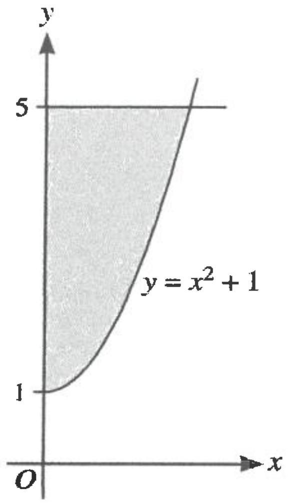
3



$\rightarrow x^2 = y - 1$

The diagram shows part of the curve with equation $y = x^2 + 1$. The shaded region enclosed by the curve, the y-axis and the line $y = 5$ is rotated through 360° about the y-axis.

Find the volume obtained. [4]

$$\text{Volume} = \pi \int x^2 \, dy$$

$$= \pi \int_1^5 (y-1) \, dy$$

$$= \pi \left[\tfrac{1}{2} y^2 - y \right]_1^5$$

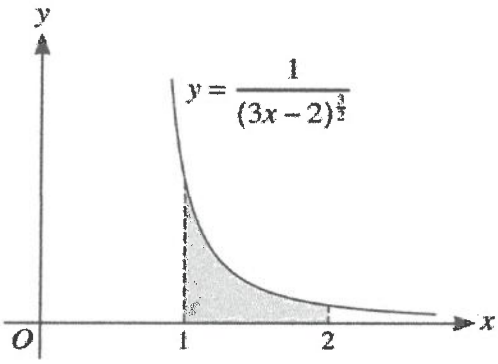
$$= \pi \left[\tfrac{1}{2}(5)^2 - 5 \right] - \pi \left[\tfrac{1}{2}(1)^2 - 1 \right]$$

$$= \pi \left[\frac{15}{2} \right] - \pi \left[\frac{-1}{2} \right]$$

$$= \frac{15\pi}{2} + \frac{\pi}{2}$$

$$= \underline{8\pi}$$

10.



The diagram shows the curve with equation $y = \frac{1}{(3x-2)^{\frac{3}{2}}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$. The shaded region is rotated through 360° about the x-axis.

(b) Find the volume of revolution. [4]

$$\text{Volume} = \pi \int y^2 \, dx$$

$$= \pi \int_1^2 \left(\frac{1}{(3x-2)^{\frac{3}{2}}} \right)^2 dx$$

$$= \pi \int_1^2 \frac{1}{(3x-2)^3} \, dx = \pi \int_1^2 (3x-2)^{-3} \, dx$$

$$= \pi \left[\frac{-1}{2} \times \frac{1}{3} (3x-2)^{-2} \right]_1^2$$

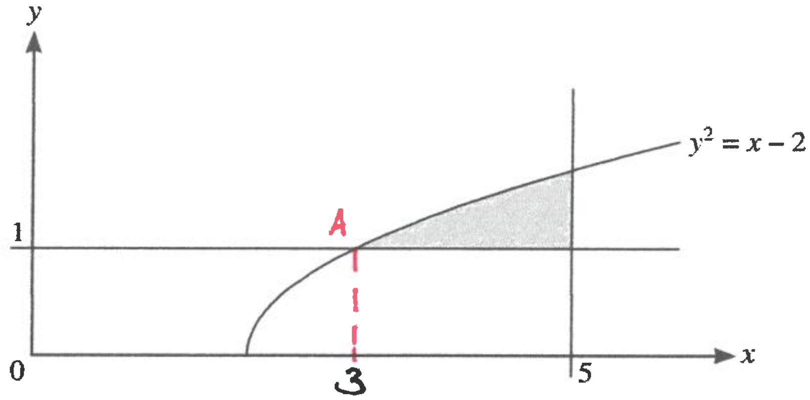
$$= \pi \left[\frac{-1}{6} (3x-2)^{-2} \right]_1^2$$

$$= \pi \left[\frac{-1}{6} (3(2)-2)^{-2} \right] - \pi \left[\frac{-1}{6} (3(1)-2)^{-2} \right]$$

$$= \pi \left[\frac{-1}{6} \times 4^{-2} \right] - \pi \left[\frac{-1}{6} \times 1^{-2} \right]$$

$$= \frac{-\pi}{96} - -\frac{\pi}{6} = \frac{5\pi}{32}$$

9



The diagram shows part of the curve with equation $y^2 = x - 2$ and the lines $x = 5$ and $y = 1$. The shaded region enclosed by the curve and the lines is rotated through 360° about the x-axis.

Find the volume obtained. [6]

find point A: sub. $y=1$: $1^2 = x - 2 \rightarrow x = 3$

Volume of shaded region = Volume bounded by $y^2=x-2$ - Volume of cylinder bounded by line $y=1$

Volume bounded by $y^2=x-2$:

$$\text{Volume} = \pi\int y^2\,dx$$

$$= \pi\int_3^5 (x-2)\,dx$$

$$= \pi\left[\tfrac{1}{2}x^2 - 2x\right]_3^5$$

$$= \pi\left[\tfrac{1}{2}(5)^2 - 2(5)\right] - \pi\left[\tfrac{1}{2}(3)^2 - 2(3)\right]$$

$$= \pi\left[\tfrac{25}{2} - 10\right] - \pi\left[\tfrac{9}{2} - 6\right]$$

$$= \frac{5\pi}{2} - -\frac{3\pi}{2} = 4\pi$$

Volume of cylinder:

$$V = \pi r^2 h$$

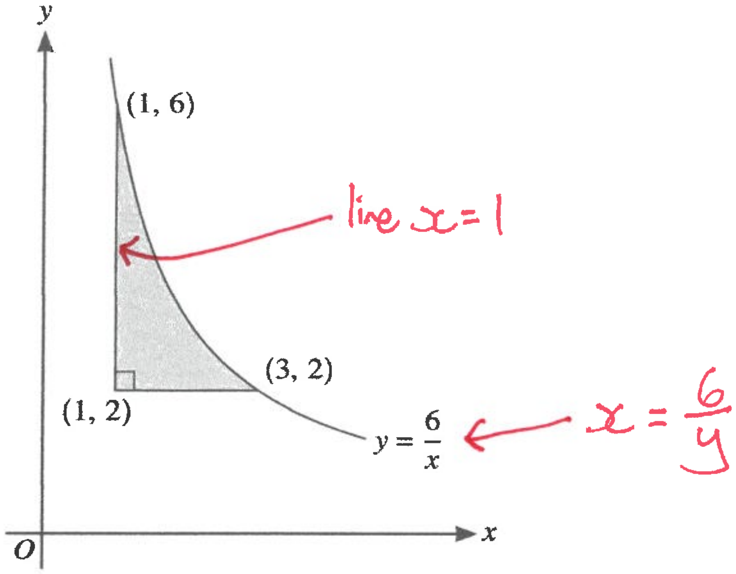
$$= \pi \times 1^2 \times 2$$

$$= 2\pi$$

Volume of shaded region:

$$4\pi - 2\pi = 2\pi$$

8



The diagram shows part of the curve $y = \frac{6}{x}$. The points (1, 6) and (3, 2) lie on the curve. The shaded region is bounded by the curve and the lines $y = 2$ and $x = 1$.

(a) Find the volume generated when the shaded region is rotated through 360° about the *y*-axis. [5]

Volume bounded by shaded region = Volume bounded by $y=\frac{6}{x}$ − volume of cylinder bounded by line $x=1$

Volume bounded by $y=\frac{6}{x}$:

$$\text{Volume} = \pi \int x^2 \, dy$$

$$= \pi \int_2^6 \left(\frac{6}{y}\right)^2 dy$$

$$= \pi \int_2^6 36y^{-2} \, dy$$

$$= \pi \left[-36y^{-1}\right]_2^6$$

$$= \pi\left[-36(6)^{-1}\right] - \pi\left[-36(2)^{-1}\right]$$

$$= -6\pi - -18\pi = \underline{12\pi}$$

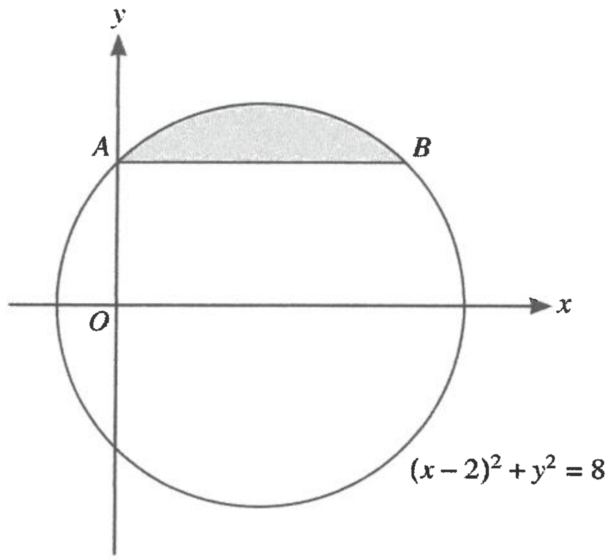
Volume of cylinder bounded by line $x=1$:

$$\text{Volume} = \pi r^2 h$$
$$= \pi \times 1^2 \times 4$$
$$= 4\pi$$

Volume bounded by shaded region:

$$12\pi - 4\pi = \underline{8\pi}$$

8



The diagram shows the circle with equation $(x-2)^2+y^2=8$. The chord AB of the circle intersects the positive y-axis at A and is parallel to the x-axis.

(a) Find, by calculation, the coordinates of A and B. [3]

At point A, $x=0$:

$(0-2)^2+y^2=8$

$(-2)^2+y^2=8$

$y^2=4$

$y=\pm 2$

$\hookrightarrow$ $y=2$ (from diagram)

At point B, $y=2$:

$(x-2)^2+2^2=8$

$x^2-4x+4+4=8$

$x^2-4x+8=8$

$x^2-4x=0$

$x(x-4)=0$

$x=0$, $x=4$

$\rightarrow$ $(0,2)^A$ $\rightarrow$ $(4,2)^B$

(b) Find the volume of revolution when the shaded segment, bounded by the circle and the chord AB, is rotated through 360° about the x-axis. [5]

Volume from shaded segment = Volume from circle − cylinder bounded by the line $y=2$

Volume from circle:

$$\text{Volume} = \pi\int y^2\,dx$$

$$= \pi\int_0^4 \left(8-(x-2)^2\right)dx$$

$$= \pi\left[8x - \tfrac{1}{3}(x-2)^3\right]_0^4$$

$$= \pi\left[8(4) - \tfrac{1}{3}(4-2)^3\right] - \pi\left[0 - \tfrac{1}{3}(0-2)^3\right]$$

$$= \pi\left[32 - \tfrac{1}{3}(2)^3\right] - \pi\left[-\tfrac{1}{3}(-2)^3\right]$$

$$= \pi\left[32 - \tfrac{8}{3}\right] - \pi\left[\tfrac{8}{3}\right]$$

$$= \frac{88\pi}{3} - \frac{8\pi}{3} = \underline{\frac{80\pi}{3}}$$

Volume of cylinder bounded by $y=2$:

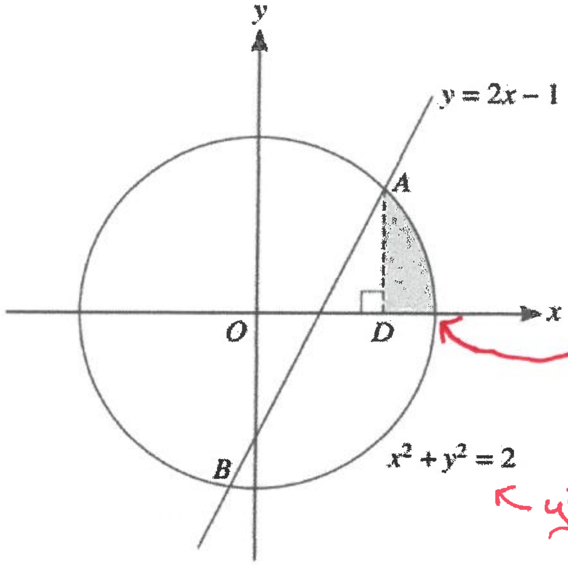
$$\text{Volume} = \pi r^2 h$$

$$= \pi \times 2^2 \times 4$$

$$= 16\pi$$

Volume from shaded segment:

$$\frac{80\pi}{3} - 16\pi = \underline{\underline{\frac{32\pi}{3}}}$$



$x^2 + 0^2 = 2$
$x^2 = 2$
$x = \sqrt{2}$

$y^2 = 2 - x^2$

from (a): $A = (1,1)$

The diagram shows the circle $x^2 + y^2 = 2$ and the straight line $y = 2x - 1$ intersecting at the points A and B. The point D on the x-axis is such that AD is perpendicular to the x-axis.

(b) Find the volume of revolution when the shaded region is rotated through 360° about the x-axis. Give your answer in the form $\frac{\pi}{a}(b\sqrt{c} - d)$, where a, b, c and d are integers. [4]

$$\text{Volume} = \pi\int_1^{\sqrt{2}} y^2\,dx = \pi\int_1^{\sqrt{2}} (2 - x^2)\,dx$$

$$= \pi\left[2x - \tfrac{1}{3}x^3\right]_1^{\sqrt{2}}$$

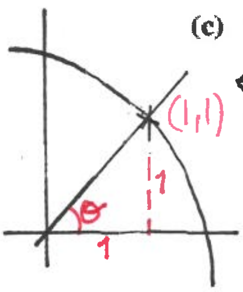
$$= \pi\left[2\sqrt{2} - \tfrac{1}{3}(\sqrt{2})^3\right] - \pi\left[2(1) - \tfrac{1}{3}\times 1^3\right]$$

$$= \pi\left[2\sqrt{2} - \tfrac{1}{3}\times 2\sqrt{2}\right] - \pi\left[2 - \tfrac{1}{3}\right]$$

$$= \pi\left[\frac{4\sqrt{2}}{3}\right] - \pi\left[\frac{5}{3}\right]$$

$$= \pi\left(\frac{4\sqrt{2}}{3} - \frac{5}{3}\right) = \frac{\pi}{3}(4\sqrt{2} - 5)$$

(c) Find an exact expression for the perimeter of the shaded region. [2]



$\tan\theta = \frac{1}{1} \rightarrow \theta = \frac{\pi}{4}$

Arc length $= r\theta$

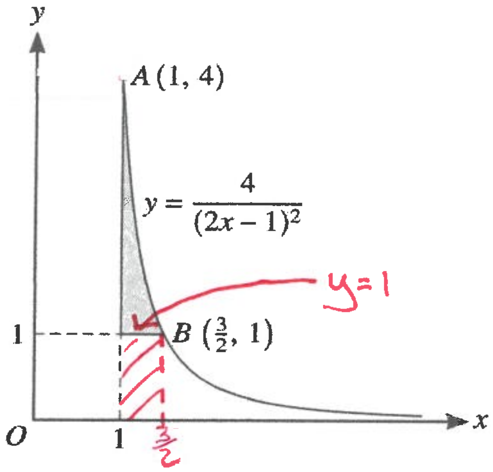
$= \sqrt{2} \times \frac{\pi}{4}$

perimeter $=$

$\frac{\pi\sqrt{2}}{4} + 1 + \sqrt{2} - 1$

$= \sqrt{2} + \frac{\pi\sqrt{2}}{4}$

10



The diagram shows part of the curve with equation $y = \frac{4}{(2x-1)^2}$ and parts of the lines $x = 1$ and $y = 1$. The curve passes through the points $A\ (1, 4)$ and $B, (\frac{3}{2}, 1)$.

(a) Find the exact volume generated when the shaded region is rotated through 360° about the x-axis. [5]

Volume of shaded region = Volume bounded by $y=\frac{4}{(2x-1)^2}$ − Volume of cylinder bounded by line $y=1$

Volume bounded by $y=\frac{4}{(2x-1)^2}$: $V = \pi\int y^2\,dx$

$$V = \pi\int_1^{\frac{3}{2}} \frac{16}{(2x-1)^4}\,dx = \pi\int_1^{\frac{3}{2}} 16(2x-1)^{-4}\,dx$$

$$= \pi\left[\frac{-16}{3}\times\frac{1}{2}(2x-1)^{-3}\right]_1^{\frac{3}{2}} = \pi\left[\frac{-8}{3}(2x-1)^{-3}\right]_1^{\frac{3}{2}}$$

$$= \pi\left[\frac{-8}{3}\left(2\left(\frac{3}{2}\right)-1\right)^{-3}\right] - \pi\left[\frac{-8}{3}(2(1)-1)^{-3}\right]$$

$$= \pi\left[\frac{-8}{3}\times 2^{-3}\right] - \pi\left[\frac{-8}{3}\times 1^{-3}\right]$$

$$= \frac{-\pi}{3} - -\frac{8\pi}{3} = \frac{7\pi}{3}$$

Volume of cylinder:

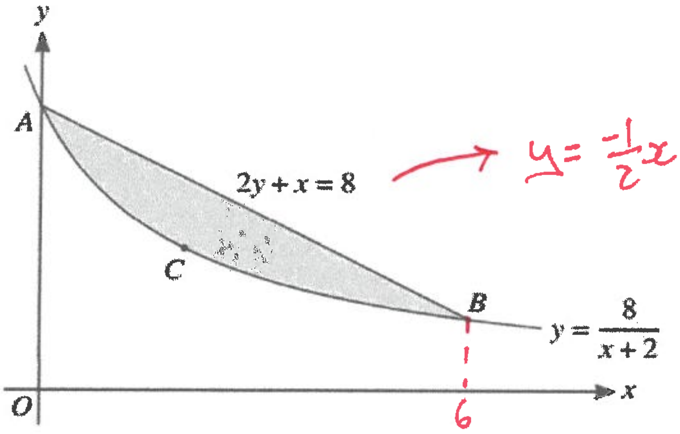
$V = \pi r^2 h$

$= \pi \times 1^2 \times 0.5$

$= \frac{\pi}{2}$

Volume of shaded region:

$\frac{7\pi}{3} - \frac{\pi}{2} = \frac{11\pi}{6}$



$\rightarrow y = \frac{-1}{2}x + 4$

from (a): $B = (6, 1)$

The diagram shows part of the curve $y = \frac{8}{x+2}$ and the line $2y + x = 8$, intersecting at points A and B. The point C lies on the curve and the tangent to the curve at C is parallel to AB.

(b) Find the volume generated when the shaded region, bounded by the curve and the line, is rotated through 360° about the x-axis. [6]

Volume bounded by shaded region = Volume bounded by $2y + x = 8$ − Volume bounded by $y = \frac{8}{x+2}$

Volume bounded by $2y + x = 8$:

$$\text{Volume} = \pi \int y^2 \, dx$$

$$= \pi \int_0^6 \left(\frac{-1}{2}x + 4\right)^2 dx$$

$$= \pi \int_0^6 \left(\frac{1}{4}x^2 - 4x + 16\right) dx$$

$$= \pi \left[\frac{1}{12}x^3 - 2x^2 + 16x\right]_0^6$$

$$= \pi \left[\frac{1}{12}(6)^3 - 2(6)^2 + 16(6)\right] - \pi[0]$$

$$= \pi [18 - 72 + 96]$$

$$= 42\pi$$

Volume bounded by $y = \frac{8}{x+2}$:

$$\text{Volume} = \pi \int y^2 \, dx$$

$$= \pi \int_0^6 \left(\frac{8}{x+2}\right)^2 dx$$

$$= \pi \int_0^6 \frac{64}{(x+2)^2} \, dx$$

$$= \pi \int_0^6 64(x+2)^{-2} \, dx$$

$$= \pi \left[-64(x+2)^{-1}\right]_0^6$$

$$= \pi\left[-64(6+2)^{-1}\right] - \pi\left[-64(0+2)^{-1}\right]$$

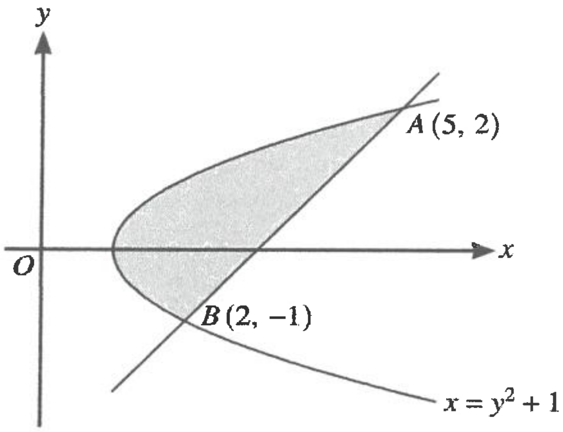
$$= \pi\left[-64 \times \frac{1}{8}\right] - \pi\left[-64 \times \frac{1}{2}\right]$$

$$= -8\pi - -32\pi = \underline{24\pi}$$

Volume bounded by shaded region:

$$42\pi - 24\pi = \underline{18\pi}$$

11



The diagram shows the curve with equation $x = y^2 + 1$. The points $A\,(5, 2)$ and $B\,(2, -1)$ lie on the curve.

(a) Find an equation of the line AB. [2]

Gradient AB: $m = \dfrac{2--1}{5-2}$

$= \dfrac{3}{3} = \underline{1}$

$y - 2 = 1(x - 5)$

$y - 2 = x - 5$

$\underline{\underline{y = x - 3}}$

(b) Find the volume of revolution when the region between the curve and the line AB is rotated through 360° about the **y-axis**. [9]

Volume from shaded region = Volume bounded by $y = x - 3$ − Volume bounded by $x = y^2 + 1$

Volume bounded by $y = x - 3$:

$\text{Volume} = \pi \int x^2 \, dy$

$= \pi \int_{-1}^{2} (y+3)^2 \, dy$

$= \pi \int_{-1}^{2} (y^2 + 6y + 9) \, dy$

$= \pi\left[\frac{1}{3}y^3 + 3y^2 + 9y\right]_{-1}^{2}$

$= \pi\left[\frac{1}{3}(2)^3 + 3(2)^2 + 9(2)\right] - \pi\left[\frac{1}{3}(-1)^3 + 3(-1)^2 + 9(-1)\right]$

$= \pi\left[\frac{8}{3} + 12 + 18\right] - \pi\left[\frac{-1}{3} + 3 - 9\right]$

$= \frac{98\pi}{3} - -\frac{19\pi}{3} = \underline{39\pi}$

Volume bounded by $x = y^2 + 1$:

$\text{Volume} = \pi\int x^2\,dy$

$= \pi\int_{-1}^{2}(y^2+1)^2\,dy$

$= \pi\int_{-1}^{2}(y^4 + 2y^2 + 1)\,dy$

$= \pi\left[\frac{1}{5}y^5 + \frac{2}{3}y^3 + y\right]_{-1}^{2}$

$= \pi\left[\frac{1}{5}(2)^5 + \frac{2}{3}(2)^3 + 2\right] - \pi\left[\frac{1}{5}(-1)^5 + \frac{2}{3}(-1)^3 + -1\right]$

$= \pi\left[\frac{32}{5} + \frac{16}{3} + 2\right] - \pi\left[\frac{-1}{5} - \frac{2}{3} - 1\right]$

$= \frac{206\pi}{15} - -\frac{28\pi}{15} = \underline{\frac{78\pi}{5}}$

Volume from shaded region: $39\pi - \frac{78\pi}{5} = \underline{\frac{117\pi}{5}}$