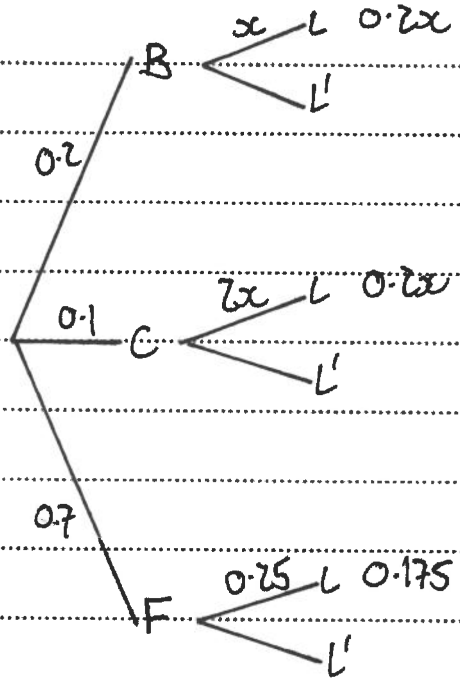
1 On any day, Kino travels to school by bus, by car or on foot with probabilities 0.2, 0.1 and 0.7 respectively. The probability that he is late when he travels by bus is x. The probability that he is late when he travels by car is $2x$ and the probability that he is late when he travels on foot is 0.25.

The probability that, on a randomly chosen day, Kino is late is 0.235.

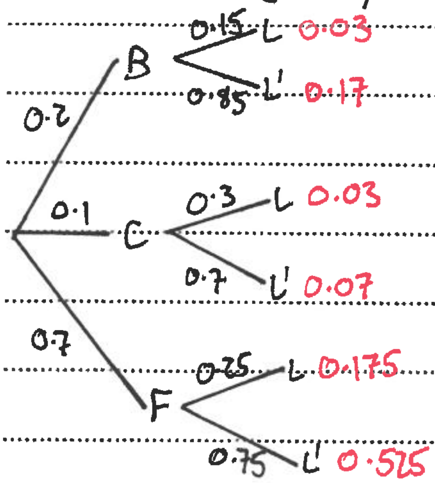
(a) Find the value of x. [3]



$$0.2x + 0.2x + 0.175 = 0.235$$
$$0.4x + 0.175 = 0.235$$
$$0.4x = 0.06$$
$$x = 0.15$$

(b) Find the probability that, on a randomly chosen day, Kino travels to school by car given that he is not late. [2]

$$P(C \cap L') = P(C|L') \times P(L')$$
$$P(C|L') = \frac{P(C \cap L')}{P(L')}$$



$$P(C|L') = \frac{0.07}{0.17 + 0.07 + 0.525}$$

or $1 - 0.235$

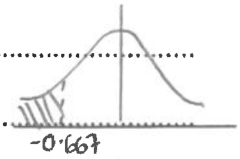
$$= \frac{0.07}{0.765}$$
$$= 0.0915$$

2 The lengths of the rods produced by a company are normally distributed with mean 55.6 mm and standard deviation 1.2 mm.

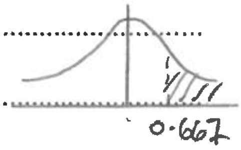
(a) In a random sample of 400 of these rods, how many would you expect to have length less than 54.8 mm? [4]

$P(L < 54.8) = P\left(Z < \frac{54.8 - 55.6}{1.2}\right)$

$= P(Z < -0.667)$



$= 1 - \Phi(0.667)$



$= 1 - 0.7477$

$= 0.2523$

Expected number $= 0.2523 \times 400$

$= 100.92$

$= \underline{101}$

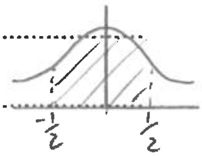
(b) Find the probability that a randomly chosen rod produced by this company has a length that is within half a standard deviation of the mean. [3]

$55.6 + 0.5(1.2) = 56.2$

$55.6 - 0.5(1.2) = 55$

$P(55 < L < 56.2) = P\left(\frac{55 - 55.6}{1.2} < Z < \frac{56.2 - 55.6}{1.2}\right)$

can skip straight to here $\rightarrow$ $= P\left(-\frac{1}{2} < Z < \frac{1}{2}\right)$



$= P(Z < \tfrac{1}{2}) - P(Z < -\tfrac{1}{2})$

$= \Phi(\tfrac{1}{2}) - (1 - \Phi(\tfrac{1}{2}))$

$= 0.6915 - (1 - 0.6915)$

$= \underline{0.383}$

3 Three fair 6-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time repeatedly. The score on each throw is the sum of the numbers on the uppermost faces.

(a) Find the probability that a score of 17 or more is first obtained on the 6th throw. [3]

A	B	C
6	6	6
6	6	5
6	5	6
5	6	6

$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{216}$ $\quad 4 \times \frac{1}{216} = \frac{1}{54}$

$X \sim \text{Geo}\left(\frac{1}{54}\right)$

$P(X=6) = \left(\frac{1}{54}\right) \times \left(\frac{53}{54}\right)^5$

$= 0.0169$

(b) Find the probability that a score of 17 or more is obtained in fewer than 8 throws. [2]

$P(X<8) = P(X \leq 7)$

$= 1 - q^7$

probability of 7 failures

$= 1 - \left(\frac{53}{54}\right)^7$

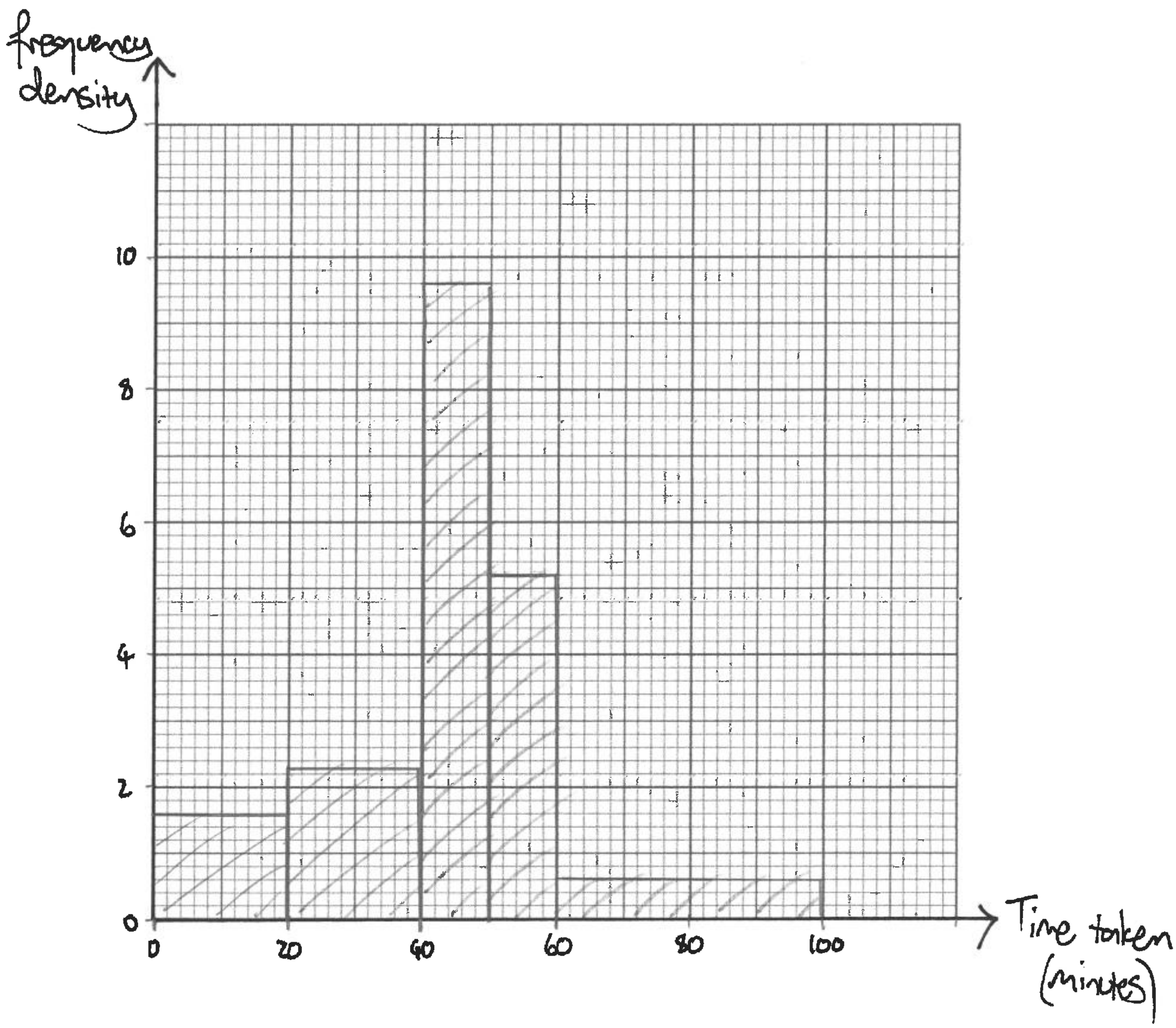
$= 0.123$

4 The times taken, in minutes, to complete a word processing task by 250 employees at a particular company are summarised in the table.

Time taken (t minutes)	$0 \leqslant t < 20$	$20 \leqslant t < 40$	$40 \leqslant t < 50$	$50 \leqslant t < 60$	$60 \leqslant t < 100$
Frequency	32	46	96	52	24

(a) Draw a histogram to represent this information. [4]

Class width	20	20	10	10	40
f.d.	1.6	2.3	9.6	5.2	0.6



From the data, the estimate of the mean time taken by these 250 employees is 43.2 minutes.

(b) Calculate an estimate for the standard deviation of these times. [3]

Mid-points: 10, 30, 45, 55, 80

$$\text{Var}: \frac{10^2 \times 32 + 30^2 \times 46 + 45^2 \times 96 + 55^2 \times 52 + 80^2 \times 24}{250} - 43.2^2$$

$= 333.36$

$\sigma = \sqrt{333.36}$

$= 18.3$

5 Eric has three coins. One of the coins is fair. The other two coins are each biased so that the probability of obtaining a head on any throw is $\frac{1}{4}$, independently of all other throws. Eric throws all three coins at the same time.

Events A and B are defined as follows.

A: all three coins show the same result

B: at least one of the biased coins shows a head

(a) Show that $\mathrm{P}(B) = \frac{7}{16}$. [2]

F	B_1	B_2	
H	H	H	$\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{32}$
H	H	T	$\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{32}$
H	T	H	$\frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} = \frac{3}{32}$
T	H	H	$\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{32}$
T	T	H	$\frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} = \frac{3}{32}$
T	H	T	$\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{32}$

$+ \;= \frac{14}{32}$

$= \frac{7}{16}$ QED

(b) Find $\mathrm{P}(A \mid B)$. [2]

$P(A \cap B) = P(A|B) \times P(B)$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$P(A \cap B) = HHH$

$= \frac{1}{32}$

$P(B) = \frac{7}{16}$ (part (a))

$P(A|B) = \dfrac{\frac{1}{32}}{\frac{7}{16}}$

$= \dfrac{1}{14}$

The random variable X is the number of heads obtained when Eric throws the three coins.

(c) Draw up the probability distribution table for X. [3]

F B$_1$ B$_2$

3H: H H H $\frac{1}{32}$

2H: H H T $\frac{3}{32}$

H T H $\frac{3}{32}$

T H H $\frac{1}{32}$

1H: T T H $\frac{3}{32}$

T H T $\frac{3}{32}$

H T T $\frac{1}{2} \times \frac{3}{4} \times \frac{3}{4} = \frac{9}{32}$

0H: T T T $\frac{1}{2} \times \frac{3}{4} \times \frac{3}{4} = \frac{9}{32}$

x	0	1	2	3
$P(X=x)$	$\frac{9}{32}$	$\frac{15}{32}$	$\frac{7}{32}$	$\frac{1}{32}$

6 At a company's call centre, 90% of callers are connected immediately to a representative.

A random sample of 12 callers is chosen.

(a) Find the probability that fewer than 10 of these callers are connected immediately. [3]

$C \sim B(12, 0.9)$

$P(C < 10) = 1 - (P(10) + P(11) + P(12))$

$= 1 - ({}^{12}C_{10} \times 0.9^{10} \times 0.1^{2} + {}^{12}C_{11} \times 0.9^{11} \times 0.1 + {}^{12}C_{12} \times 0.9^{12} \times 0.1^{0})$

$= \underline{0.111}$

A random sample of 80 callers is chosen.

(b) Use an approximation to find the probability that more than 69 of these callers are connected immediately. [5]

$C \sim B(80, 0.9)$

$\mu = 80 \times 0.9 \qquad \sigma^2 = 72(0.1)$

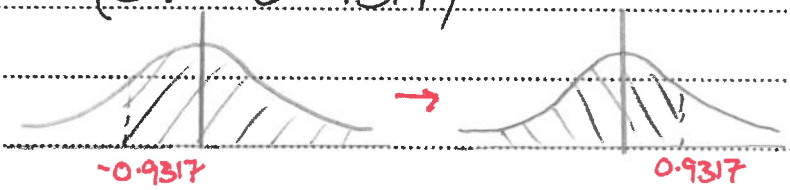
$= 72 \qquad\qquad = 7.2$

$C \sim N(72, 7.2)$

$P(C > 69) \rightarrow P(C > 69.5)$ (continuity correction)

$= P\left(Z > \frac{69.5 - 72}{\sqrt{7.2}}\right)$

$= P(Z > -0.9317)$



$= \Phi(0.932)$

$= 0.8243$

(c) Justify the use of your approximation in part **(b)**. [1]

$np = 80 \times 0.9 \qquad nq = 80 \times 0.1$

$= 72 \qquad\qquad = 8$

Both are greater than 5 so we can use the Normal approximation

7 (a) Find the number of different arrangements of the 9 letters in the word ALLIGATOR in which the two As are together and the two Ls are together. [2]

AALLIGTOR

AA LL IGTOR

one object   one object

7 objects in 7 spaces : 7! = 5040

(b) The 9 letters in the word ALLIGATOR are arranged in a random order.

Find the probability that the two Ls are together and there are exactly 6 letters between the two As. [5]

LL AAIGTOR

one object

LL can't go here as that only leaves 5 letters to go between A and A.

① A _ _ _ _ _ A _

② _ A _ _ _ _ _ A

A _ _ _ _ _ A _

pick one of IGTOR for this space : $^5C_1$

A ___ ___ ___ ___ ___ A ___

Now permutate the remaining 4 from IGTOR with the LL into the 5 spaces: 5!

$= {}^5C_4 \times 5!$

Same working for ___ A ___ ___ ___ ___ ___ A

$\rightarrow {}^5C_4 \times 5! \times 2 = 1200$

Without restrictions: $\frac{9!}{2! \times 2!} = 90720$

2Ls 2As

$\frac{1200}{90720} = \frac{5}{378}$

(c) Find the number of different selections of 5 letters from the 9 letters in the word ALLIGATOR which contain at least one A and at most one L. [3]

Number of different selections, so As and Ls are indistinguishable. Also not permutating as it says selections.

A ___ ___ ___ ___ ${}^5C_4 = 5$

pick 4 letters from IGTOR

A A ___ ___ ___ ${}^5C_3 = 10$

A L ___ ___ ___ ${}^5C_3 = 10$

A A L ___ ___ ${}^5C_2 = 10$

$10 + 10 + 10 + 5 = 35$