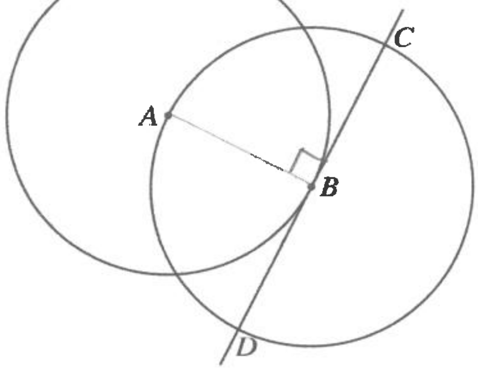
9



The diagram shows a circle with centre A passing through the point B. A second circle has centre B and passes through A. The tangent at B to the first circle intersects the second circle at C and D.

The coordinates of A are $(-1, 4)$ and the coordinates of B are $(3, 2)$.

(a) Find the equation of the tangent CBD. [2]

Gradient of $AB = \frac{2-4}{3--1}$

$= \frac{-2}{4} = \frac{-1}{2}$

Gradient of tangent $= 2$

$y - y_1 = m(x - x_1)$

$y - 2 = 2(x - 3)$

$y - 2 = 2x - 6$

$\underline{y = 2x - 4}$

(b) Find an equation of the circle with centre B. [3]

$(x-3)^2 + (y-2)^2 = r^2$

Sub. $(-1, 4)$:

$(-1-3)^2 + (4-2)^2 = r^2$

$(-4)^2 + 2^2 = r^2$

$16 + 4 = r^2$

$r^2 = 20$

$\rightarrow \underline{(x-3)^2 + (y-2)^2 = 20}$

(c) Find, by calculation, the x-coordinates of C and D. [3]

Sub. $y = 2x - 4$ into eq^n of circle:

$(x-3)^2 + (2x-4-2)^2 = 20$

$(x-3)^2 + (2x-6)^2 = 20$

$x^2 - 6x + 9 + 4x^2 - 24x + 36 = 20$

$5x^2 - 30x + 25 = 0$

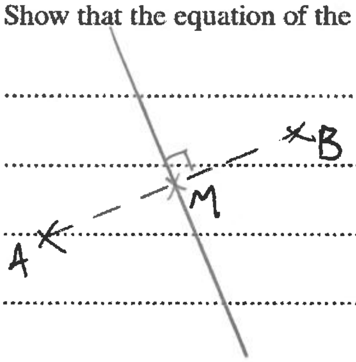
$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$\underline{x = 5}$ or $\underline{x = 1}$

10 **(a)** The coordinates of two points A and B are $(-7, 3)$ and $(5, 11)$ respectively.

Show that the equation of the perpendicular bisector of AB is $3x + 2y = 11$. [4]



M = mid-point of AB

$= \left(\frac{-7+5}{2}, \frac{3+11}{2}\right)$

$= (-1, 7)$

Gradient of $AB = \frac{11-3}{5--7}$

$= \frac{8}{12}$

$= \frac{2}{3}$

Gradient of perpendicular bisector $= \frac{-3}{2}$

$y - y_1 = m(x - x_1)$

$y - 7 = \frac{-3}{2}(x - -1)$

$y - 7 = \frac{-3}{2}(x + 1)$

×2 ×2

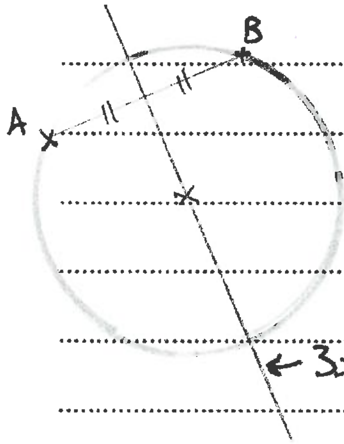
$2y - 14 = -3(x + 1)$

$2y - 14 = -3x - 3$

$3x + 2y = 11$ QED

(b) A circle passes through A and B and its centre lies on the line $12x - 5y = 70$.

Find an equation of the circle. [5]



Centre of circle lies on both $3x + 2y = 11$ and $12x - 5y = 70$

Solve simultaneously:

$3x + 2y = 11 \quad \times 5: \quad 15x + 10y = 55$ (1)

$12x - 5y = 70 \quad \times 2: \quad 24x - 10y = 140$ (2)

(1) + (2): $39x = 195$

$x = 5$

$\rightarrow 3x + 2y = 11$

$3(5) + 2y = 11$

$15 + 2y = 11$

$2y = -4$

$y = -2$

$\rightarrow$ Centre $= (5, -2)$

$(x-5)^2 + (y+2)^2 = r^2$

Sub. A or B: $(5, 11)$: $(5-5)^2 + (11+2)^2 = r^2$

$0^2 + 13^2 = r^2$

$r^2 = 169$

$\rightarrow (x-5)^2 + (y+2)^2 = 169$

9 The equation of a circle is $x^2 + y^2 + 6x - 2y - 26 = 0$.

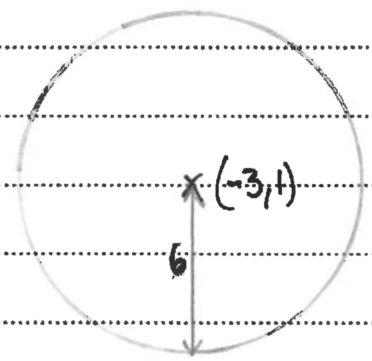
(a) Find the coordinates of the centre of the circle and the radius. Hence find the coordinates of the lowest point on the circle. [4]

$x^2 + 6x + y^2 - 2y - 26 = 0$

$(x+3)^2 - 9 + (y-1)^2 - 1 - 26 = 0$

$(x+3)^2 + (y-1)^2 = 36$

Centre = $(-3, 1)$ radius = 6



Lowest point = $(-3, -5)$

9 A circle has centre at the point $B(5, 1)$. The point $A(-1, -2)$ lies on the circle.

(a) Find the equation of the circle. [3]

$(x-5)^2 + (y-1)^2 = r^2$

sub. $(-1,-2)$: $(-1-5)^2 + (-2-1)^2 = r^2$

$(-6)^2 + (-3)^2 = r^2$

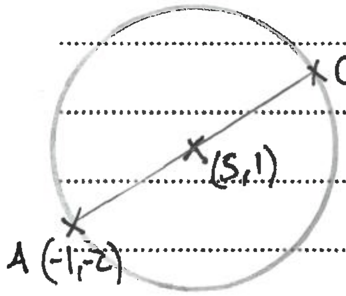
$36 + 9 = r^2$

$r^2 = 45$

$\rightarrow (x-5)^2 + (y-1)^2 = 45$

Point C is such that AC is a diameter of the circle. Point D has coordinates $(5, 16)$.

(b) Show that DC is a tangent to the circle. [4]



$(5,1)$ is the mid-point of AC:

$(5,1) = \left(\frac{x+-1}{2}, \frac{y+-2}{2}\right)$

$\frac{x-1}{2} = 5 \qquad \frac{y-2}{2} = 1$

$x - 1 = 10 \qquad y - 2 = 2$

$x = 11 \qquad y = 4 \quad \rightarrow C = (11,4)$

gradient of $DC = \frac{16-4}{5-11}$

$= \frac{12}{-6} = -2$

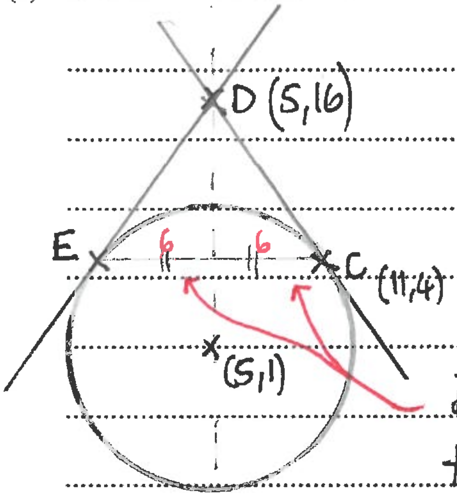
gradient of $AC = \frac{4--2}{11--1}$

$= \frac{6}{12} = \frac{1}{2}$

$-2 \times \frac{1}{2} = -1$, therefore DC is perpendicular to AC, So DC is a tangent to the circle.

The other tangent from D to the circle touches the circle at E.

(c) Find the coordinates of E. [2]

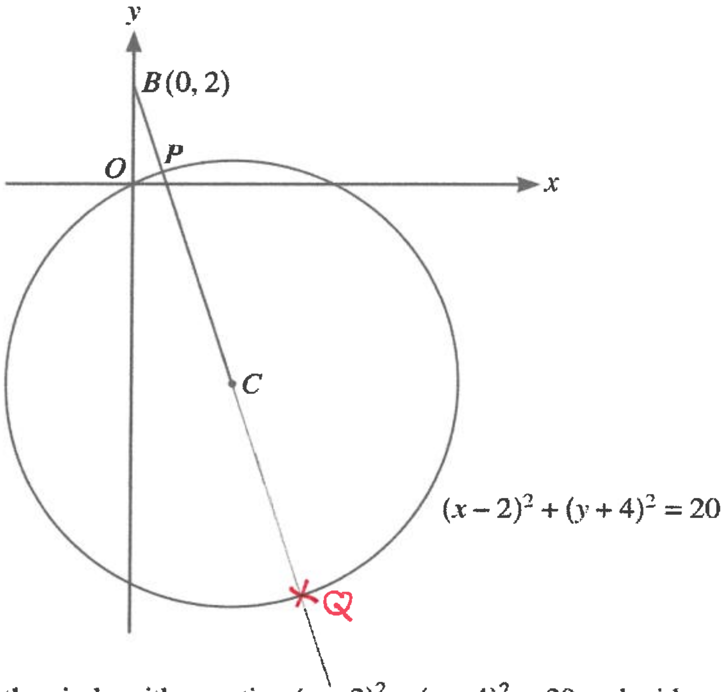


By symmetry, y-coordinate of E is the same as y-coordinate of C: $y=4$

By symmetry and using the fact that the radius bisects chord EC, x-coordinate $= 5-6$
$= -1$

$E = \underline{(-1, 4)}$

7



The diagram shows the circle with equation $(x-2)^2+(y+4)^2=20$ and with centre C. The point B has coordinates $(0, 2)$ and the line segment BC intersects the circle at P.

(a) Find the equation of BC. [2]

Centre $C = (2, -4)$

Gradient of $BC = \frac{-4-2}{2-0}$

$= \frac{-6}{2}$

$= -3$

$y - y_1 = m(x - x_1)$

$y - 2 = -3(x - 0)$

$y - 2 = -3x$

$y = -3x + 2$

(b) Hence find the coordinates of P, giving your answer in exact form. [5]

Sub. $y = -3x + 2$ into eq^n of circle:

$$(x-2)^2 + (-3x+2+4)^2 = 20$$
$$(x-2)^2 + (-3x+6)^2 = 20$$
$$x^2 - 4x + 4 + 9x^2 - 36x + 36 = 20$$
$$10x^2 - 40x + 40 = 20$$
$$10x^2 - 40x + 20 = 0$$
$$x^2 - 4x + 2 = 0$$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$
$$x = 2 + \sqrt{2} \quad \text{or} \quad \underline{x = 2 - \sqrt{2}}$$

further to the right, so this would be point Q on the diagram.

$$y = -3x + 2$$
$$= -3(2 - \sqrt{2}) + 2$$
$$= -6 + 3\sqrt{2} + 2$$
$$= \underline{-4 + 3\sqrt{2}}$$

$$\rightarrow \underline{(2 - \sqrt{2},\ 3\sqrt{2} - 4)}$$

7 The point A has coordinates $(1, 5)$ and the line l has gradient $-\frac{2}{3}$ and passes through A. A circle has centre $(5, 11)$ and radius $\sqrt{52}$.

(a) Show that l is the tangent to the circle at A. [2]

eq^n of circle: $(x-5)^2 + (y-11)^2 = 52$ ①

eq^n of line: $y - y_1 = m(x - x_1)$

$y - 5 = -\frac{2}{3}(x-1)$

$y - 5 = -\frac{2}{3}x + \frac{2}{3}$

$y = -\frac{2}{3}x + \frac{17}{3}$ ②

Sub. ② into ①: $(x-5)^2 + \left(-\frac{2}{3}x + \frac{17}{3} - 11\right)^2 = 52$

$(x-5)^2 + \left(-\frac{2}{3}x - \frac{16}{3}\right)^2 = 52$

$x^2 - 10x + 25 + \frac{4}{9}x^2 + \frac{64}{9}x + \frac{256}{9} = 52$

$9x^2 - 90x + 225 + 4x^2 + 64x + 256 = 468$

$13x^2 - 26x + 13 = 0$

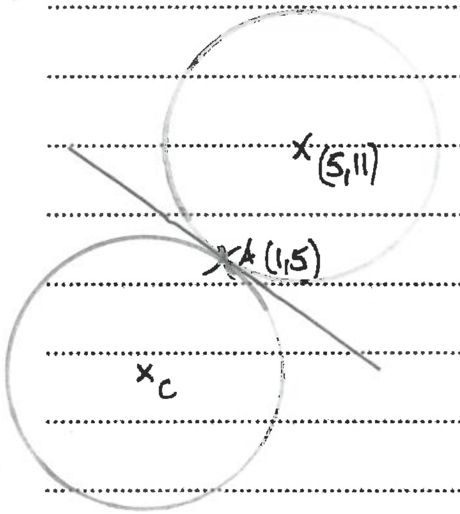
$x^2 - 2x + 1 = 0$

$b^2 - 4ac$: $(-2)^2 - 4(1)(1)$

$= 4 - 4 = 0$

discriminant $= 0$, so there is only one intersection, so l is a tangent.

(b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also the tangent at A. [3]



A is the mid-point of $(5,11)$ and C:

$(1,5) = \left(\frac{5+x}{2}, \frac{11+y}{2}\right)$

$\frac{5+x}{2} = 1$ $\quad$ $\frac{11+y}{2} = 5$

$5 + x = 2$ $\quad$ $11 + y = 10$

$x = -3$ $\quad$ $y = -1$

$\rightarrow$ centre: $(-3, -1)$

$\rightarrow$ $(x+3)^2 + (y+1)^2 = 52$

12 A diameter of a circle C_1 has end-points at $(-3, -5)$ and $(7, 3)$.

(a) Find an equation of the circle C_1. [3]

mid-point: $\left(\frac{-3+7}{2}, \frac{-5+3}{2}\right) : (2, -1)$

$\rightarrow (x-2)^2 + (y+1)^2 = r^2$

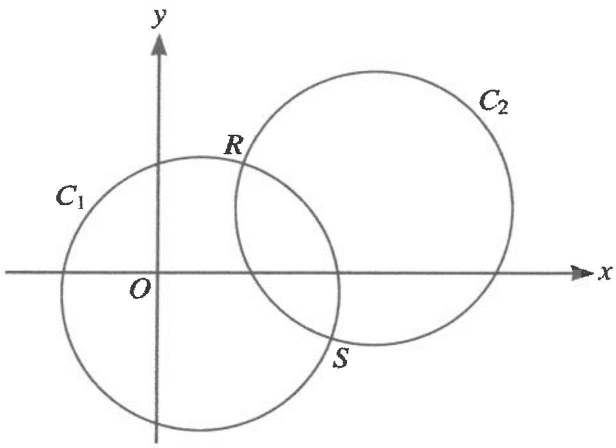
sub. $(7,3)$: $(7-2)^2 + (3+1)^2 = r^2$

$5^2 + 4^2 = r^2$

$25 + 16 = r^2$

$\rightarrow r^2 = 41$

$\underline{(x-2)^2 + (y+1)^2 = 41}$



The circle C_1 is translated by $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ to give circle C_2, as shown in the diagram.

(b) Find an equation of the circle C_2. [2]

Translate centre by $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$: Centre of C_2: $(10, 3)$

Radius is unchanged:

$\underline{(x-10)^2 + (y-3)^2 = 41}$

The two circles intersect at points R and S.

(c) Show that the equation of the line RS is $y = -2x + 13$. [4]

Solve C_1 and C_2 simultaneously:

$$(x-2)^2 + (y+1)^2 = (x-10)^2 + (y-3)^2$$

(we can do this because they are both $= 41$)

$$\cancel{x^2} - 4x + 4 + \cancel{y^2} + 2y + 1 = \cancel{x^2} - 20x + 100 + \cancel{y^2} - 6y + 9$$

$$-4x + 2y + 5 = -20x - 6y + 109$$

$$8y = -16x + 104$$

$$y = -2x + 13 \quad \text{QED}$$

(d) Hence show that the x-coordinates of R and S satisfy the equation $5x^2 - 60x + 159 = 0$. [2]

Sub. $y = -2x + 13$ into C_1 or C_2:

$$(x-2)^2 + (-2x + 13 + 1)^2 = 41$$

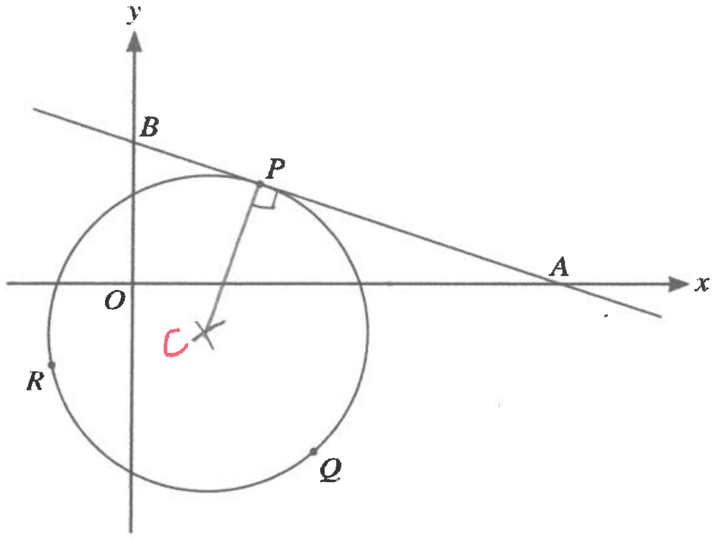
$$(x-2)^2 + (-2x + 14)^2 = 41$$

$$x^2 - 4x + 4 + 4x^2 - 56x + 196 = 41$$

$$5x^2 - 60x + 200 = 41$$

$$5x^2 - 60x + 159 = 0 \quad \text{QED}$$

12



The diagram shows the circle with equation $x^2 + y^2 - 6x + 4y - 27 = 0$ and the tangent to the circle at the point $P(5, 4)$.

(a) The tangent to the circle at P meets the x-axis at A and the y-axis at B.

Find the area of triangle OAB, where O is the origin. [5]

find C: $x^2 - 6x + y^2 + 4y - 27 = 0$

$(x-3)^2 - 9 + (y+2)^2 - 4 - 27 = 0$

$(x-3)^2 + (y+2)^2 = 40$

Centre $= (3, -2)$

Gradient of CP $= \dfrac{4 - -2}{5-3} = \dfrac{6}{2} = 3$

Gradient of tangent $= \dfrac{-1}{3}$

$y - y_1 = m(x - x_1)$

$y - 4 = \dfrac{-1}{3}(x - 5)$

$y - 4 = \dfrac{-1}{3}x + \dfrac{5}{3}$

$y = \dfrac{-1}{3}x + \dfrac{17}{3}$

$\rightarrow B = \left(0, \dfrac{17}{3}\right)$

A: $0 = \dfrac{-1}{3}x + \dfrac{17}{3}$

$\dfrac{1}{3}x = \dfrac{17}{3}$

$x = 17$

$\rightarrow A = (17, 0)$

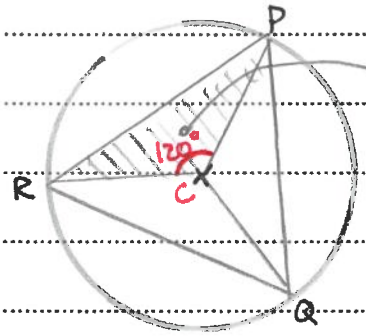
Area $= \dfrac{1}{2} \times b \times h$

$= \dfrac{1}{2} \times 17 \times \dfrac{17}{3}$

$= \dfrac{289}{6}$

(b) Points Q and R also lie on the circle, such that PQR is an equilateral triangle.

Find the exact area of triangle PQR. [3]



Area CPR $= \frac{1}{2} ab\sin C$

radius

$= \frac{1}{2} \times \sqrt{40} \times \sqrt{40} \sin(120)$

$= 10\sqrt{3}$

$3 \times 10\sqrt{3} = \underline{30\sqrt{3}}$

11 The equation of a circle with centre C is $x^2 + y^2 - 8x + 4y - 5 = 0$.

(a) Find the radius of the circle and the coordinates of C. [3]

$x^2 - 8x + y^2 + 4y - 5 = 0$

$(x-4)^2 - 16 + (y+2)^2 - 4 - 5 = 0$

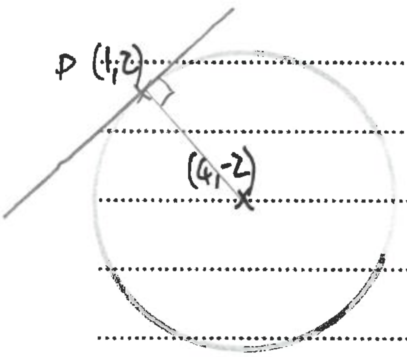
$(x-4)^2 + (y+2)^2 - 25 = 0$

$(x-4)^2 + (y+2)^2 = 25$

Centre $= (4, -2)$ Radius $= 5$

The point $P(1, 2)$ lies on the circle.

(b) Show that the equation of the tangent to the circle at P is $4y = 3x + 5$. [3]



Gradient of radius $= \frac{-2-2}{4-1}$

$= \frac{-4}{3}$

Gradient of tangent $= \frac{3}{4}$

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{3}{4}(x-1)$

$\times 4$ $\times 4$

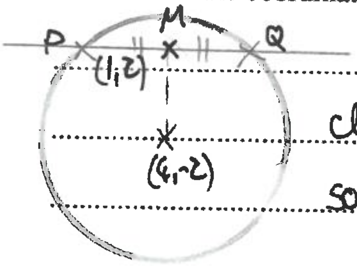
$4y - 8 = 3(x-1)$

$4y - 8 = 3x - 3$

$4y = 3x + 5$ 1

The point Q also lies on the circle and PQ is parallel to the x-axis.

(c) Write down the coordinates of Q. [2]



y-coord of Q = y-coord of P = 2

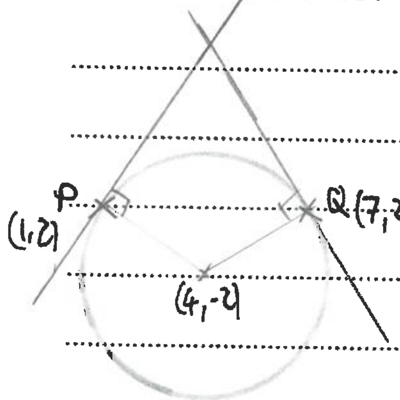
chord PQ bisects radius so PM = MQ = 3

so x-coord of Q = 7

$\rightarrow Q: (7, 2)$

The tangents to the circle at P and Q meet at T.

(d) Find the coordinates of T. [3]



Eqn of tangent at Q:

gradient of radius $= \dfrac{2 - -2}{7 - 4}$

$= \dfrac{4}{3}$

gradient of tangent $= \dfrac{-3}{4}$

$y - y_1 = m(x - x_1)$

$y - 2 = \dfrac{-3}{4}(x - 7)$

$\times 4 \qquad \times 4$

$4y - 8 = -3(x - 7)$

$4y - 8 = -3x + 21$

$4y = -3x + 29$ ②

Solve ① + ② simultaneously:

① = ②: $3x + 5 = -3x + 29$

$6x = 24$

$x = 4$

Sub into ①: $4y = 3(4) + 5$

$4y = 12 + 5 \rightarrow 4y = 17$

$y = \dfrac{17}{4}$

$\left(4, \dfrac{17}{4}\right)$

11 The coordinates of points A, B and C are $A(5, -2)$, $B(10, 3)$ and $C(2p, p)$, where p is a constant.

(a) Given that AC and BC are equal in length, find the value of the fraction p. [3]

$|AC| = \sqrt{(2p-5)^2 + (p - -2)^2}$

$= \sqrt{4p^2 - 20p + 25 + p^2 + 4p + 4}$

$= \sqrt{5p^2 - 16p + 29}$

$|BC| = \sqrt{(2p-10)^2 + (p-3)^2}$

$= \sqrt{4p^2 - 40p + 100 + p^2 - 6p + 9}$

$= \sqrt{5p^2 - 46p + 109}$

$|AC| = |BC|$: $\sqrt{5p^2 - 16p + 29} = \sqrt{5p^2 - 46p + 109}$

$\cancel{5p^2} - 16p + 29 = \cancel{5p^2} - 46p + 109$

$30p = 80$

$p = \frac{8}{3}$

(b) It is now given instead that AC is perpendicular to BC and that p is an integer.

(i) Find the value of p. [4]

gradient of $AC = \frac{p - -2}{2p-5} = \frac{p+2}{2p-5}$

gradient of $BC = \frac{p-3}{2p-10}$

AC and BC are perpendicular, so $m_{AC} \times m_{BC} = -1$:

$\frac{p+2}{2p-5} \times \frac{p-3}{2p-10} = -1$

$\frac{p^2 - p - 6}{4p^2 - 30p + 50} = -1$

$p^2 - p - 6 = -4p^2 + 30p - 50$

$5p^2 - 31p + 44 = 0$

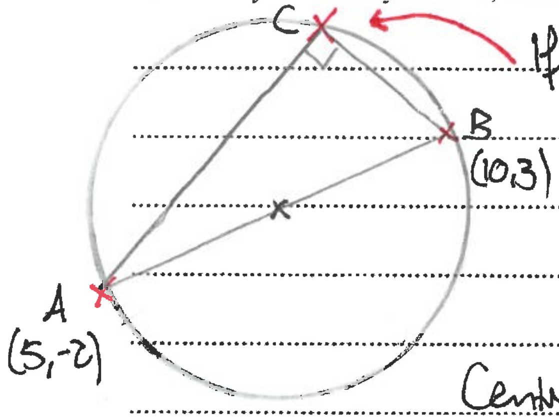
Cont...

$$p = \frac{-(-31) \pm \sqrt{(-31)^2 - 4(5)(44)}}{2(5)}$$

$p = 4$ or $p = \frac{11}{5}$

p is an integer, so $p = 4$

(ii) Find the equation of the circle which passes through A, B and C, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants. [4]



If $p = 4$, coordinates of $C = (8, 4)$

Given ACB is a right-angle, AB is a diameter.

Centre = mid-point of $AB = \left(\frac{5+10}{2}, \frac{-2+3}{2}\right)$
$= \left(\frac{15}{2}, \frac{1}{2}\right)$

$$\left(x - \frac{15}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = r^2$$

Sub. A, B or C: (8,4):

$$\left(8 - \frac{15}{2}\right)^2 + \left(4 - \frac{1}{2}\right)^2 = r^2$$

$$\left(\frac{1}{2}\right)^2 + \left(\frac{7}{2}\right)^2 = r^2$$

$$\frac{1}{4} + \frac{49}{4} = r^2$$

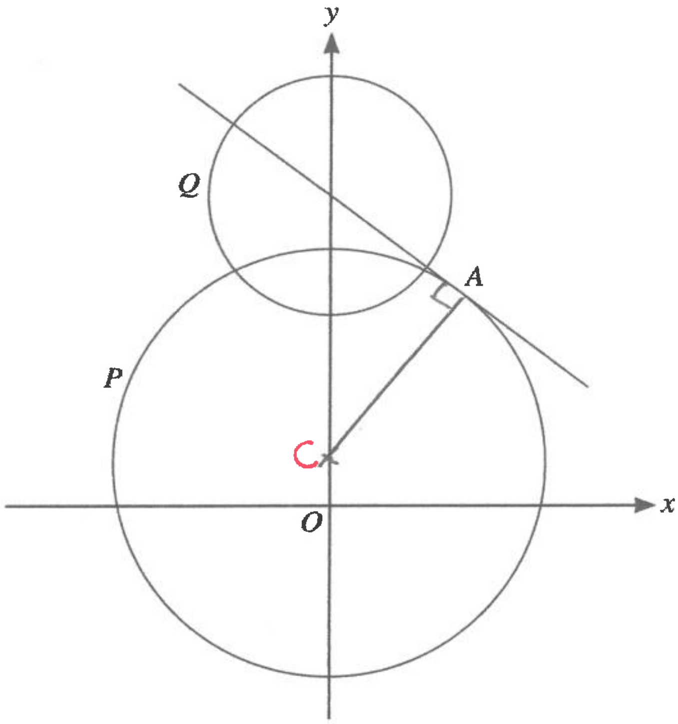
$$r^2 = \frac{50}{4}$$

$$\left(x - \frac{15}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{50}{4}$$

$$x^2 - 15x + \frac{225}{4} + y^2 - y + \frac{1}{4} = \frac{50}{4}$$

$$x^2 + y^2 - 15x - y + 44 = 0$$

12



The diagram shows a circle P with centre $(0, 2)$ and radius 10 and the tangent to the circle at the point A with coordinates $(6, 10)$. It also shows a second circle Q with centre at the point where this tangent meets the y-axis and with radius $\frac{5}{2}\sqrt{5}$.

(a) Write down the equation of circle P. [1]

$(x-0)^2 + (y-2)^2 = 10^2$

$x^2 + (y-2)^2 = 100$ 1

(b) Find the equation of the tangent to the circle P at A. [2]

Gradient of radius AC $= \frac{10-2}{6-0}$

$= \frac{8}{6} = \frac{4}{3}$

Gradient of tangent $= \frac{-3}{4}$

$y - y_1 = m(x - x_1)$

$y - 10 = \frac{-3}{4}(x-6)$

$y - 10 = \frac{-3}{4}x + \frac{9}{2}$

$y = \frac{-3}{4}x + \frac{29}{2}$

(c) Find the equation of circle Q and hence verify that the y-coordinates of both of the points of intersection of the two circles are 11. [3]

y-intercept of line in part (a)

Centre of Q: $\left(0, \frac{29}{2}\right)$ radius $= \frac{5}{2}\sqrt{5}$ (given)

$$(x-0)^2 + \left(y - \frac{29}{2}\right)^2 = \left(\frac{5}{2}\sqrt{5}\right)^2$$

$$x^2 + \left(y - \frac{29}{2}\right)^2 = \frac{125}{4} \quad ②$$

from eqⁿ ①: $x^2 = 100 - (y-2)^2$

sub into ②: $100 - (y-2)^2 + \left(y - \frac{29}{2}\right)^2 = \frac{125}{4}$

$$100 - (y^2 - 4y + 4) + y^2 - 29y + \frac{841}{4} = \frac{125}{4}$$

$$100 - \cancel{y^2} + 4y - 4 + \cancel{y^2} - 29y + \frac{841}{4} = \frac{125}{4}$$

$$-25y + \frac{1225}{4} = \frac{125}{4}$$

$$-25y = -275$$

$$y = 11 \quad \text{QED}$$

(d) Find the coordinates of the points of intersection of the tangent and circle Q, giving the answers in surd form. [3]

Sub. $y = \frac{-3}{4}x + \frac{29}{2}$ into: $x^2 + \left(y - \frac{29}{2}\right)^2 = \frac{125}{4}$

$$x^2 + \left(\frac{-3}{4}x + \frac{29}{2} - \frac{29}{2}\right)^2 = \frac{125}{4}$$

$$x^2 + \left(\frac{-3}{4}x\right)^2 = \frac{125}{4}$$

$$x^2 + \frac{9}{16}x^2 = \frac{125}{4}$$

$$\frac{25}{16}x^2 = \frac{125}{4}$$

$$x^2 = 20$$

$$x = \pm\sqrt{20}$$

$x = \sqrt{20}$: $y = \frac{-3}{4}(\sqrt{20}) + \frac{29}{2} = \frac{29}{2} - \frac{3\sqrt{20}}{4}$

$\left(\sqrt{20}, \frac{29}{2} - \frac{3\sqrt{20}}{4}\right)$

$x = -\sqrt{20}$: $y = \frac{-3}{4}(-\sqrt{20}) + \frac{29}{2} = \frac{29}{2} + \frac{3\sqrt{20}}{4}$

$\left(-\sqrt{20}, \frac{29}{2} + \frac{3\sqrt{20}}{4}\right)$

11 A circle with centre C has equation $(x-8)^2 + (y-4)^2 = 100$.

(a) Show that the point $T(-6, 6)$ is outside the circle. [3]

Sub. $(-6,6)$: $(-6-8)^2 + (6-4)^2$

$= (-14)^2 + 2^2$

$= 196 + 4$

$= 200$

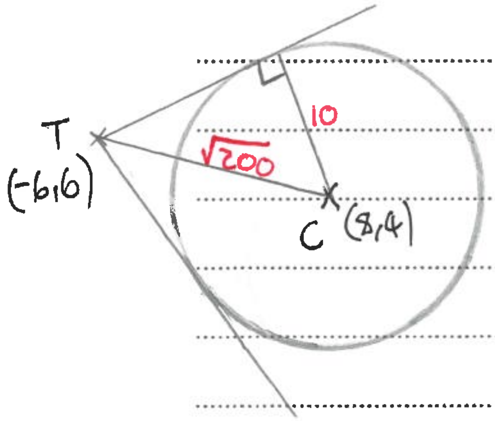
distance from centre to $(-6,6) = \sqrt{200}$

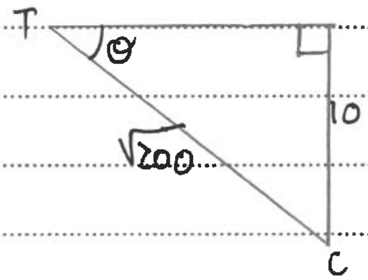
$\sqrt{200} > 10$ ← radius of circle

So T is outside circle.

Two tangents from T to the circle are drawn.

(b) Show that the angle between one of the tangents and CT is exactly 45°. [2]





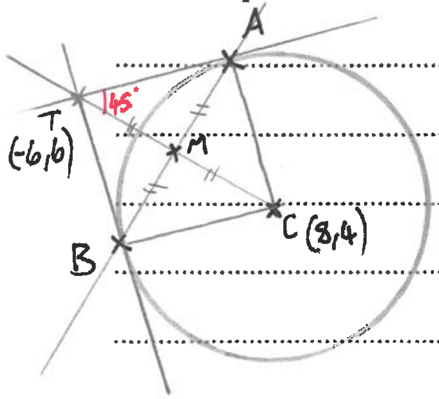
$\sin\theta = \dfrac{10}{\sqrt{200}}$

$\theta = \sin^{-1}\left(\dfrac{10}{\sqrt{200}}\right)$

$= 45°$ QED

The two tangents touch the circle at A and B.

(c) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [4]



from (b) ATB is a right-angle, so ACBT is a square.

Find point M: $\left(\frac{8+-6}{2}, \frac{4+6}{2}\right) = (1,5)$

Gradient of CT $= \frac{4-6}{8--6}$

$= \frac{-2}{14} = \frac{-1}{7}$

Gradient of AB $= 7$

$y - y_1 = m(x - x_1)$

$y - 5 = 7(x-1)$

$y - 5 = 7x - 7$

$y = 7x - 2$

(d) Find the x-coordinates of A and B. [3]

Sub. $y = 7x-2$ into eqⁿ of circle:

$(x-8)^2 + (7x-2-4)^2 = 100$

$(x-8)^2 + (7x-6)^2 = 100$

$x^2 - 16x + 64 + 49x^2 - 84x + 36 = 100$

$50x^2 - 100x + 100 = 100$

$50x^2 - 100x = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0$ or $x = 2$

10 The equation of a circle is $(x-a)^2 + (y-3)^2 = 20$. The line $y = \frac{1}{2}x + 6$ is a tangent to the circle at the point P.

(a) Show that one possible value of a is 4 and find the other possible value. [5]

Sub. $y = \frac{1}{2}x + 6$ into eqⁿ of circle:

$$(x-a)^2 + \left(\tfrac{1}{2}x + 6 - 3\right)^2 = 20$$

$$(x-a)^2 + \left(\tfrac{1}{2}x + 3\right)^2 = 20$$

$$x^2 - 2ax + a^2 + \tfrac{1}{4}x^2 + 3x + 9 = 20$$

$$\tfrac{5}{4}x^2 + 3x - 2ax + a^2 + 9 - 20 = 0$$

$$\underset{a}{\tfrac{5}{4}}x^2 + \underset{b}{(3-2a)}x + \underset{c}{a^2 - 11} = 0$$

If the line is a tangent, $b^2 - 4ac = 0$:

$$(3-2a)^2 - 4\left(\tfrac{5}{4}\right)(a^2 - 11) = 0$$

$$9 - 12a + 4a^2 - 5(a^2 - 11) = 0$$

$$9 - 12a + 4a^2 - 5a^2 + 55 = 0$$

$$-a^2 - 12a + 64 = 0$$

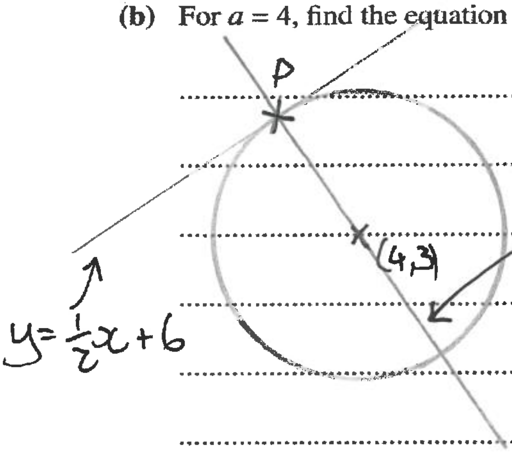
$\times -1$ $\times -1$ $\times -1$ $\times -1$

$$a^2 + 12a - 64 = 0$$

$$(a + 16)(a - 4) = 0$$

$$a = -16 \qquad a = 4$$

(b) For $a = 4$, find the equation of the normal to the circle at P. [4]



Centre of circle: $(4,3)$

Gradient of tangent $= \frac{1}{2}$

Gradient of normal $= -2$

$m = -2$, point on line $= (4,3)$

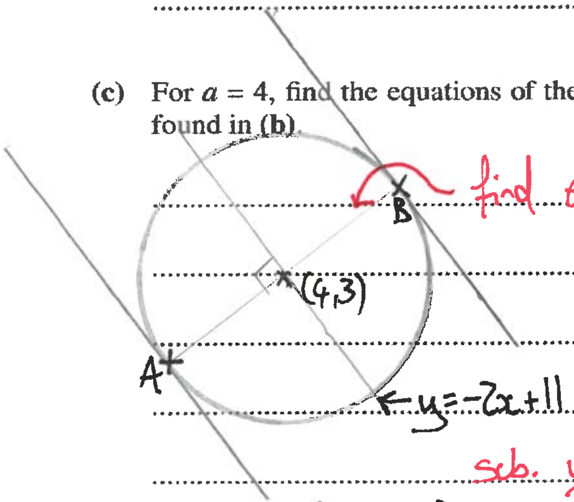
$y - y_1 = m(x - x_1)$

$y - 3 = -2(x - 4)$

$y - 3 = -2x + 8$

$y = -2x + 11$

(c) For $a = 4$, find the equations of the two tangents to the circle which are parallel to the normal found in **(b)**. [4]



find eqn of this line: gradient $= \frac{1}{2}$

$\rightarrow y - 3 = \frac{1}{2}(x - 4)$

$y - 3 = \frac{1}{2}x - 2$

$y = \frac{1}{2}x + 1$

sub. $y = \frac{1}{2}x + 1$ into eqn of circle to find A & B:

$(x-4)^2 + (\frac{1}{2}x + 1 - 3)^2 = 20$

$(x-4)^2 + (\frac{1}{2}x - 2)^2 = 20$

$x^2 - 8x + 16 + \frac{1}{4}x^2 - 2x + 4 = 20$

$\frac{5}{4}x^2 - 10x + 20 = 20$

$\frac{5}{4}x^2 - 10x = 0$

$5x^2 - 40x = 0$

$5x(x - 8) = 0$

$x = 0$ or $x = 8$

$x = 0$: $y = \frac{1}{2}(0) + 1$

$y = 1 \rightarrow (0,1)$ ①

$x = 8$: $y = \frac{1}{2}(8) + 1$

$y = 5$ $(8,5)$ ②

①: $y - 1 = -2(x - 0)$

$y - 1 = -2x$

$y = -2x + 1$

②: $y - 5 = -2(x - 8)$

$y - 5 = -2x + 16$

$y = -2x + 21$