1 Solve the equation $3x + 2 = \frac{2}{x-1}$. [3]

$$3x + 2 = \frac{2}{x-1}$$

$\times(x-1)$ $\times(x-1)$

$$(3x+2)(x-1) = 2$$

$$3x^2 - 3x + 2x - 2 = 2$$

a b c

$$3x^2 - x - 4 = 0$$

factorise: $ac = -12 \rightarrow -4, 3$

$$3x^2 + 3x - 4x - 4 = 0$$

$$3x(x+1) - 4(x+1) = 0$$

$$(3x-4)(x+1) = 0$$

$3x - 4 = 0$ or $x + 1 = 0$

$3x = 4$ $\qquad$ $x = -1$

$x = \frac{4}{3}$

1 (a) Express $x^2 - 8x + 11$ in the form $(x+p)^2 + q$ where p and q are constants. [2]

$(x-4)^2 - 16 + 11$

$(x-4)^2 - 5$

ALT METHOD:

$(x+p)^2 + q = x^2 + 2px + p^2 + q$

compare coefficients

x^2: $1 = 1$

x: $2p = -8$

$p = -4$

constant: $p^2 + q = 11$

$(-4)^2 + q = 11$

$16 + q = 11$

$q = -5$

(b) Hence find the exact solutions of the equation $x^2 - 8x + 11 = 1$. [2]

$x^2 - 8x + 11 = 1$

from (a): $(x-4)^2 - 5 = 1$

$(x-4)^2 = 6$

$(x-4) = \pm\sqrt{6}$

$x = 4 \pm \sqrt{6}$

5 (a) Express $2x^2 - 8x + 14$ in the form $2[(x-a)^2 + b]$. [2]

$2[(x-a)^2 + b] = 2[x^2 - 2ax + a^2 + b]$

$= 2x^2 - 4ax + 2a^2 + 2b$

$2x^2 - 4ax + 2a^2 + 2b = 2x^2 - 8x + 14$

Compare coefficients: x^2: $2 = 2$

x: $-4a = -8$

$a = 2$

constant: $2a^2 + 2b = 14$

$2(2)^2 + 2b = 14$

$8 + 2b = 14$

$2b = 6$

$b = 3$

$\rightarrow 2[(x-2)^2 + 3]$

ALT METHOD: $2x^2 - 8x + 14$

$2[x^2 - 4x + 7]$

$2[(x-2)^2 - 4 + 7]$

$2[(x-2)^2 + 3]$

3 (a) Express $4x^2 - 24x + p$ in the form $a(x + b)^2 + c$, where a and b are integers and c is to be given in terms of the constant p. [2]

$4[x^2 - 6x] + p$

$4[(x-3)^2 - 9] + p$

$4(x-3)^2 - 36 + p$

(b) Hence or otherwise find the set of values of p for which the equation $4x^2 - 24x + p = 0$ has no real roots. [1]

$b^2 - 4ac < 0$:

$(-24)^2 - 4(4)(p) < 0$

$576 - 16p < 0$

$-16p < -576$

$16p > 576$

$p > 36$

1 (a) Express $16x^2 - 24x + 10$ in the form $(4x + a)^2 + b$. [2]

$(4x+a)^2 + b = 16x^2 + 8ax + a^2 + b$

$\rightarrow 16x^2 + 8ax + a^2 + b = 16x^2 - 24x + 10$

compare coefficients

$\cancel{x^2}$: $16 = 16$ $\quad$ $\cancel{x}$: $8a = -24$ $\quad$ constant: $a^2 + b = 10$

$a = -3$ $\quad$ $(-3)^2 + b = 10$

$9 + b = 10$

$b = 1$

$\rightarrow (4x - 3)^2 + 1$

(b) It is given that the equation $16x^2 - 24x + 10 = k$, where k is a constant, has exactly one root.

Find the value of this root. [2]

$\overset{a}{16}x^2 - \overset{b}{24}x + \overset{c}{10} - k = 0$

$b^2 - 4ac = 0$

$(-24)^2 - 4(16)(10-k) = 0$

$576 - 64(10-k) = 0$

$576 - 640 + 64k = 0$

$-64 + 64k = 0$

$64k = 64$

$k = 1$

$\rightarrow 16x^2 - 24x + 10 = 1$

from (a): $(4x - 3)^2 + 1 = 1$

$(4x-3)^2 = 0$

$4x - 3 = 0$

$4x = 3$

$x = \frac{3}{4}$

9 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 4x + 9,$$
$$g(x) = 2x^2 + 4x + 12.$$

(a) Express $f(x)$ in the form $(x-a)^2 + b$. [1]

$x^2 - 4x + 9 = (x-2)^2 - 4 + 9$

$= (x-2)^2 + 5$

(b) Express $g(x)$ in the form $2[(x+c)^2 + d]$. [2]

$2x^2 + 4x + 12 = 2[x^2 + 2x + 6]$

$= 2[(x+1)^2 - 1 + 6]$

$= 2[(x+1)^2 + 5]$

3 (a) Express $5y^2 - 30y + 50$ in the form $5(y + a)^2 + b$, where a and b are constants. [2]

$5[y^2 - 6y] + 50$

$5[(y-3)^2 - 9] + 50$

$5(y-3)^2 - 45 + 50$

$\underline{5(y-3)^2 + 5}$

ALT METHOD: $5(y+a)^2 + b = 5(y^2 + 2ay + a^2) + b$

$= \underline{5y^2 + 10ay + 5a^2 + b}$

$5y^2 + 10ay + 5a^2 + b = 5y^2 - 30y + 50$

compare coefficients

$\underline{y^2}$: $5 = 5$

$\underline{y}$: $10a = -30$

$\underline{a = -3}$

$\underline{\text{constant}}$:

$5a^2 + b = 50$

$5(-3)^2 + b = 50$

$45 + b = 50$

$\underline{b = 5}$

3 **(a)** Find the set of values of k for which the equation $\overset{a}{8}x^2 + \overset{b}{k}x + \overset{c}{2} = 0$ has no real roots. [2]

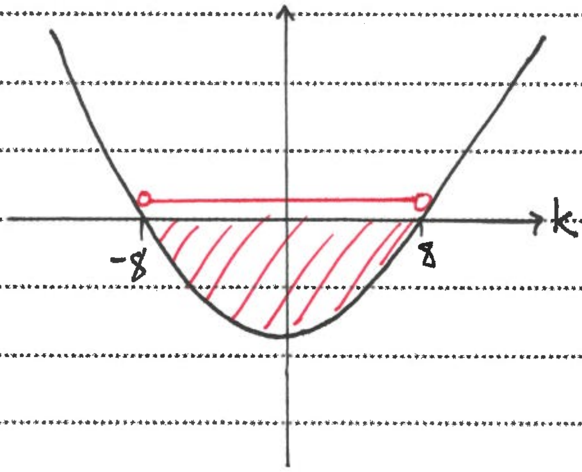
$b^2 - 4ac < 0$:

$k^2 - 4(8)(2) < 0$

$k^2 - 64 < 0$

$(k+8)(k-8) < 0$

Critical values : $k = -8$ and $k = 8$:



$\underline{-8 < k < 8}$

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6 The equation of a curve is $y = 4x^2 + 20x + 6$.

(a) Express the equation in the form $y = a(x+b)^2 + c$, where a, b and c are constants. [3]

$y = 4\left[x^2 + 5x\right] + 6$

$= 4\left[\left(x + \frac{5}{2}\right)^2 - \frac{25}{4}\right] + 6$

$= 4\left(x + \frac{5}{2}\right)^2 - 25 + 6$

$= 4\left(x + \frac{5}{2}\right)^2 - 19$

(b) Hence solve the equation $4x^2 + 20x + 6 = 45$. [3]

$4\left(x + \frac{5}{2}\right)^2 - 19 = 45$

$4\left(x + \frac{5}{2}\right)^2 = 64$

$\left(x + \frac{5}{2}\right)^2 = 16$

$x + \frac{5}{2} = \pm 4$

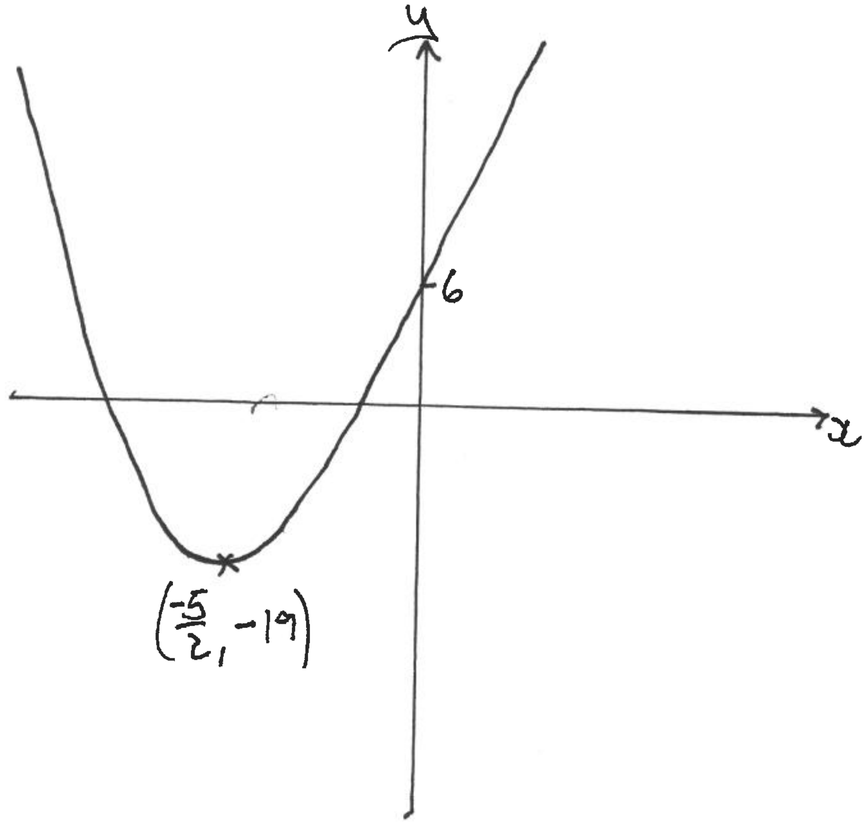
$x = -\frac{5}{2} + 4$ or $x = -\frac{5}{2} - 4$

$x = \frac{3}{2}$ $x = \frac{-13}{2}$

(c) Sketch the graph of $y = 4x^2 + 20x + 6$ showing the coordinates of the stationary point. You are not required to indicate where the curve crosses the x- and y-axes. [3]

$y = 4\left(x + \frac{5}{2}\right)^2 - 19$

← minimum point at $\left(\frac{-5}{2}, -19\right)$



5 **(a)** Solve the equation $6\sqrt{y} + \frac{2}{\sqrt{y}} - 7 = 0$. [4]

$$6\sqrt{y} + \frac{2}{\sqrt{y}} - 7 = 0$$

$$\times\sqrt{y} \quad \times\sqrt{y} \quad \times\sqrt{y} \quad \times\sqrt{y}$$

$$6y + 2 - 7\sqrt{y} = 0$$

$$6y - 7y^{\frac{1}{2}} + 2 = 0$$

$$a \quad b \quad c$$

Let $X = y^{\frac{1}{2}}$: $6X^2 - 7X + 2 = 0$

factorising:

$ac: 12 \rightarrow -3, -4$

$$6X^2 - 3X - 4X + 2 = 0$$

$$3X(2X - 1) - 2(2X - 1) = 0$$

$$(3X - 2)(2X - 1) = 0$$

$3X - 2 = 0$ or $2X - 1 = 0$

$X = \frac{2}{3}$ $\quad$ $X = \frac{1}{2}$

$y^{\frac{1}{2}} = \frac{2}{3}$ $\quad$ $y^{\frac{1}{2}} = \frac{1}{2}$

$\underline{y = \frac{4}{9}}$ $\quad$ $\underline{y = \frac{1}{4}}$

4 Solve the equation $8x^6 + 215x^3 - 27 = 0$. [3]

$$8x^6 + 215x^3 - 27 = 0$$

Let $Y = x^3$:

$$8Y^2 + 215Y - 27 = 0$$

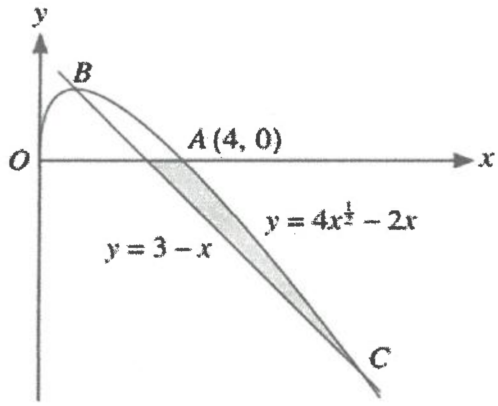
$$Y = \frac{-215 \pm \sqrt{(215)^2 - 4(8)(-27)}}{2(8)}$$

$Y = \frac{1}{8}$ or $Y = -27$

$x^3 = \frac{1}{8}$ $\quad$ $x^3 = -27$

$\sqrt[3]{\ }$ $\quad$ $\sqrt[3]{\ }$

$x = \frac{1}{2}$ $\quad$ $x = -3$



The diagram shows a curve with equation $y = 4x^{\frac{1}{2}} - 2x$ for $x \geqslant 0$, and a straight line with equation $y = 3 - x$. The curve crosses the x-axis at $A\,(4, 0)$ and crosses the straight line at B and C.

(a) Find, by calculation, the x-coordinates of B and C. **[4]**

$$4x^{\frac{1}{2}} - 2x = 3 - x$$
$$4x^{\frac{1}{2}} = 3 + x$$
$$0 = 3 + x - 4x^{\frac{1}{2}}$$
$$x - 4x^{\frac{1}{2}} + 3 = 0$$

Let $Y = x^{\frac{1}{2}}$:

$$Y^2 - 4Y + 3 = 0$$
$$(Y - 3)(Y - 1) = 0$$
$$Y = 3 \quad \text{or} \quad Y = 1$$
$$x^{\frac{1}{2}} = 3 \qquad x^{\frac{1}{2}} = 1$$
$$\underline{x = 9} \qquad \underline{x = 1}$$

2 By using a suitable substitution, solve the equation

$$(2x-3)^2 - \frac{4}{(2x-3)^2} - 3 = 0.$$ [4]

$(2x-3)^2 - \frac{4}{(2x-3)^2} - 3 = 0$

$\times(2x-3)^2 \quad \times(2x-3)^2 \quad \times(2x-3)^2 \quad \times(2x-3)^2$

$(2x-3)^4 - 4 - 3(2x-3)^2 = 0$

$(2x-3)^4 - 3(2x-3)^2 - 4 = 0$

Let $Y = (2x-3)^2$:

$Y^2 - 3Y - 4 = 0$

$(Y-4)(Y+1) = 0$

$Y - 4 = 0$ or $Y + 1 = 0$

$Y = 4$ or $Y = -1$

$(2x-3)^2 = 4$ or $(2x-3)^2 = -1$

no real sol^s

$2x - 3 = 2$ or $2x - 3 = -2$

$2x = 5$ $\quad 2x = 1$

$x = \frac{5}{2}$ $\quad x = \frac{1}{2}$

2 The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 - 6x + c$, where c is a constant. It is given that $f(x) > 2$ for all values of x.

Find the set of possible values of c. [4]

$$x^2 - 6x + c > 2$$

$$x^2 - 6x + c - 2 > 0$$

$$(x-3)^2 - 9 + c - 2 > 0$$

$$(x-3)^2 + c - 11 > 0$$

This is always ≥ 0, because we're squaring a real number. Its minimum value is 0, so let's use this:

$$0 + c - 11 > 0$$

$$c - 11 > 0$$

$$c > 11$$