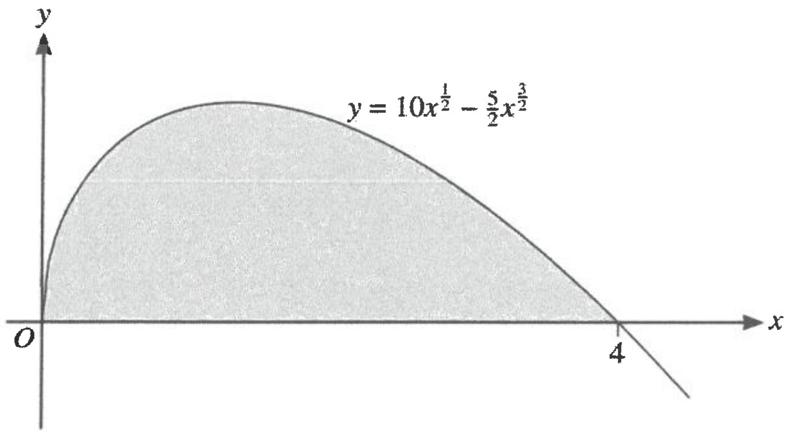
5



The diagram shows the curve with equation $y = 10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$ for $x > 0$. The curve meets the x-axis at the points $(0, 0)$ and $(4, 0)$.

Find the area of the shaded region. [4]

$$\int_0^4 \left(10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}\right) dx$$

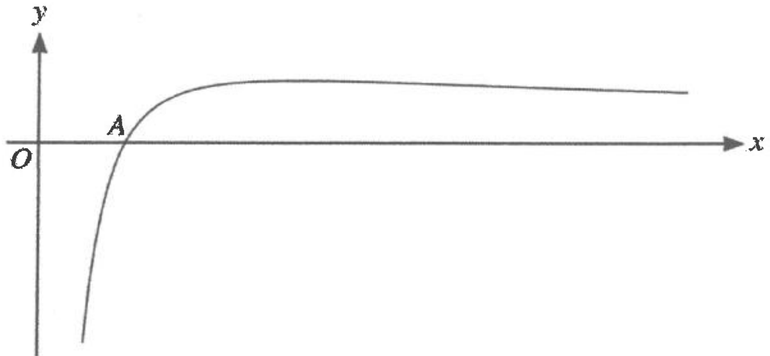
$$= \left[\frac{2}{3} \times 10x^{\frac{3}{2}} - \frac{2}{5} \times \frac{5}{2} x^{\frac{5}{2}}\right]_0^4$$

$$= \left[\frac{20}{3}x^{\frac{3}{2}} - x^{\frac{5}{2}}\right]_0^4$$

$$= \left[\frac{20}{3}(4)^{\frac{3}{2}} - (4)^{\frac{5}{2}}\right] - [0]$$

$$= \frac{20}{3} \times 8 - 32$$

$$= \frac{64}{3}$$



from part (a) $A = (4, 0)$

The diagram shows the curve with equation $y = 9(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}})$. The curve crosses the x-axis at the point A.

(d) Find the area of the region bounded by the curve, the x-axis and the line $x = 9$. [4]

$$\int_4^9 9(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}})\,dx$$

$$9\int_4^9 (x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}})\,dx$$

$$9\left[2x^{\frac{1}{2}} + 8x^{-\frac{1}{2}}\right]_4^9$$

$$= 9\left[\left(2(9)^{\frac{1}{2}} + 8(9)^{-\frac{1}{2}}\right) - \left(2(4)^{\frac{1}{2}} + 8(4)^{-\frac{1}{2}}\right)\right]$$

$$= 9\left[\left(2\times 3 + 8\times\frac{1}{3}\right) - \left(2\times 2 + 8\times\frac{1}{2}\right)\right]$$

$$= 9\left[\left(6 + \frac{8}{3}\right) - (4 + 4)\right]$$

$$= 9\left[6 + \frac{8}{3} - 8\right]$$

$$= 9\left[\frac{2}{3}\right]$$

$$= \underline{6}$$

11 The equation of a curve is $y = 2\sqrt{3x+4} - x$.

(d) Find the exact area of the region bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$. [4]

$$\int_0^4 \left[2(3x+4)^{\frac{1}{2}} - x\right] dx$$

$$\left[\frac{2}{3} \times 2 \times \frac{1}{3}(3x+4)^{\frac{3}{2}} - \frac{1}{2}x^2\right]_0^4$$

$$= \left[\frac{4}{9}(3x+4)^{\frac{3}{2}} - \frac{1}{2}x^2\right]_0^4$$

$$= \left[\frac{4}{9}(3(4)+4)^{\frac{3}{2}} - \frac{1}{2}(4)^2\right] - \left[\frac{4}{9}(3(0)+4)^{\frac{3}{2}} - 0\right]$$

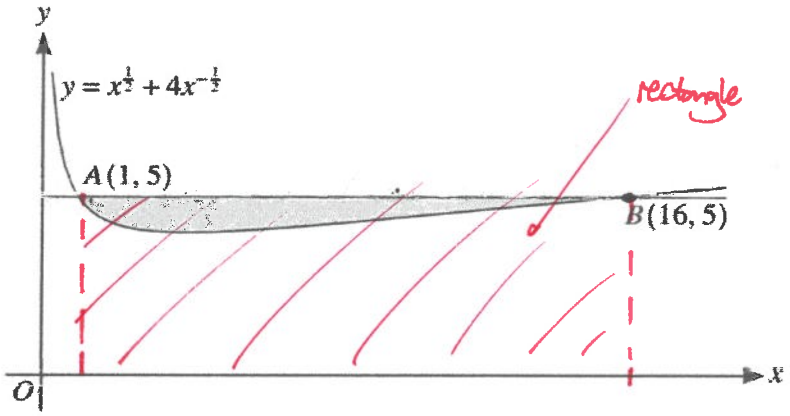
$$= \left[\frac{4}{9}(16)^{\frac{3}{2}} - \frac{1}{2} \times 16\right] - \left[\frac{4}{9}(4)^{\frac{3}{2}}\right]$$

$$= \left[\frac{4}{9} \times 64 - 8\right] - \left[\frac{4}{9} \times 8\right]$$

$$= \frac{256}{9} - 8 - \frac{32}{9}$$

$$= \frac{152}{9}$$

9709/11/M/J/21



The diagram shows the curve with equation $y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$. The line $y = 5$ intersects the curve at the points $A(1, 5)$ and $B(16, 5)$.

(b) Calculate the area of the shaded region. **[4]**

Area under curve: $\int_1^{16} \left(x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}\right) dx$

$= \left[\frac{2}{3}x^{3/2} + 8x^{\frac{1}{2}}\right]_1^{16}$

$= \left[\frac{2}{3}(16)^{3/2} + 8(16)^{\frac{1}{2}}\right] - \left[\frac{2}{3}(1)^{\frac{3}{2}} + 8(1)^{\frac{1}{2}}\right]$

$= \left[\frac{2}{3} \times 64 + 8 \times 4\right] - \left[\frac{2}{3} \times 1 + 8 \times 1\right]$

$= \left[\frac{128}{3} + 32\right] - \left[\frac{2}{3} + 8\right]$

$= \left[\frac{224}{3}\right] - \left[\frac{26}{3}\right]$

$= 66$

Area of rectangle: Area $=$ base $\times$ height

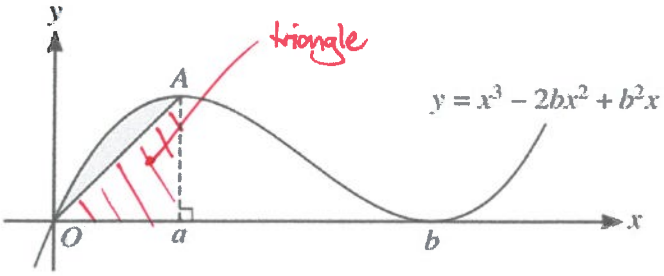
$= 15 \times 5$

$= 75$

Shaded Area: $75 - 66 = \underline{9}$

9709/13/M/J/22

11



from (a): b = 3a

The diagram shows part of the curve with equation $y = x^3 - 2bx^2 + b^2x$ and the line OA, where A is the maximum point on the curve. The x-coordinate of A is a and the curve has a minimum point at $(b, 0)$, where a and b are positive constants.

(b) Show that the area of the shaded region between the line and the curve is ka^4, where k is a fraction to be found. [7]

$$y = x^3 - 6ax^2 + 9a^2x$$

Area under curve:

$$\int_0^a (x^3 - 6ax^2 + 9a^2x)\,dx$$

$$= \left[\frac{1}{4}x^4 - 2ax^3 + \frac{9}{2}a^2x^2\right]_0^a$$

$$= \left[\frac{1}{4}a^4 - 2a^4 + \frac{9}{2}a^4\right] - [0]$$

$$= \underline{\frac{11}{4}a^4}$$

Area of triangle:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

For height, need y-coordinate of A:

Sub. $x = a$ into y:

$$y = a^3 - 6a \times a^2 + 9a^2 \times a$$

$$= a^3 - 6a^3 + 9a^3$$

$$= \underline{4a^3}$$

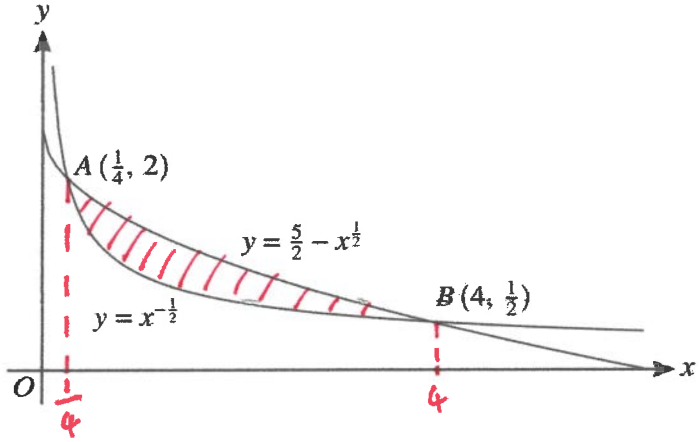
$$\rightarrow \text{Area} = \frac{1}{2} \times a \times 4a^3$$

$$= 2a^4$$

Shaded Area: $\frac{11}{4}a^4 - 2a^4 = \underline{\frac{3}{4}a^4}$

9709/13/M/J/20

8



The diagram shows the curves with equations $y = x^{-\frac{1}{2}}$ and $y = \frac{5}{2} - x^{\frac{1}{2}}$. The curves intersect at the points $A\left(\frac{1}{4}, 2\right)$ and $B\left(4, \frac{1}{2}\right)$.

(a) Find the area of the region between the two curves. [6]

$$\int_{\frac{1}{4}}^{4} \left[\left(\frac{5}{2} - x^{\frac{1}{2}}\right) - x^{-\frac{1}{2}}\right] dx$$

$$= \int_{\frac{1}{4}}^{4} \left(\frac{5}{2} - x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right) dx$$

$$\left[\frac{5}{2}x - \frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}}\right]_{\frac{1}{4}}^{4}$$

$$= \left[\frac{5}{2}(4) - \frac{2}{3}(4)^{\frac{3}{2}} - 2(4)^{\frac{1}{2}}\right] - \left[\frac{5}{2}\left(\frac{1}{4}\right) - \frac{2}{3}\left(\frac{1}{4}\right)^{\frac{3}{2}} - 2\left(\frac{1}{4}\right)^{\frac{1}{2}}\right]$$

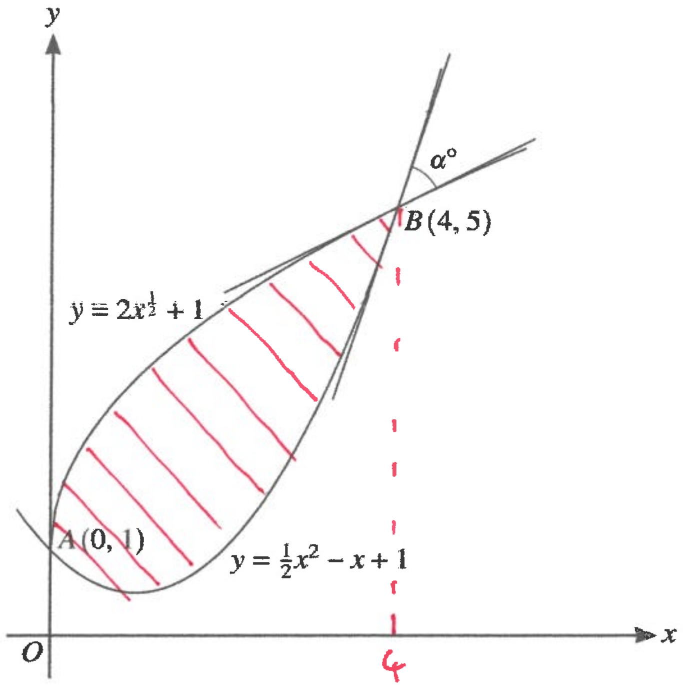
$$= \left[10 - \frac{2}{3} \times 8 - 2 \times 2\right] - \left[\frac{5}{8} - \frac{2}{3} \times \frac{1}{8} - 2 \times \frac{1}{2}\right]$$

$$= \left[10 - \frac{16}{3} - 4\right] - \left[\frac{5}{8} - \frac{1}{12} - 1\right]$$

$$= \left[\frac{2}{3}\right] - \left[\frac{-11}{24}\right]$$

$$= \frac{9}{8}$$

10



Curves with equations $y = 2x^{\frac{1}{2}} + 1$ and $y = \frac{1}{2}x^2 - x + 1$ intersect at $A\,(0, 1)$ and $B\,(4, 5)$, as shown in the diagram.

(a) Find the area of the region between the two curves. [5]

$$\int_0^4 \left[\left(2x^{\frac{1}{2}} + 1\right) - \left(\tfrac{1}{2}x^2 - x + 1\right)\right] dx$$

$$\int_0^4 \left(2x^{\frac{1}{2}} - \tfrac{1}{2}x^2 + x\right) dx$$

$$= \left[\tfrac{4}{3}x^{\frac{3}{2}} - \tfrac{1}{6}x^3 + \tfrac{1}{2}x^2\right]_0^4$$

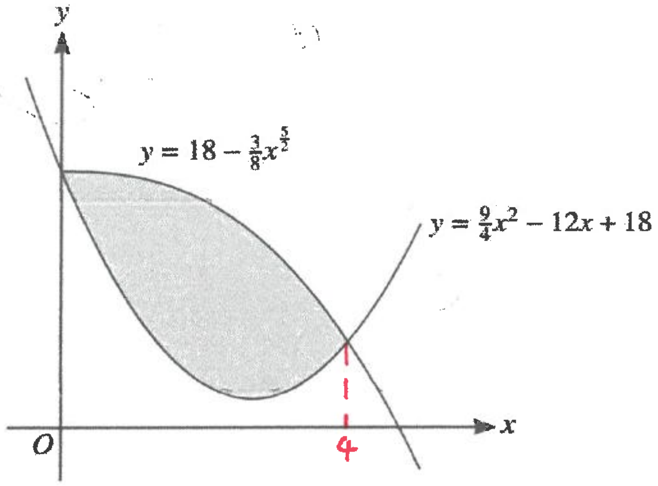
$$= \left[\tfrac{4}{3}(4)^{\frac{3}{2}} - \tfrac{1}{6}(4)^3 + \tfrac{1}{2}(4)^2\right] - [0]$$

$$= \tfrac{4}{3} \times 8 - \tfrac{1}{6} \times 64 + \tfrac{1}{2} \times 16$$

$$= \frac{32}{3} - \frac{32}{3} + 8$$

$$= \underline{8}$$

11



The diagram shows the curves with equations $y = \frac{9}{4}x^2 - 12x + 18$ and $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$. The curves intersect at the points (0, 18) and (4, 6).

(b) Find the area of the shaded region. [5]

$$\int_0^4 \left[\left(18 - \frac{3}{8}x^{5/2}\right) - \left(\frac{9}{4}x^2 - 12x + 18\right)\right] dx$$

$$= \int_0^4 \left(-\frac{3}{8}x^{5/2} - \frac{9}{4}x^2 + 12x\right) dx$$

$$= \left[\frac{2}{7} \times \frac{-3}{8}x^{7/2} - \frac{1}{3} \times \frac{9}{4}x^3 + 6x^2\right]_0^4$$

$$= \left[\frac{-3}{28}x^{7/2} - \frac{3}{4}x^3 + 6x^2\right]_0^4$$

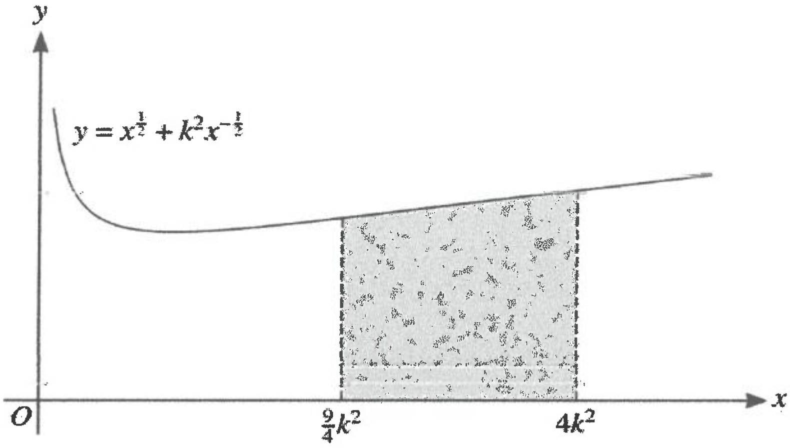
$$= \left[\frac{-3}{28}(4)^{7/2} - \frac{3}{4}(4)^3 + 6(4)^2\right] - [0]$$

$$= \frac{-3}{28} \times 128 - \frac{3}{4} \times 64 + 6 \times 16$$

$$= -\frac{96}{7} - 48 + 96$$

$$= \frac{240}{7}$$

11



The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$, where k is a positive constant.

The shaded region is bounded by the curve, the x-axis and the lines $x = \frac{9}{4}k^2$ and $x = 4k^2$.

(c) Find the area of the shaded region in terms of k. [3]

$$\int_{\frac{9}{4}k^2}^{4k^2} \left(x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}\right) dx$$

$$= \left[\frac{2}{3}x^{\frac{3}{2}} + 2k^2x^{\frac{1}{2}}\right]_{\frac{9}{4}k^2}^{4k^2}$$

$$= \left[\frac{2}{3}(4k^2)^{\frac{3}{2}} + 2k^2(4k^2)^{\frac{1}{2}}\right] - \left[\frac{2}{3}\left(\frac{9}{4}k^2\right)^{\frac{3}{2}} + 2k^2\left(\frac{9}{4}k^2\right)^{\frac{1}{2}}\right]$$

$$= \left[\frac{2}{3}(2k)^3 + 2k^2(2k)\right] - \left[\frac{2}{3}\left(\frac{3}{2}k\right)^3 + 2k^2\left(\frac{3}{2}k\right)\right]$$

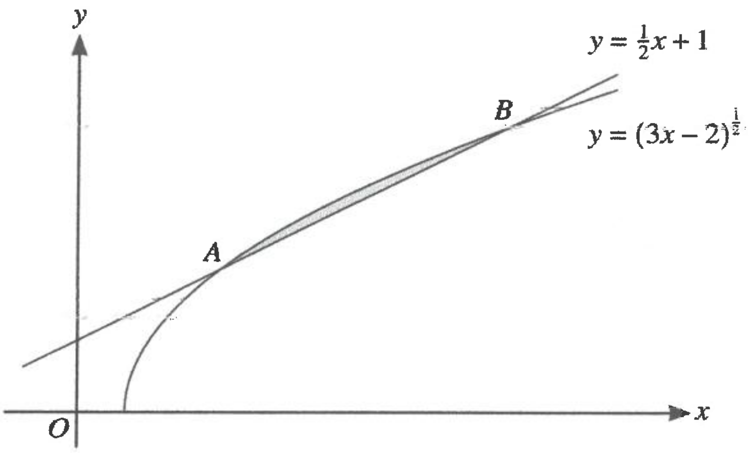
$$= \left[\frac{2}{3}\times 8k^3 + 4k^3\right] - \left[\frac{2}{3}\times\frac{27}{8}k^3 + 3k^3\right]$$

$$= \frac{16}{3}k^3 + 4k^3 - \frac{9}{4}k^3 - 3k^3$$

$$= \underline{\frac{49}{12}k^3}$$

9709/13/M/J/21

7



The diagram shows the curve with equation $y = (3x-2)^{\frac{1}{2}}$ and the line $y = \frac{1}{2}x + 1$. The curve and the line intersect at points A and B.

(a) Find the coordinates of A and B. [4]

Solve simultaneously:

$(3x-2)^{\frac{1}{2}} = \frac{1}{2}x + 1$

$\sqrt{3x-2} = \frac{x}{2} + 1$

$3x - 2 = \left(\frac{x}{2} + 1\right)^2$

$3x - 2 = \frac{x^2}{4} + x + 1$

$\frac{x^2}{4} - 2x + 3 = 0$

$\times 4 \quad \times 4 \quad \times 4 \quad \times 4$

$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$x = 6$ or $x = 2$

sub. $x=2$ into y:

$y = (3\times 2 - 2)^{\frac{1}{2}}$

$= (4)^{\frac{1}{2}}$

$= 2$

$\rightarrow (2,2)$ A

sub. $x=6$ into y:

$y = (3\times 6 - 2)^{\frac{1}{2}}$

$= (16)^{\frac{1}{2}}$

$= 4$

$\rightarrow (6,4)$ B

(b) Hence find the area of the region enclosed between the curve and the line. [5]

$$\int_2^6 \left[(3x-2)^{\frac{1}{2}} - \left(\tfrac{1}{2}x+1\right)\right] dx$$

↑ curve ↑ line

$$= \left[\frac{2}{3}\times\frac{1}{3}(3x-2)^{\frac{3}{2}} - \left(\frac{1}{4}x^2+x\right)\right]_2^6$$

$$= \left[\frac{2}{9}(3x-2)^{\frac{3}{2}} - \frac{1}{4}x^2 - x\right]_2^6$$

$$= \left[\frac{2}{9}(3(6)-2)^{\frac{3}{2}} - \frac{1}{4}(6)^2 - 6\right] - \left[\frac{2}{9}(3(2)-2)^{\frac{3}{2}} - \frac{1}{4}(2)^2 - 2\right]$$

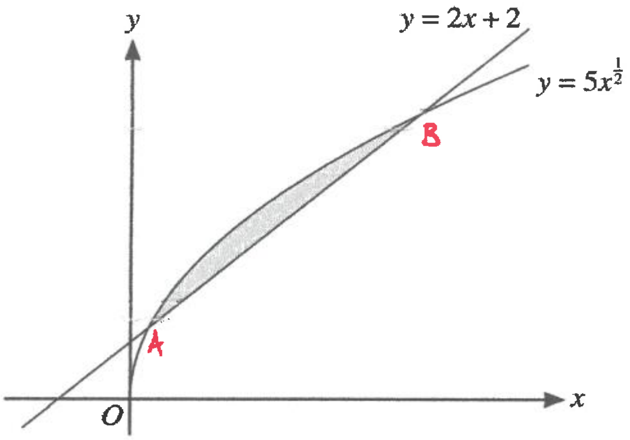
$$= \left[\frac{2}{9}(16)^{\frac{3}{2}} - \frac{1}{4}\times 36 - 6\right] - \left[\frac{2}{9}(4)^{\frac{3}{2}} - \frac{1}{4}\times 4 - 2\right]$$

$$= \left[\frac{2}{9}\times 64 - 9 - 6\right] - \left[\frac{2}{9}\times 8 - 1 - 2\right]$$

$$= \left[\frac{-7}{9}\right] - \left[\frac{-11}{9}\right]$$

$$= \frac{4}{9}$$

6



The diagram shows the curve with equation $y = 5x^{\frac{1}{2}}$ and the line with equation $y = 2x + 2$.

Find the exact area of the shaded region which is bounded by the line and the curve. [5]

Find points of intersection:

$$5x^{\frac{1}{2}} = 2x + 2$$

$$x^{\frac{1}{2}} = \frac{2x+2}{5}$$

Square both sides...

$$x = \frac{4x^2 + 8x + 4}{25}$$

$$4x^2 + 8x + 4 = 25x$$

$$4x^2 - 17x + 4 = 0$$

$$(4x - 1)(x - 4) = 0$$

$$x = \tfrac{1}{4} \quad \text{or} \quad x = 4$$

$$\rightarrow \int_{\frac{1}{4}}^{4} \left[5x^{\frac{1}{2}} - (2x + 2)\right] dx$$

↑ curve ↑ line

$$\left[\frac{2}{3} \times 5x^{\frac{3}{2}} - x^2 - 2x\right]_{\frac{1}{4}}^{4}$$

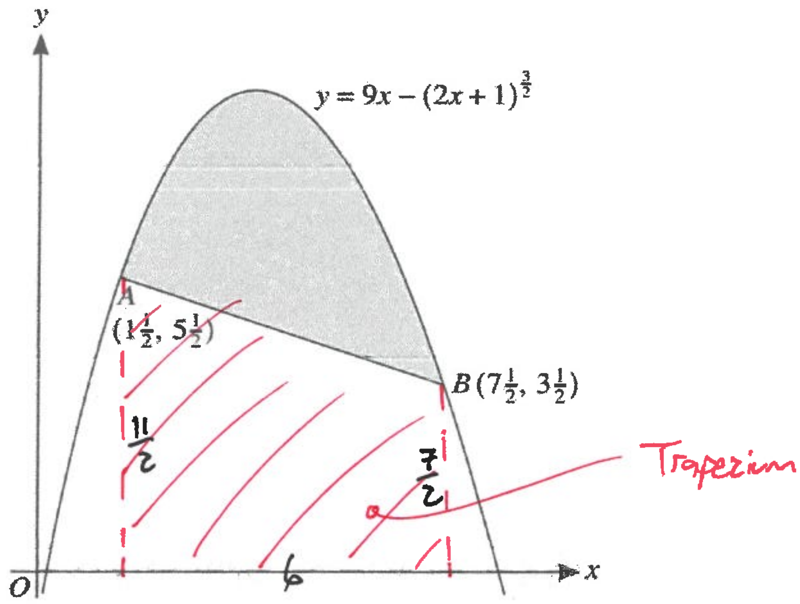
$$\left[\frac{10}{3}x^{\frac{3}{2}} - x^2 - 2x\right]_{\frac{1}{4}}^{4}$$

$$\left[\frac{10}{3}(4)^{\frac{3}{2}} - (4)^2 - 2(4)\right] - \left[\frac{10}{3}\left(\frac{1}{4}\right)^{\frac{3}{2}} - \left(\frac{1}{4}\right)^2 - 2\left(\frac{1}{4}\right)\right]$$

$$= \left[\frac{10}{3} \times 8 - 16 - 8\right] - \left[\frac{10}{3} \times \frac{1}{8} - \frac{1}{16} - \frac{1}{2}\right]$$

$$= \left[\frac{8}{3}\right] - \left[-\frac{7}{48}\right]$$

$$= \underline{\frac{45}{16}}$$



The diagram shows the points $A\,(1\frac{1}{2}, 5\frac{1}{2})$ and $B\,(7\frac{1}{2}, 3\frac{1}{2})$ lying on the curve with equation $y = 9x - (2x+1)^{\frac{3}{2}}$.

(c) Find the area of the shaded region. [5]

Area under curve between A & B:

$$\int_{3/2}^{15/2} \left(9x - (2x+1)^{3/2}\right) dx$$

$$= \left[\frac{9}{2}x^2 - \frac{2}{5}\times\frac{1}{2}(2x+1)^{5/2}\right]_{3/2}^{15/2}$$

$$= \left[\frac{9}{2}\left(\frac{15}{2}\right)^2 - \frac{1}{5}\left(2\left(\frac{15}{2}\right)+1\right)^{\frac{5}{2}}\right] - \left[\frac{9}{2}\left(\frac{3}{2}\right)^2 - \frac{1}{5}\left(2\left(\frac{3}{2}\right)+1\right)^{\frac{5}{2}}\right]$$

$$= \left[\frac{9}{2}\times\frac{225}{4} - \frac{1}{5}(16)^{5/2}\right] - \left[\frac{9}{2}\times\frac{9}{4} - \frac{1}{5}(4)^{\frac{5}{2}}\right]$$

$$= \left[\frac{2025}{8} - \frac{1024}{5}\right] - \left[\frac{81}{8} - \frac{32}{5}\right]$$

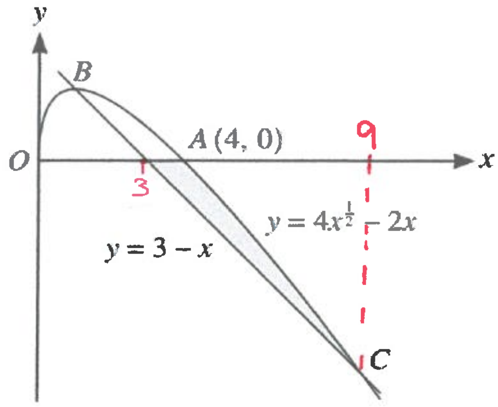
$$= \left[\frac{1933}{40}\right] - \left[\frac{149}{40}\right] = \frac{223}{5}$$

Area of trapezium:

$$\text{Area} = \frac{1}{2}(a+b)\times h$$
$$= \frac{1}{2}\left(\frac{11}{2}+\frac{7}{2}\right)\times 6$$
$$= \frac{1}{2}(9)\times 6$$
$$= 27$$

Shaded Area:

$$\frac{223}{5} - 27 = 17.6$$



from (a): x-coordinate of C = 9

Shaded Area = Area between $y=3-x$ and x-axis − Area between $y=4x^{\frac{1}{2}}-2x$ and x-axis

The diagram shows a curve with equation $y = 4x^{\frac{1}{2}} - 2x$ for $x \geqslant 0$, and a straight line with equation $y = 3 - x$. The curve crosses the x-axis at $A\ (4, 0)$ and crosses the straight line at B and C.

(c) Find the area of the shaded region. **[6]**

Area of triangle made by $y=3-x$:

x-coord of x-intercept: $0=3-x \rightarrow \underline{x=3}$

y-coord of point C: $y=3-9 \rightarrow \underline{y=6}$

$\rightarrow$ Area of triangle $= \frac{1}{2}\times b\times h$

$= \frac{1}{2}\times 6\times 6$

$= \underline{18}$

Area between curve and x-axis: (limits are $x=4$ and $x=9$)

$$\int_4^9 \left(4x^{\frac{1}{2}} - 2x\right)dx$$

$$= \left[\tfrac{2}{3}\times 4x^{3/2} - x^2\right]_4^9$$

$$= \left[\tfrac{2}{3}\times 4(9)^{3/2} - (9)^2\right] - \left[\tfrac{2}{3}\times 4(4)^{3/2} - (4)^2\right]$$

$$= \left[\tfrac{8}{3}\times 27 - 81\right] - \left[\tfrac{8}{3}\times 8 - 16\right]$$

$$= \left[-9\right] - \left[\tfrac{16}{3}\right]$$

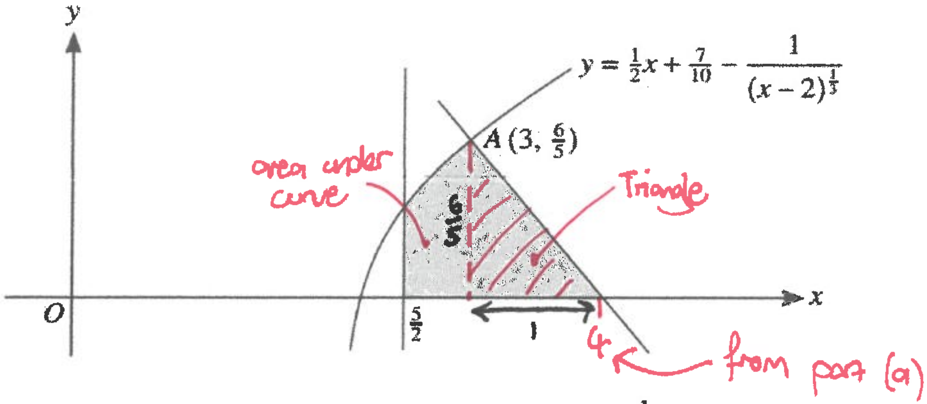
$$= -\frac{43}{3}$$

negative because below axis, so area $= \frac{43}{3}$

$\rightarrow$ Shaded area:

$18 - \frac{43}{3}$

$= \underline{\frac{11}{3}}$



The diagram shows the line $x = \frac{5}{2}$, part of the curve $y = \frac{1}{2}x + \frac{7}{10} - \frac{1}{(x-2)^{\frac{1}{3}}}$ and the normal to the curve at the point $A\,(3, \frac{6}{5})$.

(b) Find the area of the shaded region, giving your answer correct to 2 decimal places. [6]

Area of triangle: $\text{Area} = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 1 \times \frac{6}{5}$

$= 0.6$

Area under curve: $\int_{\frac{5}{2}}^{3} \left(\frac{1}{2}x + \frac{7}{10} - (x-2)^{-\frac{1}{3}}\right) dx$

$= \left[\frac{1}{4}x^2 + \frac{7}{10}x - \frac{3}{2}(x-2)^{2/3}\right]_{5/2}^{3}$

$= \left[\frac{1}{4}(3)^2 + \frac{7}{10}(3) - \frac{3}{2}(1)^{2/3}\right] - \left[\frac{1}{4}\left(\frac{5}{2}\right)^2 + \frac{7}{10}\left(\frac{5}{2}\right) - \frac{3}{2}\left(\frac{1}{2}\right)^{2/3}\right]$

$= \left[\frac{9}{4} + \frac{21}{10} - \frac{3}{2}\right] - \left[\frac{1}{4} \times \frac{25}{4} + \frac{35}{20} - \frac{3}{2} \times 0.5^{2/3}\right]$

$= \left[\frac{57}{20}\right] - \left[\frac{53}{16} - \frac{3}{2}(0.5)^{2/3}\right]$

$= \frac{-37}{80} + \frac{3}{2}(0.5)^{2/3}$

$= 0.482_{\text{STO}}$

Shaded Area: $0.6 + 0.482 = \underline{\underline{1.08}}$

9709/12/O/N/21