1 Solve the equation $8\sin^2\theta + 6\cos\theta + 1 = 0$ for $0° < \theta < 180°$. [3]

$8\sin^2\theta + 6\cos\theta + 1 = 0$

$\uparrow 1-\cos^2\theta$

$8(1-\cos^2\theta) + 6\cos\theta + 1 = 0$

$8 - 8\cos^2\theta + 6\cos\theta + 1 = 0$

$-8\cos^2\theta + 6\cos\theta + 9 = 0$

$8\cos^2\theta - 6\cos\theta - 9 = 0$

Let $Y = \cos\theta$:

$8Y^2 - 6Y - 9 = 0$

factorise: $ac = -72 \rightarrow -12, 6$

$8Y^2 - 12Y + 6Y - 9 = 0$

$4Y(2Y-3) + 3(2Y-3) = 0$

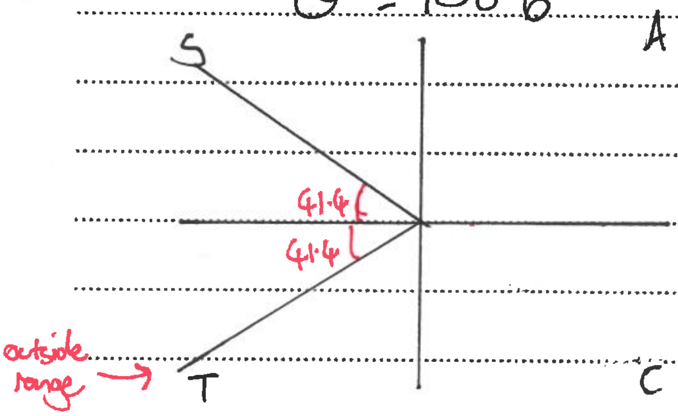
$(4Y+3)(2Y-3) = 0$

$4Y + 3 = 0$ or $2Y - 3 = 0$

$Y = \frac{-3}{4}$ $\quad$ $Y = \frac{3}{2}$

$\cos\theta = -3/4$ $\quad$ $\cos\theta = 3/2$

$\theta = 138.6°$ $\quad$ $\uparrow$ no solutions



$\theta = 138.6°$

5 Solve the equation

$$\frac{\tan\theta + 3\sin\theta + 2}{\tan\theta - 3\sin\theta + 1} = 2$$

for $0° \leqslant \theta \leqslant 90°$. [5]

$\tan\theta + 3\sin\theta + 2 = 2(\tan\theta - 3\sin\theta + 1)$

$\tan\theta + 3\sin\theta + \cancel{2} = 2\tan\theta - 6\sin\theta + \cancel{2}$

$9\sin\theta - \tan\theta = 0$

$9\sin\theta - \frac{\sin\theta}{\cos\theta} = 0$

$\times\cos\theta$ $\times\cos\theta$ $\times\cos\theta$

$9\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(9\cos\theta - 1) = 0$

$\sin\theta = 0$ or $9\cos\theta - 1 = 0$

$\theta = 0$ $\cos\theta = \frac{1}{9}$

$\theta = 83.6°$

$\theta = 0°, 83.6°$

- all other answers are beyond range. $(0 \leqslant \theta \leqslant 90)$

1 Solve the equation $2\cos\theta = 7 - \dfrac{3}{\cos\theta}$ for $-90° < \theta < 90°$. [4]

$\times \cos\theta$ $\times \cos\theta$

$2\cos^2\theta = 7\cos\theta - 3$

$2\cos^2\theta - 7\cos\theta + 3 = 0$

Let $Y = \cos\theta$:

$2Y^2 - 7Y + 3 = 0$

$(2Y-1)(Y-3) = 0$

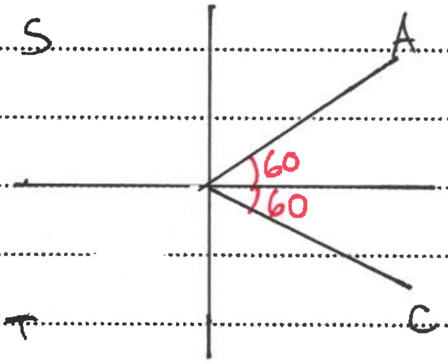
$2Y - 1 = 0$ or $Y - 3 = 0$

$Y = \frac{1}{2}$ $Y = 3$

$\cos\theta = \frac{1}{2}$ $\cos\theta = 3$

↳ no solutions

$\theta = 60°$



$\theta = 60°, -60°$

1 Solve the equation $4\sin\theta + \tan\theta = 0$ for $0° < \theta < 180°$. [3]

$$4\sin\theta + \frac{\sin\theta}{\cos\theta} = 0$$

$\times\cos\theta$ $\times\cos\theta$ $\times\cos\theta$

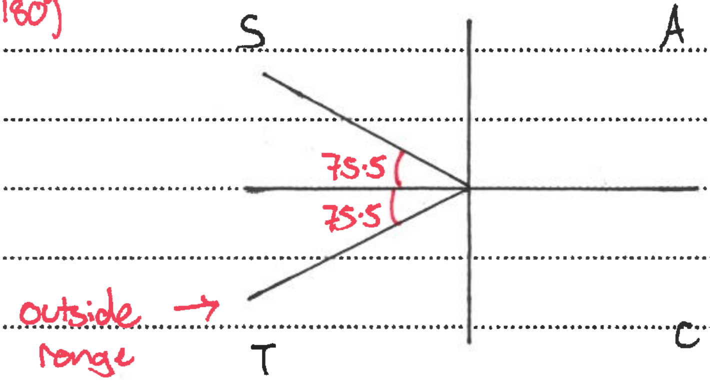
$$4\sin\theta\cos\theta + \sin\theta = 0$$

$$\sin\theta(4\cos\theta + 1) = 0$$

$\sin\theta = 0$ or $4\cos\theta + 1 = 0$

$\theta = 0°$ $\quad\quad$ $\cos\theta = \frac{-1}{4}$

outside range (and so is 180) $\quad\quad$ $\theta = 104.5°$



$$\underline{\theta = 104.5°}$$

6 **(a)** Show that the equation

$$\frac{1}{\sin\theta + \cos\theta} + \frac{1}{\sin\theta - \cos\theta} = 1$$

may be expressed in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are constants to be found. [3]

$$\frac{1}{\sin\theta + \cos\theta} \times\frac{(\sin\theta - \cos\theta)}{(\sin\theta - \cos\theta)} + \frac{1}{\sin\theta - \cos\theta} \times\frac{(\sin\theta + \cos\theta)}{(\sin\theta + \cos\theta)} = 1$$

$$\frac{\sin\theta - \cos\theta}{\sin^2\theta - \cos^2\theta} + \frac{\sin\theta + \cos\theta}{\sin^2\theta - \cos^2\theta} = 1$$

$$\frac{\sin\theta - \cancel{\cos\theta} + \sin\theta + \cancel{\cos\theta}}{\sin^2\theta - \cos^2\theta} = 1$$

$$\frac{2\sin\theta}{\sin^2\theta - \cos^2\theta} = 1$$

$$2\sin\theta = \sin^2\theta - \cos^2\theta$$

$1 - \sin^2\theta$

$$2\sin\theta = \sin^2\theta - (1 - \sin^2\theta)$$

$$2\sin\theta = \sin^2\theta - 1 + \sin^2\theta$$

$$2\sin\theta = 2\sin^2\theta - 1$$

$$2\sin^2\theta - 2\sin\theta - 1 = 0$$ QED

(b) Hence solve the equation $\dfrac{1}{\sin\theta + \cos\theta} + \dfrac{1}{\sin\theta - \cos\theta} = 1$ for $0° \leqslant \theta \leqslant 360°$. [3]

from (a): $2\sin^2\theta - 2\sin\theta - 1 = 0$

Let $Y = \sin\theta$:

$$2Y^2 - 2Y - 1 = 0$$

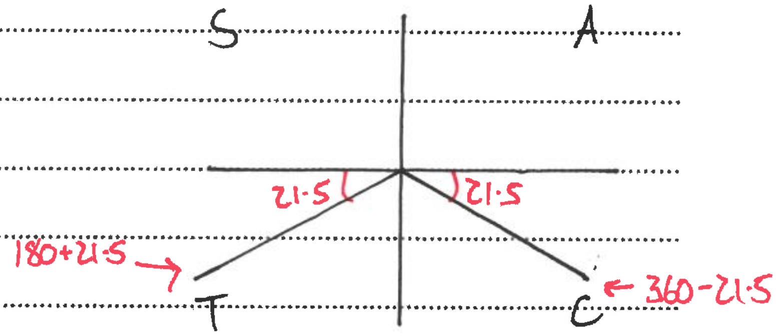
$$Y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$$

$$Y = \frac{1+\sqrt{3}}{2} \quad \text{or} \quad Y = \frac{1-\sqrt{3}}{2}$$

$$\sin\theta = \frac{1+\sqrt{3}}{2} \qquad \sin\theta = \frac{1-\sqrt{3}}{2}$$

no solutions

$$\theta = -21.5°$$



$$\underline{\theta = 201.5°, 338.5°}$$

6 (a) Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$. [4]

$$\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x} + \frac{1 \times \sin x}{1 \times \sin x}\right)$$

$$\left(\frac{1 - \sin x}{\cos x}\right)\left(\frac{1}{\sin x} + \frac{\sin x}{\sin x}\right)$$

$$\left(\frac{1 - \sin x}{\cos x}\right)\left(\frac{1 + \sin x}{\sin x}\right)$$

$$\frac{1 - \sin^2 x}{\sin x \cos x} \leftarrow \cos^2 x$$

$$\frac{\cos^{\cancel{2}} x}{\sin x \cancel{\cos x}}$$

$$\frac{\cos x}{\sin x}$$

$$\underline{\frac{1}{\tan x}} \quad \text{QED}$$

(b) Hence solve the equation $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

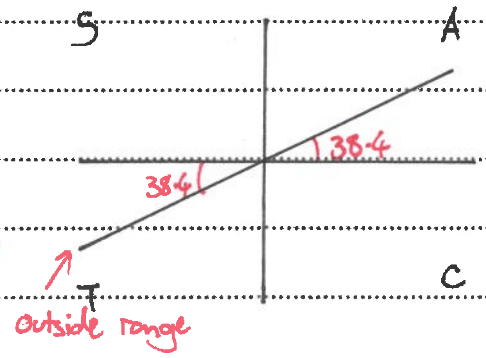
from (a): $\frac{1}{\tan x} = 2\tan^2 x$

$1 = 2\tan^3 x$

$\tan^3 x = \frac{1}{2}$

$\tan x = \sqrt[3]{\frac{1}{2}}$

$\underline{x = 38.4°}$



7 **(a)** Show that $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} \equiv 2\tan^2\theta$. [3]

$$\frac{\sin\theta}{1-\sin\theta}\,\frac{\times(1+\sin\theta)}{\times(1+\sin\theta)} - \frac{\sin\theta}{1+\sin\theta}\,\frac{\times(1-\sin\theta)}{\times(1-\sin\theta)}$$

$$\frac{\sin\theta+\sin^2\theta}{1-\sin^2\theta} - \frac{\sin\theta-\sin^2\theta}{1-\sin^2\theta}$$

$$\frac{\sin\theta+\sin^2\theta-(\sin\theta-\sin^2\theta)}{1-\sin^2\theta}$$

$$\frac{\cancel{\sin\theta}+\sin^2\theta-\cancel{\sin\theta}+\sin^2\theta}{1-\sin^2\theta}$$

$$\frac{2\sin^2\theta}{1-\sin^2\theta} \leftarrow \cos^2\theta$$

$$\frac{2\sin^2\theta}{\cos^2\theta}$$

$$2\tan^2\theta \quad \text{QED}$$

(b) Hence solve the equation $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} = 8$, for $0° < \theta < 180°$. [3]

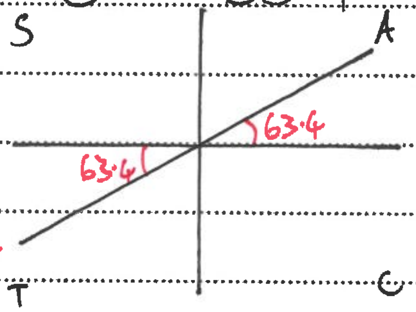
from (a): $2\tan^2\theta = 8$

$\tan^2\theta = 4$

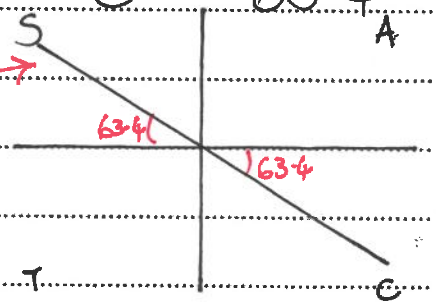
$\tan\theta = 2$ or $\tan\theta = -2$

$\theta = 63.4°$ $\qquad$ $\theta = -63.4°$

outside range



180-63.4



$\theta = 63.4$ $\qquad$ $\theta = 116.6°$

$\underline{\theta = 63.4°, 116.6°}$

10 **(a)** Prove the identity $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} \equiv \dfrac{4\tan x}{\cos x}$. [4]

$$\frac{1+\sin x}{1-\sin x} \times \frac{(1+\sin x)}{(1+\sin x)} - \frac{1-\sin x}{1+\sin x} \times \frac{(1-\sin x)}{(1-\sin x)}$$

$$\frac{1+2\sin x+\sin^2 x}{1-\sin^2 x} - \frac{1-2\sin x+\sin^2 x}{1-\sin^2 x}$$

$$\frac{1+2\sin x+\sin^2 x-(1-2\sin x+\sin^2 x)}{1-\sin^2 x}$$

$$\frac{\cancel{1}+2\sin x+\cancel{\sin^2 x}\cancel{-1}+2\sin x-\cancel{\sin^2 x}}{1-\sin^2 x}$$

$$\frac{4\sin x}{1-\sin^2 x} \leftarrow \cos^2 x$$

$$\frac{4\sin x}{\cos^2 x}$$

$$\frac{4\sin x}{\cos x} \times \frac{1}{\cos x}$$

$$= 4\tan x \times \frac{1}{\cos x}$$

$$= \frac{4\tan x}{\cos x}$$ QED

(b) Hence solve the equation $\frac{1+\sin x}{1-\sin x} - \frac{1-\sin x}{1+\sin x} = 8\tan x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [3]

from (a): $\frac{4\tan x}{\cos x} = 8\tan x$

$4\tan x = 8\tan x\cos x$

$4\tan x - 8\tan x\cos x = 0$

$4\tan x(1 - 2\cos x) = 0$

$4\tan x = 0$ or $1 - 2\cos x = 0$

$\tan x = 0$ $\cos x = \frac{1}{2}$

$x = 0$ $x = \frac{\pi}{3}$

$x = 0, \frac{\pi}{3}$

7 **(a)** Prove the identity $\dfrac{1-2\sin^2\theta}{1-\sin^2\theta} \equiv 1-\tan^2\theta$. [2]

$$\frac{1-2\sin^2\theta}{1-\sin^2\theta} \leftarrow \cos^2\theta$$

$$\frac{1-2\sin^2\theta}{\cos^2\theta}$$

$$\frac{\overbrace{1-\sin^2\theta}^{\cos^2\theta} - \sin^2\theta}{\cos^2\theta}$$

$$\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}$$

$$\frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

$$1 - \tan^2\theta \quad \text{QED}$$

(b) Hence solve the equation $\dfrac{1-2\sin^2\theta}{1-\sin^2\theta} = 2\tan^4\theta$ for $0° \leqslant \theta \leqslant 180°$. [3]

from (a): $1 - \tan^2\theta = 2\tan^4\theta$

$2\tan^4\theta + \tan^2\theta - 1 = 0$

Let $Y = \tan^2\theta$:

$2Y^2 + Y - 1 = 0$

$(2Y-1)(Y+1) = 0$

$2Y - 1 = 0$ or $Y + 1 = 0$

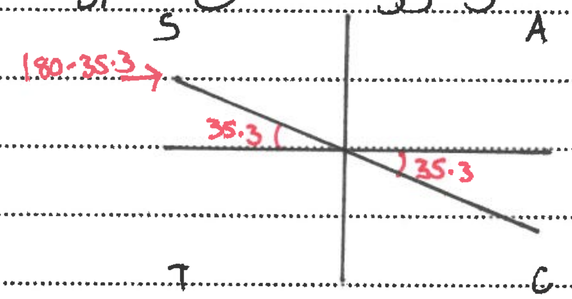
$Y = \frac{1}{2}$ $\qquad Y = -1$

$\tan^2\theta = \frac{1}{2}$ $\qquad \tan^2\theta = -1$

no solutions

$\tan\theta = \sqrt{\frac{1}{2}}$ or $\tan\theta = -\sqrt{\frac{1}{2}}$

$\theta = 35.3°$ or $\theta = -35.3°$



$\theta = 144.7°$

$\theta = 35.3°, 144.7°$

7 **(a)** Show that the equation $\dfrac{\tan x + \cos x}{\tan x - \cos x} = k$, where k is a constant, can be expressed as

$$(k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0.$$

[4]

$$\frac{\tan x + \cos x}{\tan x - \cos x} = k$$

$$\tan x + \cos x = k\tan x - k\cos x$$

$$\frac{\sin x}{\cos x} + \cos x = k\frac{\sin x}{\cos x} - k\cos x$$

$\times\cos x$ $\quad\times\cos x$ $\quad\times\cos x$ $\quad\times\cos x$

$$\sin x + \underbrace{\cos^2 x}_{1-\sin^2 x} = k\sin x - k\underbrace{\cos^2 x}_{1-\sin^2 x}$$

$$\sin x + 1 - \sin^2 x = k\sin x - k(1-\sin^2 x)$$

$$\sin x + 1 - \sin^2 x = k\sin x - k + k\sin^2 x$$

$$k\sin^2 x + \sin^2 x + k\sin x - \sin x - k - 1 = 0$$

$$(k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0 \quad \text{QED}$$

(b) Hence solve the equation $\dfrac{\tan x + \cos x}{\tan x - \cos x} = 4$ for $0° \leqslant x \leqslant 360°$. [4]

$k = 4$:

$$5\sin^2 x + 3\sin x - 5 = 0$$

Let $y = \sin x$:

$$5y^2 + 3y - 5 = 0$$

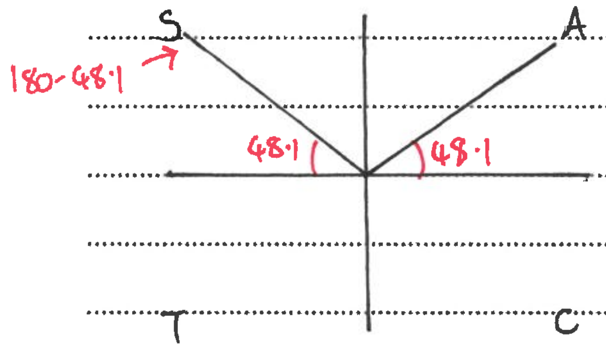
$$y = \frac{-3 \pm \sqrt{3^2 - 4(5)(-5)}}{2(5)}$$

$$y = \frac{-3 + \sqrt{109}}{10} \quad \text{or} \quad y = \frac{-3 - \sqrt{109}}{10}$$

$$\sin x = \frac{-3 + \sqrt{109}}{10} \qquad \sin x = \frac{-3 - \sqrt{109}}{10}$$

no solutions

$$x = 48.1°$$



$$\underline{x = 48.1°,\ 131.9°}$$

4 (a) Show that the equation

$$3\tan^2 x - 3\sin^2 x - 4 = 0$$

may be expressed in the form $a\cos^4 x + b\cos^2 x + c = 0$, where a, b and c are constants to be found. [3]

$\frac{3\sin^2 x}{\cos^2 x} - 3\sin^2 x - 4 = 0$

$\times\cos^2 x \quad \times\cos^2 x \quad \times\cos^2 x \quad \times\cos^2 x$

$3\sin^2 x - 3\sin^2 x\cos^2 x - 4\cos^2 x = 0$

$\uparrow 1-\cos^2 x \quad \uparrow 1-\cos^2 x$

$3(1-\cos^2 x) - 3(1-\cos^2 x)\cos^2 x - 4\cos^2 x = 0$

$3 - 3\cos^2 x - 3(\cos^2 x - \cos^4 x) - 4\cos^2 x = 0$

$3 - 3\cos^2 x - 3\cos^2 x + 3\cos^4 x - 4\cos^2 x = 0$

$\underline{3\cos^4 x - 10\cos^2 x + 3 = 0}$

(b) Hence solve the equation $3\tan^2 x - 3\sin^2 x - 4 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

Let $Y = \cos^2 x$: $3Y^2 - 10Y + 3 = 0$

$(3Y - 1)(Y - 3) = 0$

$3Y - 1 = 0$ or $Y - 3 = 0$

$Y = \frac{1}{3}$ $\quad Y = 3$

$\cos^2 x = \frac{1}{3}$ $\quad \cos^2 x = 3$

$\cos x = \sqrt{\frac{1}{3}}$ or $\cos x = -\sqrt{\frac{1}{3}}$ $\quad \cos x = \sqrt{3}$ or $\cos x = -\sqrt{3}$

$x = \underline{54.7°}$ $\quad x = \underline{125.3°}$ $\quad$ no solutions

4 (a) Prove the identity $\frac{\sin^3\theta}{\sin\theta - 1} - \frac{\sin^2\theta}{1+\sin\theta} \equiv -\tan^2\theta(1+\sin^2\theta)$. [4]

$$\frac{\sin^3\theta}{\sin\theta - 1} \times \frac{(1+\sin\theta)}{(1+\sin\theta)} - \frac{\sin^2\theta}{1+\sin\theta} \times \frac{(\sin\theta - 1)}{(\sin\theta - 1)}$$

$$\frac{\sin^3\theta + \sin^4\theta}{\cancel{\sin\theta} + \sin^2\theta - 1 - \cancel{\sin\theta}} - \frac{\sin^3\theta - \sin^2\theta}{\cancel{\sin\theta} + \sin^2\theta - 1 - \cancel{\sin\theta}}$$

$$\frac{\sin^3\theta + \sin^4\theta - (\sin^3\theta - \sin^2\theta)}{\sin^2\theta - 1}$$

$$\frac{\cancel{\sin^3\theta} + \sin^4\theta - \cancel{\sin^3\theta} + \sin^2\theta}{\sin^2\theta - 1}$$

$$\frac{\sin^4\theta + \sin^2\theta}{\sin^2\theta - 1}$$

$$\frac{\sin^2\theta(\sin^2\theta + 1)}{\sin^2\theta - 1}$$

$\cos^2\theta = 1 - \sin^2\theta$

$-\cos^2\theta = -1 + \sin^2\theta$

$$\frac{\sin^2\theta(\sin^2\theta + 1)}{-\cos^2\theta}$$

$$\frac{-\sin^2\theta}{\cos^2\theta} \times (\sin^2\theta + 1)$$

$$-\tan^2\theta(1+\sin^2\theta)$$ QED

(b) Hence solve the equation

$$\frac{\sin^3\theta}{\sin\theta - 1} - \frac{\sin^2\theta}{1+\sin\theta} = \tan^2\theta(1-\sin^2\theta)$$

for $0 < \theta < 2\pi$. [2]

from (a): $-\tan^2\theta(1+\sin^2\theta) = \tan^2\theta(1-\sin^2\theta)$

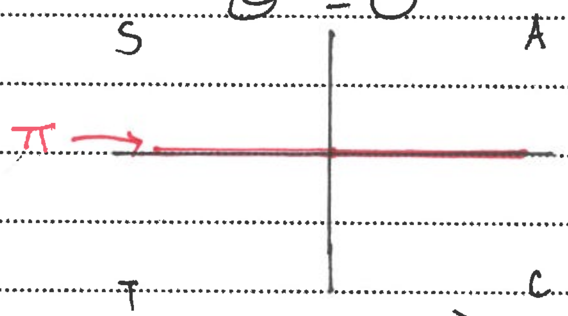
$-\tan^2\theta - \cancel{\tan^2\theta\sin^2\theta} = \tan^2\theta - \cancel{\tan^2\theta\sin^2\theta}$

$2\tan^2\theta = 0$

$\tan^2\theta = 0$

$\tan\theta = 0$

$\theta = 0$



$\theta = 0, \pi, 2\pi$

outside range   outside range

$\underline{\theta = \pi}$

8. **(b)** Find the exact solutions of the equation $4\sin\left(\frac{1}{2}x - 30°\right) = 2\sqrt{2}$ for $0° \leqslant x \leqslant 360°$. [3]

$$4\sin\left(\frac{x}{2} - 30\right) = 2\sqrt{2}$$

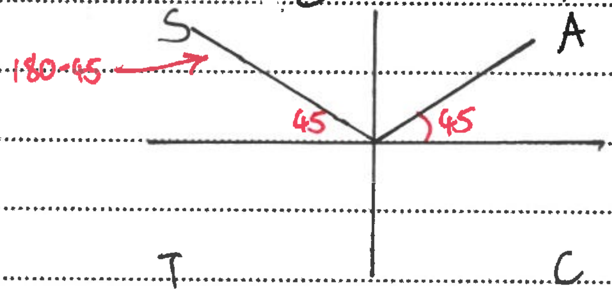
$$0 \leq x \leq 360$$

$$0 \leq \frac{x}{2} \leq 180$$

$$\boxed{-30 \leq \frac{x}{2} - 30 \leq 150}$$

$$\sin\left(\frac{x}{2} - 30\right) = \frac{\sqrt{2}}{2}$$

$$\frac{x}{2} - 30 = 45°$$



$$\frac{x}{2} - 30 = 45°, 135°$$

$$\frac{x}{2} = 75°, 165°$$

$$x = 150°, 330°$$

7 (a) (i) By first expanding $(\cos\theta + \sin\theta)^2$, find the three solutions of the equation

$$(\cos\theta + \sin\theta)^2 = 1$$

for $0 \leqslant \theta \leqslant \pi$. [3]

$(\cos\theta + \sin\theta)^2 = \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta$

$\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta = 1$

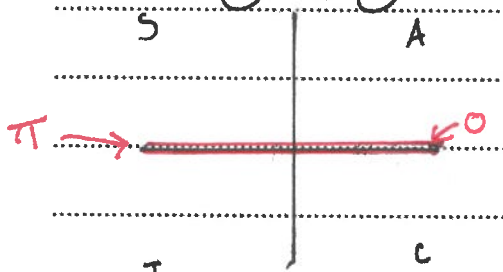
$\underbrace{\cos^2\theta + \sin^2\theta}_{=1} + 2\sin\theta\cos\theta = 1$

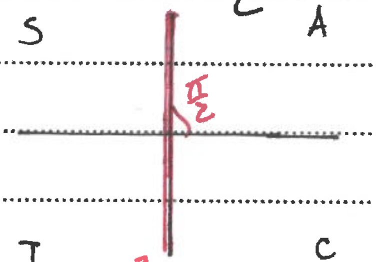
$1 + 2\sin\theta\cos\theta = 1$

$2\sin\theta\cos\theta = 0$

$\sin\theta = 0$ or $\cos\theta = 0$

$\theta = 0$ $\qquad$ $\theta = \frac{\pi}{2}$





$\theta = 0, \pi$ $\qquad$ outside range $\quad \theta = \frac{\pi}{2}$

$\theta = 0, \frac{\pi}{2}, \pi$

(ii) Hence verify that the only solutions of the equation $\cos\theta + \sin\theta = 1$ for $0 \leqslant \theta \leqslant \pi$ are 0 and $\frac{1}{2}\pi$. [2]

$\theta = 0$: $\cos(0) + \sin(0)$
$= 1 + 0$
$= 1$

$\theta = \frac{\pi}{2}$: $\cos\left(\frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right)$
$= 0 + 1$
$= 1$

both $= 1$

$\theta = \pi$: $\cos(\pi) + \sin(\pi)$
$= -1 + 0$
$= -1$

$= -1$

(b) Prove the identity $\dfrac{\sin\theta}{\cos\theta+\sin\theta}+\dfrac{1-\cos\theta}{\cos\theta-\sin\theta}\equiv\dfrac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}$. [3]

$$\frac{\sin\theta}{\cos\theta+\sin\theta}\times\frac{(\cos\theta-\sin\theta)}{(\cos\theta-\sin\theta)}+\frac{1-\cos\theta}{\cos\theta-\sin\theta}\times\frac{(\cos\theta+\sin\theta)}{(\cos\theta+\sin\theta)}$$

$$\frac{\sin\theta\cos\theta-\sin^2\theta}{\cos^2\theta-\sin^2\theta}+\frac{\cos\theta+\sin\theta-\cos^2\theta-\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$$

$$\frac{\cancel{\sin\theta\cos\theta}-\sin^2\theta+\cos\theta+\sin\theta-\cos^2\theta-\cancel{\sin\theta\cos\theta}}{\cos^2\theta-\sin^2\theta}$$

$$\frac{\cos\theta+\sin\theta-\underbrace{(\sin^2\theta+\cos^2\theta)}_{=1}}{\cos^2\theta-\sin^2\theta}$$

$$\frac{\cos\theta+\sin\theta-1}{\cos^2\theta-\sin^2\theta}$$

$1-\sin^2\theta$

$$\frac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}$$ QED

(c) Using the results of **(a)(ii)** and **(b)**, solve the equation

$$\frac{\sin\theta}{\cos\theta+\sin\theta}+\frac{1-\cos\theta}{\cos\theta-\sin\theta}=2(\cos\theta+\sin\theta-1)$$

for $0\leqslant\theta\leqslant\pi$. [3]

from (b): $\dfrac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}=2(\cos\theta+\sin\theta-1)$

$(\cos\theta+\sin\theta-1)=(\cos\theta+\sin\theta-1)(2-4\sin^2\theta)$

$(\cos\theta+\sin\theta-1)(2-4\sin^2\theta)-(\cos\theta+\sin\theta-1)=0$

factorise: $(\cos\theta+\sin\theta-1)[(2-4\sin^2\theta)-1]=0$

$(\cos\theta+\sin\theta-1)(1-4\sin^2\theta)=0$

$\cos\theta+\sin\theta-1=0$ or $1-4\sin^2\theta=0$

from (a)(ii) $\theta=0,\frac{\pi}{2}$

continued →

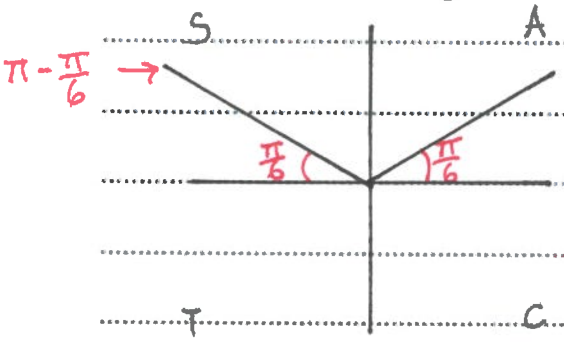
$$1 - 4\sin^2\theta = 0$$

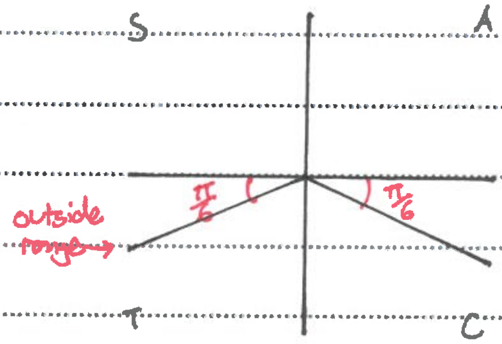
$$4\sin^2\theta = 1$$

$$\sin^2\theta = \frac{1}{4}$$

$$\sin\theta = \frac{1}{2} \quad \text{or} \quad \sin\theta = \frac{-1}{2}$$

$$\theta = \frac{\pi}{6} \qquad \theta = \frac{-\pi}{6}$$





$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\rightarrow \theta = 0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$

I cannot believe this Q is only 3 marks...