1 The equation of a curve is such that $\frac{dy}{dx} = \frac{3}{x^4} + 32x^3$. It is given that the curve passes through the point $(\frac{1}{2}, 4)$.

Find the equation of the curve. [4]

$$y = \int (3x^{-4} + 32x^3)\,dx$$

$$= -\frac{1}{3} \times 3x^{-3} + \frac{1}{4} \times 32x^4 + c$$

$$= -x^{-3} + 8x^4 + c$$

sub. $(\frac{1}{2}, 4)$:

$$4 = -(\tfrac{1}{2})^{-3} + 8(\tfrac{1}{2})^4 + c$$

$$4 = -8 + 8 \times \frac{1}{16} + c$$

$$4 = -8 + \frac{1}{2} + c$$

$$4 = -\frac{15}{2} + c$$

$$c = \frac{23}{2}$$

$$\rightarrow y = -x^{-3} + 8x^4 + \frac{23}{2}$$

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9 A curve has equation $y = \mathrm{f}(x)$, and it is given that $\mathrm{f}'(x) = 2x^2 - 7 - \dfrac{4}{x^2}$.

(a) Given that $\mathrm{f}(1) = -\frac{1}{3}$, find $\mathrm{f}(x)$. [4]

$$f(x) = \int (2x^2 - 7 - 4x^{-2})\,dx$$

$$= \frac{2}{3}x^3 - 7x + 4x^{-1} + C$$

$f(1) = \frac{-1}{3}$:

$$\frac{-1}{3} = \frac{2}{3}(1)^3 - 7(1) + 4(1)^{-1} + C$$

$$\frac{-1}{3} = \frac{2}{3} - 7 + 4 + C$$

$$\frac{-1}{3} = \frac{-7}{3} + C$$

$$C = 2$$

$$\rightarrow f(x) = \frac{2}{3}x^3 - 7x + 4x^{-1} + 2$$

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8 The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. The curve passes through the point (3, 5).

(a) Find the equation of the curve. [4]

$y = \int (3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}})\,dx$

$= \frac{2}{3} \times 3x^{\frac{3}{2}} - 2 \times 3x^{\frac{1}{2}} + C$

$= 2x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + C$

sub. (3,5):

$5 = 2(3)^{\frac{3}{2}} - 6(3)^{\frac{1}{2}} + C$

$5 = 2 \times 3\sqrt{3} - 6\sqrt{3} + C$

$5 = 6\sqrt{3} - 6\sqrt{3} + C$

$\underline{5 = C}$

$\rightarrow \underline{y = 2x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + 5}$

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1 A curve with equation $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = 2x^{-\frac{1}{3}} - x^{\frac{1}{3}}$. It is given that $\mathrm{f}(8) = 5$.

Find $\mathrm{f}(x)$. [4]

$$f(x) = \int \left(2x^{-1/3} - x^{1/3}\right) dx$$

$$= \frac{3}{2} \times 2x^{2/3} - \frac{3}{4} \times x^{4/3} + C$$

$$= 3x^{2/3} - \frac{3}{4}x^{4/3} + C$$

$f(8) = 5$:

$$5 = 3(8)^{2/3} - \frac{3}{4}(8)^{4/3} + C$$

$$5 = 3 \times 4 - \frac{3}{4} \times 16 + C$$

$$5 = 12 - 12 + C$$

$$\underline{5 = C}$$

$\rightarrow$ $$\underline{f(x) = 3x^{2/3} - \frac{3}{4}x^{4/3} + 5}$$

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1 The equation of a curve is such that $\frac{dy}{dx} = \frac{4}{(x-3)^3}$ for $x > 3$. The curve passes through the point $(4, 5)$.

Find the equation of the curve. [3]

$y = \int 4(x-3)^{-3}\,dx$

$= -2(x-3)^{-2} + C$

sub. (4,5):

$5 = -2(4-3)^{-2} + c$

$5 = -2(1)^{-2} + c$

$5 = -2 + C$

$c = 7$

$\rightarrow$ $y = -2(x-3)^{-2} + 7$

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4 A curve is such that $\frac{dy}{dx} = \frac{8}{(3x+2)^2}$. The curve passes through the point $(2, 5\frac{2}{3})$.

Find the equation of the curve. [4]

$y = \int 8(3x+2)^{-2}\,dx$

$= -8 \times \frac{1}{3}(3x+2)^{-1} + C$

$= \frac{-8}{3}(3x+2)^{-1} + C$

sub. $(2, \frac{17}{3})$:

$\frac{17}{3} = \frac{-8}{3}(3(2)+2)^{-1} + C$

$\frac{17}{3} = \frac{-8}{3}(8)^{-1} + C$

$\frac{17}{3} = \frac{-8}{3} \times \frac{1}{8} + C$

$\frac{17}{3} = \frac{-1}{3} + C$

$c = 6$

$\rightarrow$ $y = \frac{-8}{3}(3x+2)^{-1} + 6$

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2 The equation of a curve is such that $\frac{dy}{dx} = 12(\frac{1}{2}x - 1)^{-4}$. It is given that the curve passes through the point $P(6, 4)$.

(a) Find the equation of the tangent to the curve at P. [2]

gradient at (6,4): $12(\frac{1}{2}(6) - 1)^{-4}$

$= 12(3-1)^{-4}$

$= 12 \times 2^{-4}$

$= \frac{3}{4}$

eq^n of tangent:

$y - 4 = \frac{3}{4}(x - 6)$

$y - 4 = \frac{3}{4}x - \frac{9}{2}$

$y = \frac{3}{4}x - \frac{1}{2}$

(b) Find the equation of the curve. [4]

$y = \int 12(\frac{1}{2}x - 1)^{-4} \, dx$

$= \frac{-1}{3} \times 2 \times 12(\frac{1}{2}x - 1)^{-3} + C$

$= -8(\frac{1}{2}x - 1)^{-3} + C$

sub. (6,4):

$4 = -8(\frac{1}{2} \times 6 - 1)^{-3} + C$

$4 = -8(2)^{-3} + C$

$4 = -8 \times \frac{1}{8} + C$

$4 = -1 + C$

$C = 5$

$\rightarrow y = -8(\frac{1}{2}x - 1)^{-3} + 5$

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2 The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{(x-3)^2} + x$. It is given that the curve passes through the point (2, 7).

Find the equation of the curve. [4]

$$y = \int \left[(x-3)^{-2} + x \right] dx$$

$$= -(x-3)^{-1} + \frac{1}{2}x^2 + C$$

Sub. (2,7):

$$7 = -(2-3)^{-1} + \frac{1}{2}(2)^2 + C$$

$$7 = -(-1)^{-1} + \frac{1}{2} \times 4 + C$$

$$7 = 1 + 2 + C$$

$$7 = 3 + C$$

$$C = 4$$

$\rightarrow$ $$y = -(x-3)^{-1} + \frac{1}{2}x^2 + 4$$

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3 The equation of a curve is such that $\frac{dy}{dx} = 3(4x-7)^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$. It is given that the curve passes through the point $(4, \frac{5}{2})$.

Find the equation of the curve. [4]

$$y = \int \left[3(4x-7)^{\frac{1}{2}} - 4x^{-\frac{1}{2}}\right] dx$$

$$= \frac{2}{3} \times \frac{1}{4} \times 3(4x-7)^{\frac{3}{2}} - 2 \times 4x^{\frac{1}{2}} + C$$

$$= \frac{1}{2}(4x-7)^{\frac{3}{2}} - 8x^{\frac{1}{2}} + C$$

Sub. $(4, \frac{5}{2})$:

$$\frac{5}{2} = \frac{1}{2}(4 \times 4 - 7)^{\frac{3}{2}} - 8(4)^{\frac{1}{2}} + C$$

$$\frac{5}{2} = \frac{1}{2}(9)^{3/2} - 8 \times 2 + C$$

$$\frac{5}{2} = \frac{1}{2} \times 27 - 16 + C$$

$$\frac{5}{2} = -\frac{5}{2} + C$$

$$C = 5$$

$$\rightarrow \quad y = \frac{1}{2}(4x-7)^{\frac{3}{2}} - 8x^{\frac{1}{2}} + 5$$

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10 The equation of a curve is such that $\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$. The curve has a stationary point at $(-1, \frac{9}{2})$.

(a) Determine the nature of the stationary point at $(-1, \frac{9}{2})$. [1]

Sub. $x = -1$ into $\frac{d^2y}{dx^2}$: $6(-1)^2 - \frac{4}{(-1)^3}$

$= 6 \times 1 - \frac{4}{-1}$

$= 6 + 4$

$= 10$

$\frac{d^2y}{dx^2} > 0$

so minimum point

(b) Find the equation of the curve. [5]

$\frac{dy}{dx} = \int (6x^2 - 4x^{-3})\,dx$

$= 2x^3 + 2x^{-2} + c$

SP at $(-1, \frac{9}{2})$, so $\frac{dy}{dx} = 0$ when $x = -1$:

$0 = 2(-1)^3 + 2(-1)^{-2} + c$

$0 = -2 + 2 + c$

$c = 0$

$\rightarrow \frac{dy}{dx} = 2x^3 + 2x^{-2}$

$y = \int (2x^3 + 2x^{-2})\,dx$

$y = \frac{1}{2}x^4 - 2x^{-1} + c$

Sub. $(-1, \frac{9}{2})$:

$\frac{9}{2} = \frac{1}{2}(-1)^4 - 2(-1)^{-1} + c$

$\frac{9}{2} = \frac{1}{2} + 2 + c$

$\frac{9}{2} = \frac{5}{2} + c$

$c = 2$

$y = \frac{1}{2}x^4 - 2x^{-1} + 2$

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