x \leq 2\pi$, where k is a constant.

(a) Given that $k = 3$, find the exact solutions of the equation $f(x) = 0$.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b)** Use the quadratic formula to show that, when $k > 5$, the equation $f(x) = 0$ has no solutions. [5]

This image shows a full page of a document template designed for handwriting practice or general note-taking. It consists of approximately 20 evenly spaced, horizontal dotted lines running across the width of the page. The background is plain white, and there are no margins, headers, footers, or other markings present.

- 11 (a)** Solve the equation $3 \tan^2 x - 5 \tan x - 2 = 0$ for $0^\circ \leq x \leq 180^\circ$. [4]

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- (b)** Find the set of values of k for which the equation $3 \tan^2 x - 5 \tan x + k = 0$ has no solutions. [2]

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- (c) For the equation $3 \tan^2 x - 5 \tan x + k = 0$, state the value of k for which there are three solutions in the interval $0^\circ \leq x \leq 180^\circ$, and find these solutions. [3]

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