3 The function f is defined as follows:

$$f(x) = \frac{x+3}{x-1} \text{ for } x > 1.$$

(a) Find the value of ff(5). [2]

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(b) Find an expression for $f^{-1}(x)$. [3]

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6 Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto \tfrac{1}{2}x - a,$$
$$\mathrm{g} : x \mapsto 3x + b,$$

where a and b are constants.

(a) Given that $\mathrm{gg}(2) = 10$ and $\mathrm{f}^{-1}(2) = 14$, find the values of a and b. [4]

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(b) Using these values of a and b, find an expression for $\mathrm{gf}(x)$ in the form $cx + d$, where c and d are constants. [2]

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6 The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$.

(a) Express $f(x)$ in the form $2(x + a)^2 + b$. [2]

(b) Find the range of f. [1]

(c) Find an expression for $f^{-1}(x)$. [3]

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The function g is defined by $g(x) = 2x + 4$ for $x < -1$.

(d) Find and simplify an expression for $fg(x)$. [2]

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5 Functions f and g are defined by

$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$

$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

(a) Find the value of fg(7). [1]

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(b) Find the values of x for which $f^{-1}(x) = g^{-1}(x)$. [5]

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9 Functions f, g and h are defined as follows:

$$f : x \mapsto x - 4x^{\frac{1}{2}} + 1 \quad \text{for } x \geqslant 0,$$
$$g : x \mapsto mx^2 + n \quad \text{for } x \geqslant -2, \text{ where } m \text{ and } n \text{ are constants,}$$
$$h : x \mapsto x^{\frac{1}{2}} - 2 \quad \text{for } x \geqslant 0.$$

(a) Solve the equation $f(x) = 0$, giving your solutions in the form $x = a + b\sqrt{c}$, where a, b and c are integers. [4]

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(b) Given that $f(x) \equiv gh(x)$, find the values of m and n. [4]

6 The function f is defined as follows:

$$f(x) = \frac{x^2 - 4}{x^2 + 4} \quad \text{for } x > 2.$$

(a) Find an expression for $f^{-1}(x)$. [3]

(b) Show that $1 - \frac{8}{x^2 + 4}$ can be expressed as $\frac{x^2 - 4}{x^2 + 4}$ and hence state the range of f. [4]

(c) Explain why the composite function ff cannot be formed. [1]

8 The function f is defined by $f(x) = 2 - \frac{3}{4x - p}$ for $x > \frac{p}{4}$, where p is a constant.

(b) Express $f^{-1}(x)$ in the form $\frac{p}{a} - \frac{b}{cx - d}$, where a, b, c and d are integers. [4]

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(c) Hence state the value of p for which $f^{-1}(x) \equiv f(x)$. [1]

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11 The functions f and g are defined by

$$\mathrm{f}(x) = x^2 + 3 \quad \text{for } x > 0,$$
$$\mathrm{g}(x) = 2x + 1 \quad \text{for } x > -\tfrac{1}{2}.$$

(a) Find an expression for $\mathrm{fg}(x)$. [1]

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(b) Find an expression for $(\mathrm{fg})^{-1}(x)$ and state the domain of $(\mathrm{fg})^{-1}$. [4]

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(c) Solve the equation $\mathrm{fg}(x) - 3 = \mathrm{gf}(x)$. [4]

9 Functions f and g are defined by

$$f(x) = x + \frac{1}{x} \quad \text{for } x > 0,$$
$$g(x) = ax + 1 \quad \text{for } x \in \mathbb{R},$$

where a is a constant.

(a) Find an expression for $gf(x)$. [1]

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(b) Given that $gf(2) = 11$, find the value of a. [2]

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(c) Given that the graph of $y = f(x)$ has a minimum point when $x = 1$, explain whether or not f has an inverse. [1]

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It is given instead that $a = 5$.

(d) Find and simplify an expression for $g^{-1}f(x)$. [3]

(e) Explain why the composite function fg cannot be formed. [1]

9 **(a)** Express $2x^2 + 12x + 11$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

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The function f is defined by $\mathrm{f}(x) = 2x^2 + 12x + 11$ for $x \leqslant -4$.

(b) Find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

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The function g is defined by $g(x) = 2x - 3$ for $x \leqslant k$.

(c) For the case where $k = -1$, solve the equation $fg(x) = 193$. [2]

(d) State the largest value of k possible for the composition fg to be defined. [1]

9 The function f is defined by $f(x) = -3x^2 + 2$ for $x \leqslant -1$.

(a) State the range of f. [1]

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(b) Find an expression for $f^{-1}(x)$. [3]

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The function g is defined by $g(x) = -x^2 - 1$ for $x \leqslant -1$.

(c) Solve the equation $fg(x) - gf(x) + 8 = 0$. [5]

9 The functions f and g are defined by

$$\mathrm{f}(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$

$$\mathrm{g}(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

(a) Express $\mathrm{f}(x)$ in the form $(x - a)^2 + b$. [2]

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It is given that f is a one-one function.

(b) State the smallest possible value of c. [1]

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It is now given that $c = 5$.

(c) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

(d) Find an expression for $gf(x)$ and state the range of gf. [3]