1 The function f is defined by $f(x) = \frac{1}{3x+2} + x^2$ for $x < -1$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

$f(x) = (3x+2)^{-1} + x^2$

$f'(x) = -3(3x+2)^{-2} + 2x$

$= \frac{-3}{(3x+2)^2} + 2x$

negative

$\frac{-3}{(3x+2)^2} + 2x$

negative because $x < -1$

always positive

so $f'(x) < 0$

therefore decreasing function

11 (c) The function g is defined by $g(x) = -(3x + 1)^{-1} + 3x$ for $x \geqslant 0$.

Determine, making your reasoning clear, whether g is an increasing function, a decreasing function or neither. [2]

$$g(x) = -(3x+1)^{-1} + 3x$$

$$g'(x) = 3(3x+1)^{-2} + 3$$

positive

$$= \frac{3}{(3x+1)^2} + 3$$

positive

always positive

so $g'(x) > 0$

therefore increasing function

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8 The function f is defined by $f(x) = 2 - \frac{3}{4x - p}$ for $x > \frac{p}{4}$, where p is a constant.

(a) Find $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [3]

$f(x) = 2 - 3(4x-p)^{-1}$

$f'(x) = 3 \times 4(4x-p)^{-2}$

$= 12(4x-p)^{-2}$

$= \frac{12}{\underbrace{(4x-p)^2}_{\text{always positive}}}$ ← positive

so $f'(x) > 0$

therefore increasing function

3 **(a)** Express $5y^2 - 30y + 50$ in the form $5(y + a)^2 + b$, where a and b are constants. [2]

$5[y^2 - 6y] + 50$

$5[(y-3)^2 - 9] + 50$

$5(y-3)^2 - 45 + 50$

$5(y-3)^2 + 5$

(b) The function f is defined by $f(x) = x^5 - 10x^3 + 50x$ for $x \in \mathbb{R}$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

$f'(x) = 5x^4 - 30x^2 + 50$

using part (a), with $y = x^2$:

$f'(x) = \underbrace{5(x^2-3)^2}_{\text{always positive}} + \underbrace{5}_{\text{positive}}$

So $f'(x) > 0$

therefore increasing function

2 The function f is defined by $f(x) = \frac{1}{3}(2x-1)^{\frac{3}{2}} - 2x$ for $\frac{1}{2} < x < a$. It is given that f is a decreasing function.

Find the maximum possible value of the constant a. [4]

$$f(x) = \frac{1}{3}(2x-1)^{3/2} - 2x$$

$$f'(x) = \frac{1}{3} \times \frac{3}{2} \times 2(2x-1)^{\frac{1}{2}} - 2$$

$$= (2x-1)^{\frac{1}{2}} - 2$$

decreasing, so $f'(x) < 0$:

$$(2x-1)^{\frac{1}{2}} - 2 < 0$$

$$\sqrt{2x-1} < 2$$

$$2x - 1 < 4$$

$$2x < 5$$

$$x < \frac{5}{2}$$

so $a = \frac{5}{2}$