1 A curve with equation $y = f(x)$ is such that $f'(x) = 6x^2 - \frac{8}{x^2}$. It is given that the curve passes through the point (2, 7).

Find $f(x)$. [3]

$$f(x) = \int (6x^2 - 8x^{-2})\,dx$$

$$= \frac{1}{3} \times 6x^3 + 8x^{-1} + C$$

$$= 2x^3 + 8x^{-1} + C$$

sub. (2,7):

$$7 = 2(2)^3 + 8(2)^{-1} + C$$

$$7 = 2 \times 8 + 8 \times \frac{1}{2} + C$$

$$7 = 16 + 4 + C$$

$$7 = 20 + C$$

$$C = -13$$

→ $f(x) = 2x^3 + 8x^{-1} - 13$

7 The point (4, 7) lies on the curve $y = f(x)$ and it is given that $f'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

(b) Find the equation of the curve. [4]

$f(x) = \int (6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}})\,dx$

$= 12x^{\frac{1}{2}} + 8x^{-\frac{1}{2}} + C$

Sub. (4,7):

$7 = 12(4)^{\frac{1}{2}} + 8(4)^{-\frac{1}{2}} + C$

$7 = 12 \times 2 + 8 \times \frac{1}{2} + C$

$7 = 24 + 4 + C$

$7 = 28 + C$

$C = -21$

$\rightarrow$ $f(x) = 12x^{\frac{1}{2}} + 8x^{-\frac{1}{2}} - 21$

2 The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. It is given that the point (4, 7) lies on the curve.

Find the equation of the curve. [4]

$y = \int \left(3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}\right) dx$

$= \frac{2}{3} \times 3x^{\frac{3}{2}} - 2 \times 3x^{\frac{1}{2}} + c$

$= 2x^{3/2} - 6x^{\frac{1}{2}} + c$

sub. (4,7):

$7 = 2(4)^{\frac{3}{2}} - 6(4)^{\frac{1}{2}} + c$

$7 = 2 \times 8 - 6 \times 2 + c$

$7 = 16 - 12 + c$

$7 = 4 + c$

$c = 3$

→ $y = 2x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + 3$

10 At the point $(4, -1)$ on a curve, the gradient of the curve is $-\frac{3}{2}$. It is given that $\frac{dy}{dx} = x^{-\frac{1}{2}} + k$, where k is a constant.

(a) Show that $k = -2$. [1]

$\frac{dy}{dx} = -\frac{3}{2}$ at $(4,1)$: $-\frac{3}{2} = (4)^{-\frac{1}{2}} + k$

$-\frac{3}{2} = \frac{1}{2} + k$

$\underline{-2 = k}$

(b) Find the equation of the curve. [4]

$y = \int (x^{-\frac{1}{2}} - 2)\, dx$

$= 2x^{\frac{1}{2}} - 2x + c$

sub. $(4,-1)$:

$-1 = 2(4)^{\frac{1}{2}} - 2(4) + c$

$-1 = 2 \times 2 - 8 + c$

$-1 = -4 + c$

$\underline{c = 3}$

$\rightarrow$ $\underline{y = 2x^{\frac{1}{2}} - 2x + 3}$

2 The function f is defined by $f(x) = \frac{2}{(x+2)^2}$ for $x > -2$.

(b) The equation of a curve is such that $\frac{dy}{dx} = f(x)$. It is given that the point $(-1, -1)$ lies on the curve.

Find the equation of the curve. [2]

$$y = \int 2(x+2)^{-2}\,dx$$

$$= -2(x+2)^{-1} + c$$

Sub. $(-1,-1)$:

$$-1 = -2(-1+2)^{-1} + c$$

$$-1 = -2(1)^{-1} + c$$

$$-1 = -2 + c$$

$$\underline{c = 1}$$

$$\rightarrow \underline{y = -2(x+2)^{-1} + 1}$$

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6 A curve is such that $\frac{dy}{dx} = \frac{6}{(3x-2)^3}$ and $A\ (1,\ -3)$ lies on the curve. A point is moving along the curve and at A the y-coordinate of the point is increasing at 3 units per second.

(b) Find the equation of the curve. [4]

$$y = \int 6(3x-2)^{-3}\,dx$$

$$= -3 \times \frac{1}{3}(3x-2)^{-2} + c$$

$$= -(3x-2)^{-2} + c$$

Sub. (1, −3):

$$-3 = -(3-2)^{-2} + c$$

$$-3 = -(1)^{-2} + c$$

$$-3 = -1 + c$$

$$c = -2$$

→ $y = -(3x-2)^{-2} - 2$

7 The curve $y = f(x)$ is such that $f'(x) = \frac{-3}{(x+2)^4}$.

(b) Find $f(x)$ given that the curve passes through the point $(-1, 5)$. [3]

$y = \int -3(x+2)^{-4}\,dx$

$= \frac{-1}{3} \times -3(x+2)^{-3} + C$

$= (x+2)^{-3} + C$

sub. $(-1, 5)$:

$5 = (-1+2)^{-3} + C$

$5 = 1^{-3} + C$

$5 = 1 + C$

$C = 4$

$\rightarrow$ $y = (x+2)^{-3} + 4$

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10 The gradient of a curve at the point (x, y) is given by $\frac{dy}{dx} = 2(x+3)^{\frac{1}{2}} - x$. The curve has a stationary point at $(a, 14)$, where a is a positive constant.

(c) Find the equation of the curve. [4]

from (a): $a = 6$

$$y = \int \left(2(x+3)^{\frac{1}{2}} - x\right) dx$$

$$= \frac{4}{3}(x+3)^{\frac{3}{2}} - \frac{1}{2}x^2 + C$$

sub. (6,14):

$$14 = \frac{4}{3}(6+3)^{3/2} - \frac{1}{2}(6)^2 + C$$

$$14 = \frac{4}{3}(9)^{3/2} - \frac{1}{2} \times 36 + C$$

$$14 = \frac{4}{3} \times 27 - 18 + C$$

$$14 = 36 - 18 + C$$

$$14 = 18 + C$$

$$\underline{C = -4}$$

$$\rightarrow \underline{y = \frac{4}{3}(x+3)^{3/2} - \frac{1}{2}x^2 - 4}$$

11 The gradient of a curve is given by $\frac{dy}{dx} = 6(3x - 5)^3 - kx^2$, where k is a constant. The curve has a stationary point at $(2, -3.5)$.

(b) Find the equation of the curve. [4]

from (a): $k = \frac{3}{2}$

$$y = \int \left[6(3x-5)^3 - \frac{3}{2}x^2\right] dx$$

$$= \frac{1}{4} \times \frac{1}{3} \times 6(3x-5)^4 - \frac{1}{3} \times \frac{3}{2}x^3 + C$$

$$= \frac{1}{2}(3x-5)^4 - \frac{1}{2}x^3 + C$$

sub. $(2, -3.5)$:

$$-3.5 = \frac{1}{2}(3 \times 2 - 5)^4 - \frac{1}{2}(2)^3 + C$$

$$-3.5 = \frac{1}{2}(1)^4 - \frac{1}{2} \times 8 + C$$

$$-3.5 = \frac{1}{2} - 4 + C$$

$$-3.5 = -3.5 + C$$

$$C = 0$$

$\rightarrow$ $y = \frac{1}{2}(3x-5)^4 - \frac{1}{2}x^3$

9 A curve which passes through $(0, 3)$ has equation $y = \mathrm{f}(x)$. It is given that $\mathrm{f}'(x) = 1 - \dfrac{2}{(x-1)^3}$.

(a) Find the equation of the curve. [4]

$$f(x) = \int \left(1 - 2(x-1)^{-3}\right) dx$$

$$= x + (x-1)^{-2} + C$$

sub. $(0,3)$:

$$3 = 0 + (-1)^{-2} + C$$

$$3 = 1 + C$$

$$C = 2$$

$$\rightarrow f(x) = x + (x-1)^{-2} + 2$$

The tangent to the curve at (0, 3) intersects the curve again at one other point, P.

(b) Show that the x-coordinate of P satisfies the equation $(2x + 1)(x - 1)^2 - 1 = 0$. [4]

gradient at $(0,3)$:

$$f'(0) = 1 - \frac{2}{(-1)^3}$$

$$= 1 - \frac{2}{-1}$$

$$= 3$$

eqⁿ of tangent: $y - 3 = 3(x - 0)$

$$y - 3 = 3x$$

$$y = 3x + 3$$

solve simultaneously with $f(x)$:

$$3x + 3 = x + (x-1)^{-2} + 2$$

$$2x + 1 = (x-1)^{-2}$$

$$2x + 1 = \frac{1}{(x-1)^2}$$

$$(2x+1)(x-1)^2 = 1$$

$$(2x+1)(x-1)^2 - 1 = 0$$ QED

(c) Verify that $x = \frac{3}{2}$ satisfies this equation and hence find the y-coordinate of P. [2]

Sub. $x = \frac{3}{2}$: $\left(2\left(\frac{3}{2}\right) + 1\right)\left(\frac{3}{2} - 1\right)^2 - 1$

$$= (3 + 1)\left(\frac{1}{2}\right)^2 - 1$$

$$= 4 \times \frac{1}{4} - 1$$

$$= 1 - 1$$

$$= 0$$ QED

sub $x = \frac{3}{2}$ into $y = 3x + 3$:

$$y = 3\left(\frac{3}{2}\right) + 3$$

$$= \frac{9}{2} + 3$$

$$= \frac{15}{2}$$