7 The curve $y = f(x)$ is such that $f'(x) = \dfrac{-3}{(x+2)^4}$.

(a) The tangent at a point on the curve where $x = a$ has gradient $-\frac{16}{27}$.

Find the possible values of a. [4]

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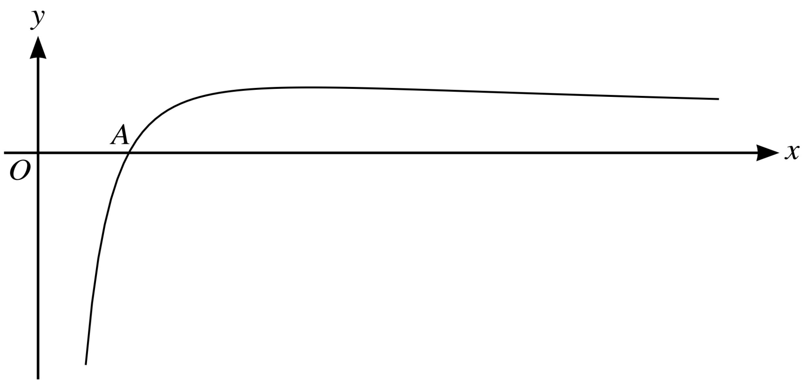
11



The diagram shows the curve with equation $y = 9\left(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}\right)$. The curve crosses the x-axis at the point A.

(a) Find the x-coordinate of A. [2]

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(b) Find the equation of the tangent to the curve at A. [4]

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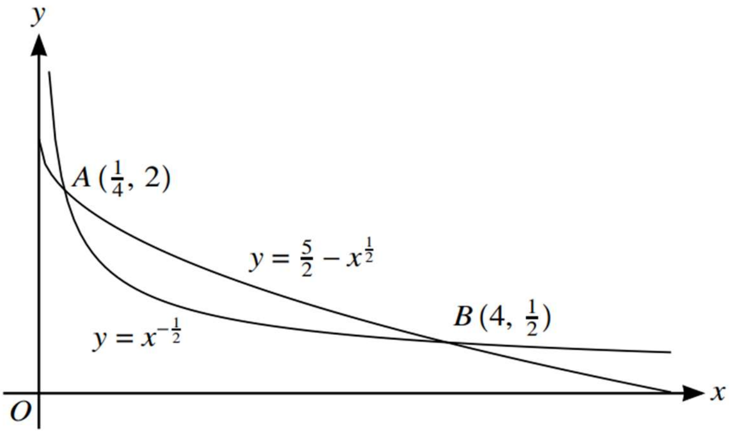

The diagram shows the curves with equations $y = x^{-\frac{1}{2}}$ and $y = \frac{5}{2} - x^{\frac{1}{2}}$. The curves intersect at the points $A\left(\frac{1}{4}, 2\right)$ and $B\left(4, \frac{1}{2}\right)$.

(b) The normal to the curve $y = x^{-\frac{1}{2}}$ at the point $(1, 1)$ intersects the y-axis at the point $(0, p)$.

Find the value of p. [4]

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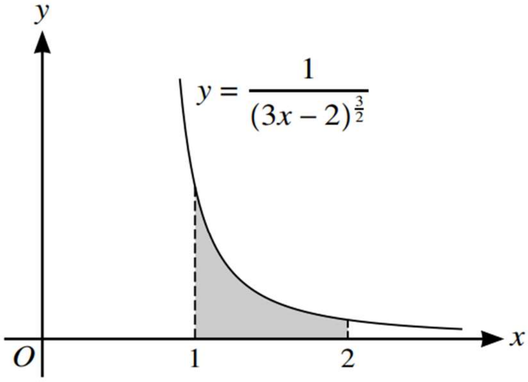

The diagram shows the curve with equation $y = \frac{1}{(3x-2)^{\frac{3}{2}}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$. The shaded region is rotated through 360° about the x-axis.

The normal to the curve at the point $(1,\ 1)$ crosses the y-axis at the point A.

(c) Find the y-coordinate of A. [4]

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10 A curve has equation $y = \frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}$ where $x > 0$ and k is a positive constant.

(a) It is given that when $x = \frac{1}{4}$, the gradient of the curve is 3.

Find the value of k. [4]

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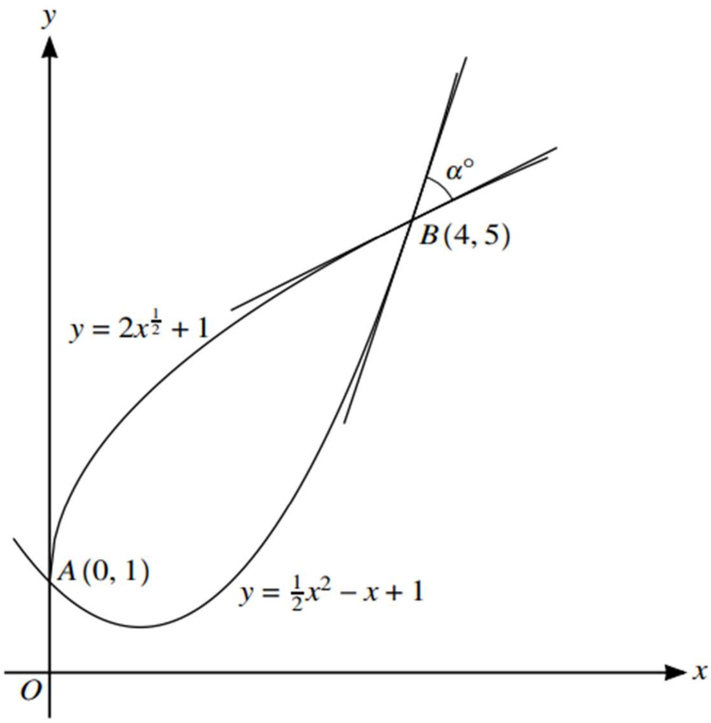

Curves with equations $y = 2x^{\frac{1}{2}} + 1$ and $y = \frac{1}{2}x^2 - x + 1$ intersect at $A\,(0,\ 1)$ and $B\,(4,\ 5)$, as shown in the diagram.

The acute angle between the two tangents at B is denoted by $\alpha°$, and the scales on the axes are the same.

(b) Find α. [5]

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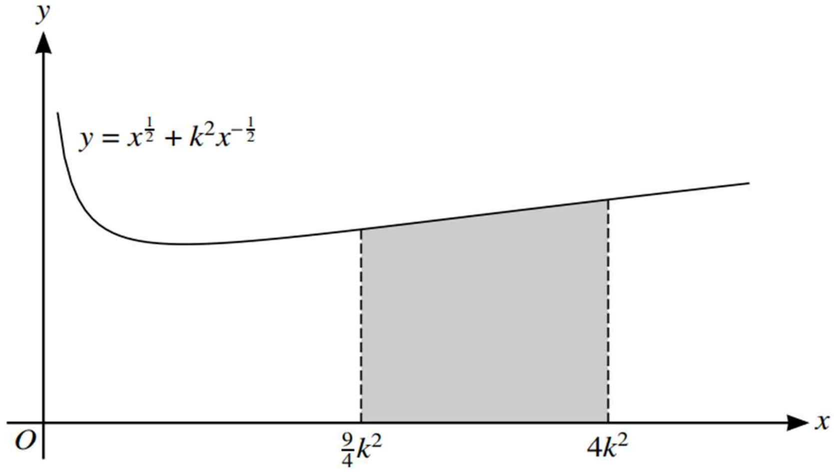

The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$, where k is a positive constant.

The tangent at the point on the curve where $x = 4k^2$ intersects the y-axis at P.

(b) Find the y-coordinate of P in terms of k. [4]

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10 Functions f and g are defined as follows:

$$f(x) = \frac{2x+1}{2x-1} \quad \text{for } x \neq \tfrac{1}{2},$$

$$g(x) = x^2 + 4 \quad \text{for } x \in \mathbb{R}.$$

(e) Show that $1 + \dfrac{2}{2x-1}$ can be expressed as $\dfrac{2x+1}{2x-1}$. Hence find the area of the triangle enclosed by the tangent to the curve $y = f(x)$ at the point where $x = 1$ and the x- and y-axes. [6]

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6 The equation of a curve is $y = 2 + \sqrt{25 - x^2}$.

Find the coordinates of the point on the curve at which the gradient is $\frac{4}{3}$. [5]

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