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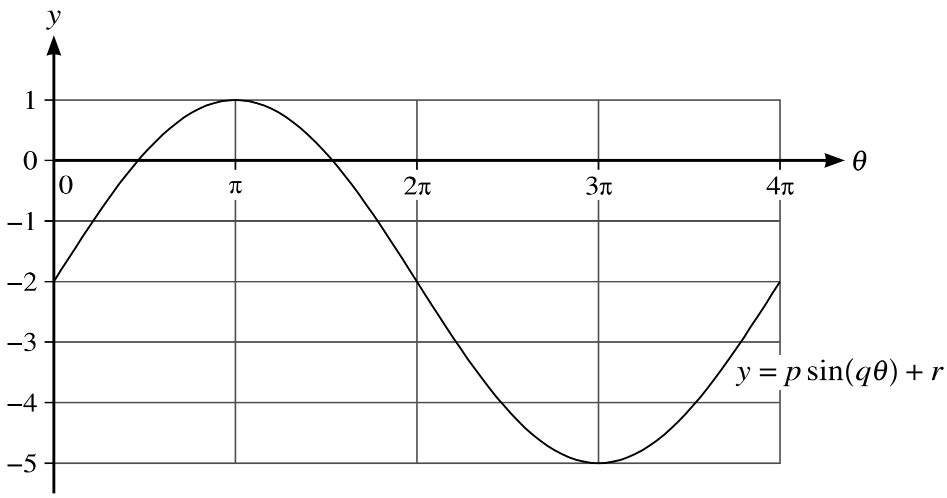

The diagram shows part of the curve with equation $y = p\sin(q\theta) + r$, where p, q and r are constants.

(a) State the value of p. [1]

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(b) State the value of q. [1]

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(c) State the value of r. [1]

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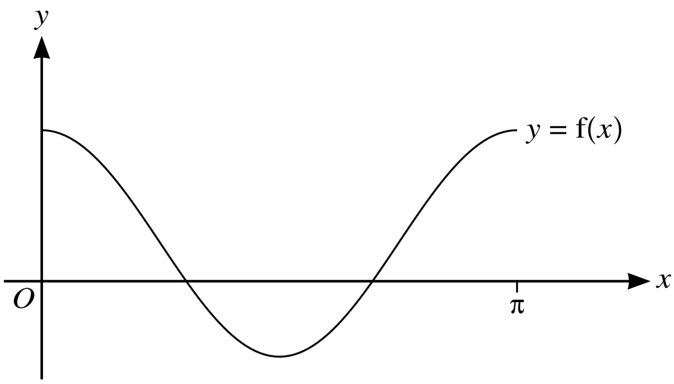

The diagram shows the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$.

(a) State the range of f. [2]

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A function g is such that $\mathrm{g}(x) = \mathrm{f}(x) + k$, where k is a positive constant. The x-axis is a tangent to the curve $y = \mathrm{g}(x)$.

(b) State the value of k and hence describe fully the transformation that maps the curve $y = \mathrm{f}(x)$ on to $y = \mathrm{g}(x)$. [2]

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(c) State the equation of the curve which is the reflection of $y = \mathrm{f}(x)$ in the x-axis. Give your answer in the form $y = a\cos 2x + b$, where a and b are constants. [1]

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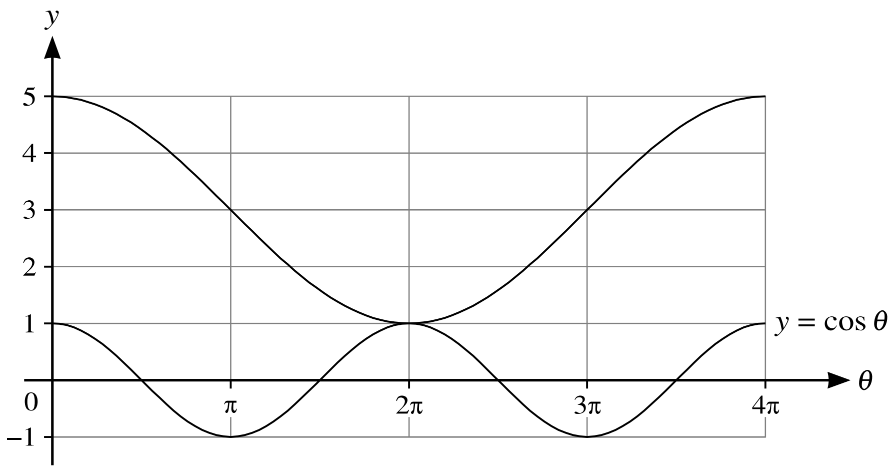

In the diagram, the lower curve has equation $y = \cos\theta$. The upper curve shows the result of applying a combination of transformations to $y = \cos\theta$.

Find, in terms of a cosine function, the equation of the upper curve. [3]

7 A curve has equation $y = 2 + 3\sin\frac{1}{2}x$ for $0 \leqslant x \leqslant 4\pi$.

(a) State greatest and least values of y. [2]

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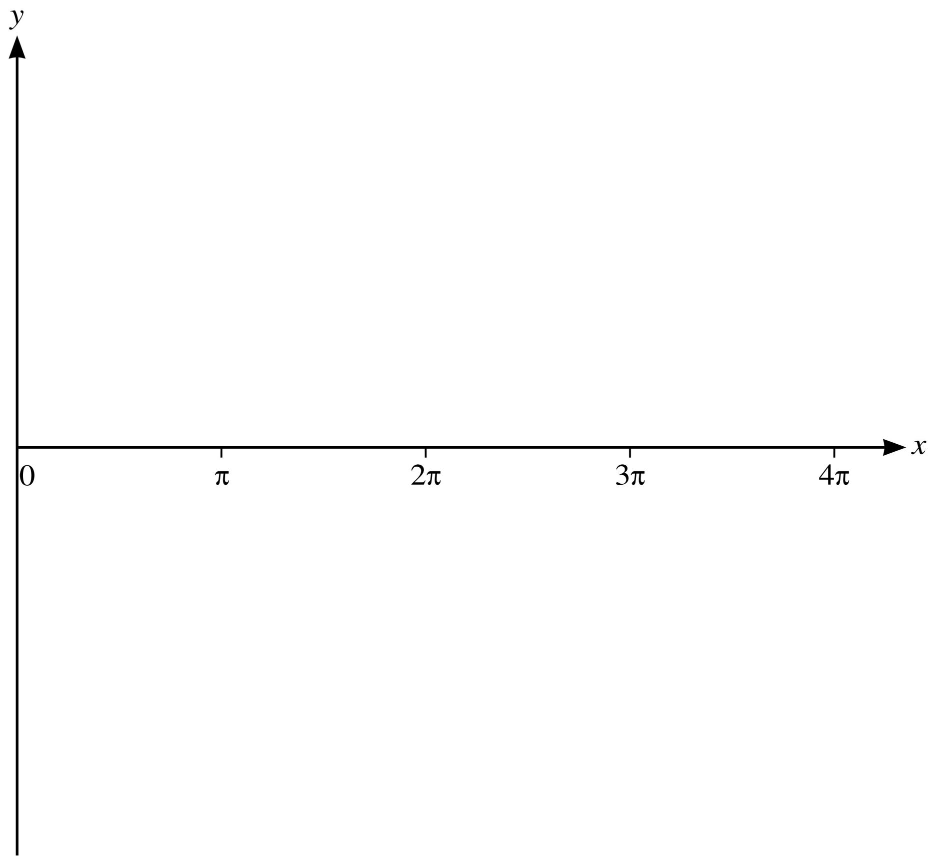
(b) Sketch the curve. [2]



(c) State the number of solutions of the equation

$$2 + 3\sin\tfrac{1}{2}x = 5 - 2x$$

for $0 \leqslant x \leqslant 4\pi$. [1]

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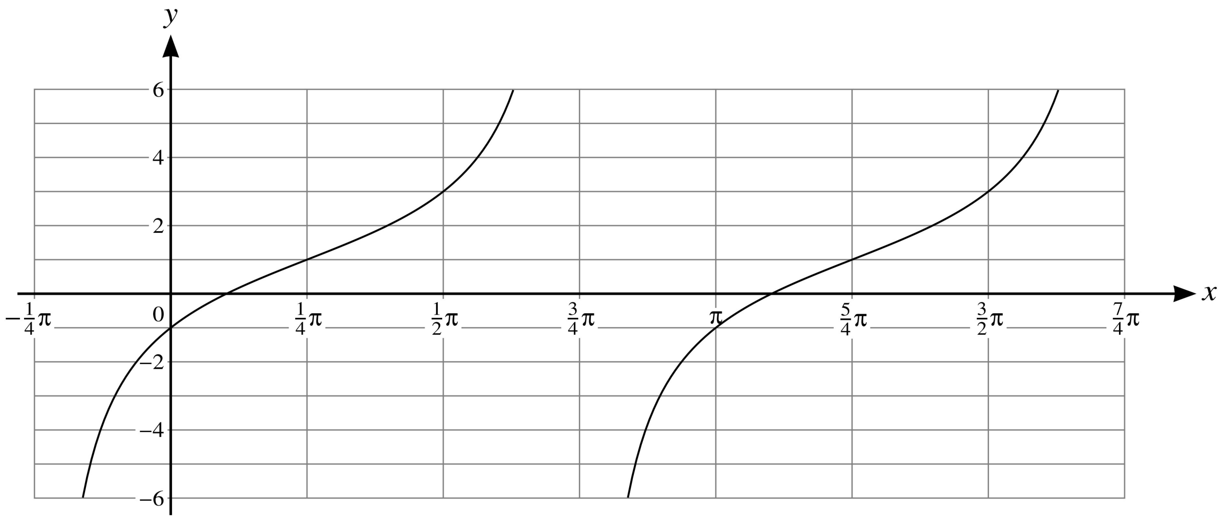

The diagram shows part of the graph of $y = a\tan(x - b) + c$.

Given that $0 < b < \pi$, state the values of the constants a, b and c. [3]

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8 **(a)** The curve $y = \sin x$ is transformed to the curve $y = 4\sin\left(\frac{1}{2}x - 30°\right)$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied. [5]

9 Functions f and g are such that

$$f(x) = 2 - 3\sin 2x \quad \text{for } 0 \leqslant x \leqslant \pi,$$
$$g(x) = -2f(x) \quad \text{for } 0 \leqslant x \leqslant \pi.$$

(a) State the ranges of f and g. [3]

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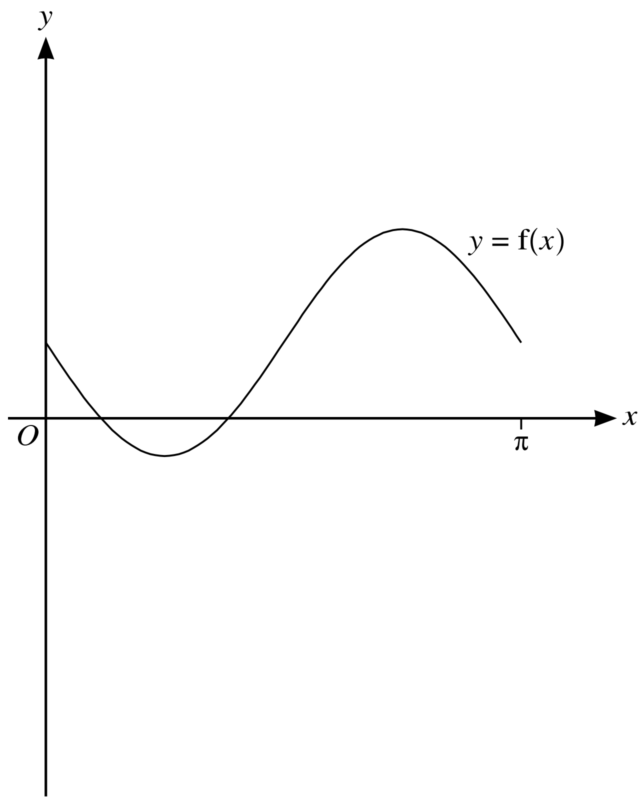
The diagram below shows the graph of $y = f(x)$.



(b) Sketch, on this diagram, the graph of $y = g(x)$. [2]

The function h is such that

$$h(x) = g(x + \pi) \quad \text{for } -\pi \leqslant x \leqslant 0.$$

(c) Describe fully a sequence of transformations that maps the curve $y = f(x)$ on to $y = h(x)$. [3]

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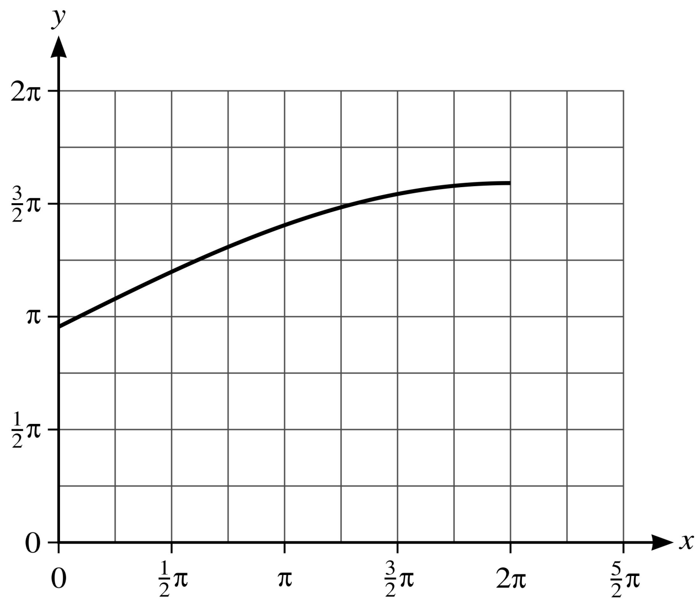

The diagram shows the graph of $y = \mathrm{f}(x)$ where the function f is defined by

$$\mathrm{f}(x) = 3 + 2\sin\tfrac{1}{4}x \text{ for } 0 \leqslant x \leqslant 2\pi.$$

(a) On the diagram above, sketch the graph of $y = \mathrm{f}^{-1}(x)$. [2]

(b) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

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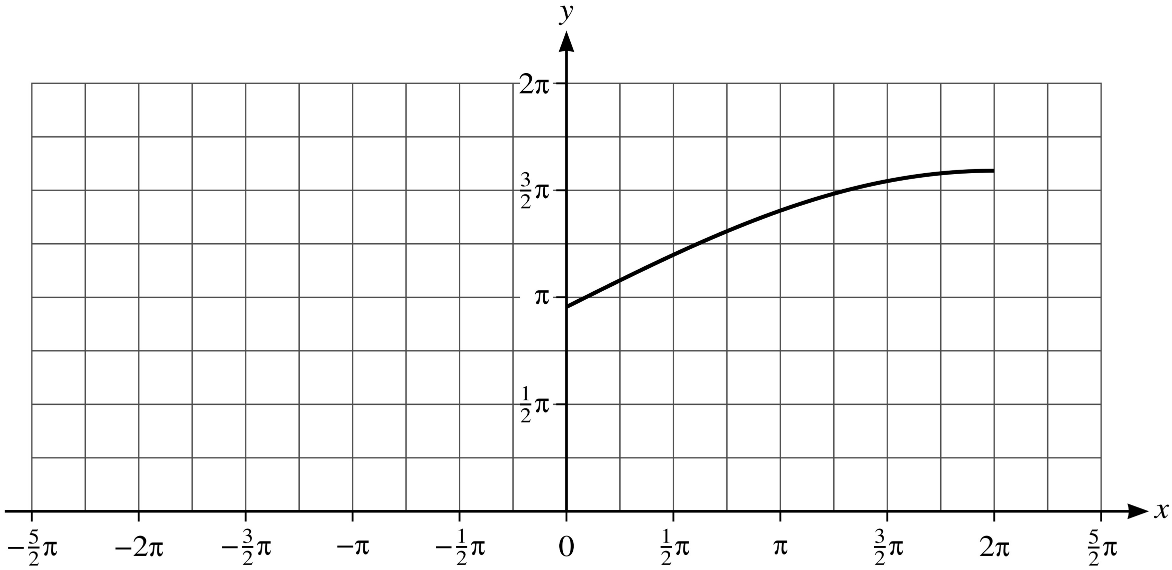
(c)



The diagram above shows part of the graph of the function $g(x) = 3 + 2\sin\frac{1}{4}x$ for $-2\pi \leqslant x \leqslant 2\pi$.

Complete the sketch of the graph of $g(x)$ on the diagram above and hence explain whether the function g has an inverse. [2]

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(d) Describe fully a sequence of three transformations which can be combined to transform the graph of $y = \sin x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ to the graph of $y = f(x)$, making clear the order in which the transformations are applied. [6]

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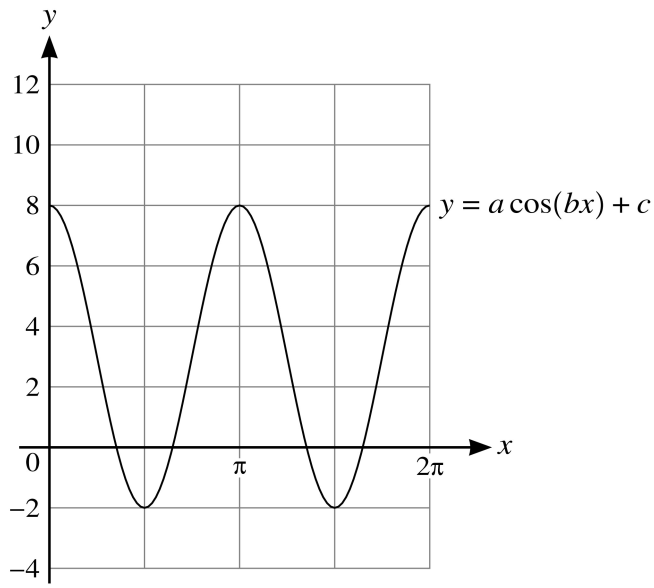

The diagram shows part of the graph of $y = a\cos(bx) + c$.

(a) Find the values of the positive integers a, b and c. [3]

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(b) For these values of a, b and c, use the given diagram to determine the number of solutions in the interval $0 \leqslant x \leqslant 2\pi$ for each of the following equations.

(i) $a\cos(bx) + c = \frac{6}{\pi}x$ [1]

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(ii) $a\cos(bx) + c = 6 - \frac{6}{\pi}x$ [1]

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11 A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

(a) State the greatest and least values of y. [2]

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(b) Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]

(c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

(i) $k = -3$ [1]

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(ii) $k = 1$ [1]

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(iii) $k = 3$ [1]

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Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$\mathrm{f}(x) = 3\cos 2x + 2,$$
$$\mathrm{g}(x) = \mathrm{f}(2x) + 4,$$
$$\mathrm{h}(x) = 2\mathrm{f}\left(x + \tfrac{1}{2}\pi\right).$$

(d) Describe fully a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ on to $y = \mathrm{g}(x)$. [2]

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(e) Describe fully a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ on to $y = \mathrm{h}(x)$. [2]

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